

Causal fermion systems as a dynamical collapse theory

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Spacetime - Matter - Quantum Mechanics

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Brief summary

- **causal fermion systems** can describe *non-smooth spacetime structures*. In particular, spacetimes involving *fluctuations*.
- **causal action principle** describes *nonlinear dynamics*

Main message of this talk

- gives rise to an **effective collapse model**
 - has similarities with CSL model, but also differences
 - In particular, stochastic background leads to **no heating** of the probe
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- joint work with *Johannes Kleiner* and *Claudio Paganini*
 - "Causal fermion systems as an effective collapse theory," arXiv:2405.19254 [math-ph], J. Phys. A: Math. Theor. 57 (2024) 395303
 - "A collapse mechanism without heating," arXiv:2511.06392 [quant-ph]
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What is a causal fermion system?

- **approach to fundamental physics**
- physical equations are formulated in **generalized spacetimes**
- Different **limiting cases**:
 - *Continuum limit*: Quantized fermionic fields interacting via classical bosonic fields
 - Last year: *QFT limit*: fermionic and bosonic quantum fields
 - This year: Geometric derivation of the *Einstein equations*

- For overview, more details (papers, books, videos, online course, physical applications,...)

www.causal-fermion-system.com

A few general words on **causal fermion systems**

- The approach is **fully quantum**
 - basic object is a family of wave functions (*physical wave functions*, complex-valued or spinorial)
 - all other spacetime structures are encoded in this family of wave functions
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How to get into the setting of causal fermion systems?

- Let $\mathcal{M}$ be four-dimensional Minkowski space.
Spacetime points $x, y \in \mathcal{M}$
- Let $\psi_1(x), \dots, \psi_f(x)$ be spinorial wave functions in spacetime (four component complex, can but need not satisfy the Dirac equation)
- Assume scalar product $\langle \cdot | \cdot \rangle_{\mathcal{H}}$. Orthonormalize: $\langle \psi_a | \psi_b \rangle_{\mathcal{H}} = \delta_{ab}$
- Introduce the **local correlation operator**: Given $x \in \mathcal{M}$,

$$F(x)_b^a := \overline{\psi_a(x)} \psi_b(x)$$

(where $\overline{\psi} := \psi^\dagger \gamma^0$ is the adjoint spinor)

- diagonal entries: density of a^{th} wave function
 - off-diagonal entries: correlation of ψ_a and ψ_b at spacetime point x .
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How to get into the setting of causal fermion systems?

- Local correlation operator can also be defined abstractly:

$$\langle \psi | F(x) \phi \rangle_{\mathcal{H}} := \prec \psi(x) | \phi(x) \succ \quad \forall \psi, \phi \in \mathcal{H}.$$

(with $\prec \psi(x) | \phi(x) \succ := \overline{\psi(x)} \phi(x)$)

- $F(x) : \mathcal{H} \rightarrow \mathcal{H}$ linear, has *finite rank*, has *at most two positive* and *at most two negative eigenvalues*.
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How to get into the setting of causal fermion systems?

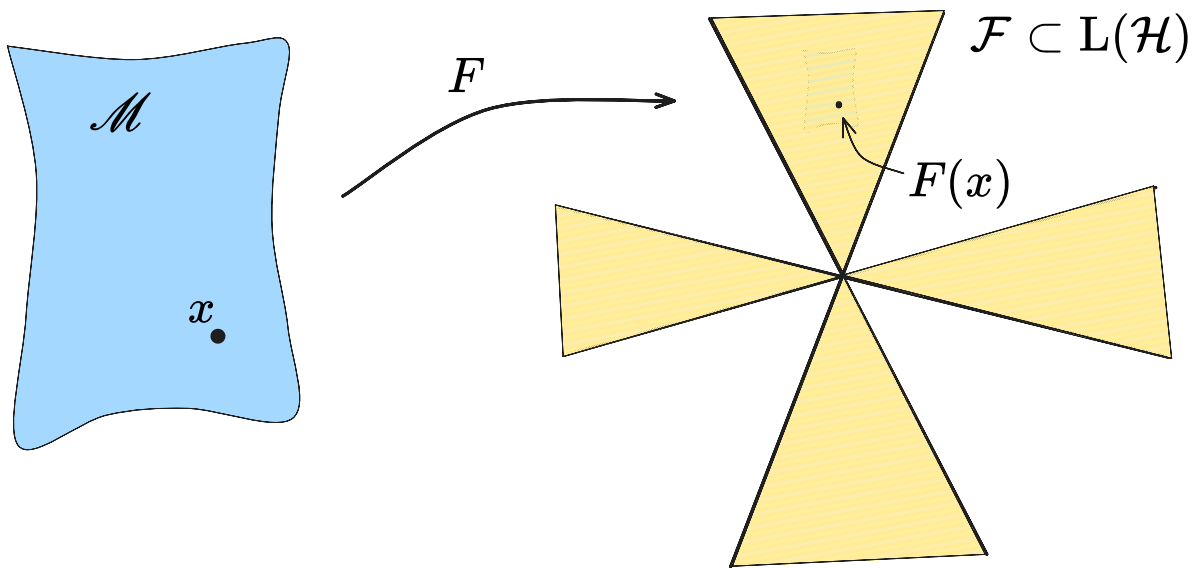
- Varying the base point x , we obtain the **local correlation map**

$$F : \mathcal{M} \rightarrow \mathcal{F} \subset \mathbf{L}(\mathcal{H}) .$$

where $\mathcal{F}$ are all the symmetric operators of rank at most four which have at most two positive and at most two negative eigenvalues.

- Assume a volume measure $\mu_{\mathcal{M}}$ on $\mathcal{M}$. Introduce the push-forward measure,

$$\rho := F_*(\mu_{\mathcal{M}}) \quad \text{i.e. } \rho(\Omega) := \mu_{\mathcal{M}}(F^{-1}(\Omega))$$



- general idea: work exclusively with objects on the right

The basic definition

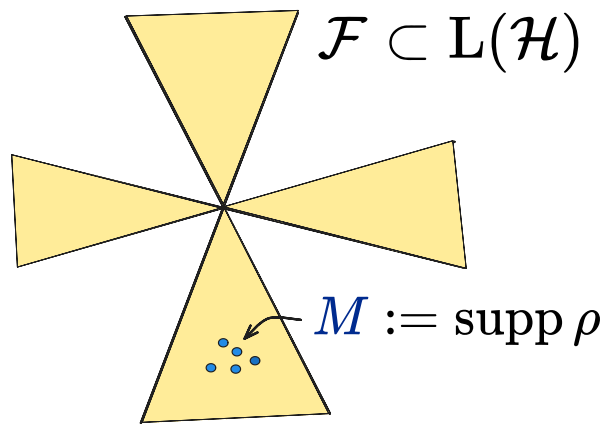
Definition (causal fermion system)

- Hilbert space $(\mathcal{H}, \langle \cdot | \cdot \rangle_{\mathcal{H}})$
 - $n \in \mathbb{N}$ (**spin dimension**)
 - $\mathcal{F} \subset \mathbf{L}(\mathcal{H})$ symmetric linear operators of finite rank, with at most n positive and at most n negative eigenvalues
 - ρ a measure on $\mathcal{F}$
- Then $(\mathcal{H}, \mathcal{F}, \rho)$ is a **causal fermion system**

- $M := \text{supp } \rho$ is **spacetime**.

- Is much more general than spacetime manifolds.

In particular, spacetime could be discrete:



The causal action principle

 **Definition** (causal Lagrangian and causal action)

$$\mathcal{L} : \mathcal{F} \times \mathcal{F} \rightarrow \mathbb{R}_0^+$$

$$\mathcal{S}(\rho) := \int_{\mathcal{F}} d\rho(x) \int_{\mathcal{F}} d\rho(y) \mathcal{L}(x, y)$$

- $\mathcal{L}(x, y)$ *formed of eigenvalues* of operator product xy .
- Is continuous and symmetric: $\mathcal{L}(x, y) = \mathcal{L}(y, x)$

 **Definition** (causal action principle)

Minimize $\mathcal{S}$ under variations of ρ under constraints:

- *volume constraint*: $\rho(\mathcal{F}) = \text{const}$
- *trace constraint* and *boundedness constraint*: here built into the Lagrangian using Lagrange multipliers (reduced causal action principle).

The causal structure

- $M := \text{supp } \rho$ is *spacetime*.

Definition

$x, y \in M$ are **timelike separated** if $\mathcal{L}(x, y) > 0$.

Effective dynamics in Minkowski space

- Return to our example of spinorial wave functions in Minkowski space.
- Basic question: How does the dynamics as described by the **causal action principle** look like?
 - The **linearized** interaction can be described effectively by a nonlocal Dirac equation
 - "Solving the linearized field equations of the causal action principle in Minkowski space," arXiv:2304.00965 [math-ph], ATMP 27 (2003) 2087-2217
 - There are **nonlinear** corrections.

Linearized fields in Minkowski space

- Begin in **one-particle description** (Fock spaces later).
- Describe the dynamics of the causal action principle in terms of a **nonlocal Dirac equation**

$$\begin{aligned}(i\partial + \mathcal{B} - m)\psi &= 0 \\ (\mathcal{B}\psi)(x) &= \int_M \mathcal{B}(x, y) \psi(y) d^4y \\ \mathcal{B}(x, y) &= \sum_{a=1}^N \gamma_j A_a^j \left(\frac{x+y}{2} \right) L_a(y-x)\end{aligned}$$

- Not just one potential, but a multitude of them!
Will later be described **stochastically**.
- All potentials satisfy the homogeneous wave equation

$$\square A_a^j = 0$$

Linearized fields in Minkowski space

$$\mathcal{B}(x, y) = \sum_{a=1}^N \gamma_j A_a^j \left(\frac{x+y}{2} \right) L_a(y-x)$$

- The kernels $L_a(y - x)$ are nonlocal on the scale $\ell_{\min}$ with

$$\ell_{\text{Planck}} \ll \ell_{\min} \ll \ell_{\text{macro}}$$

(and ℓ_{Planck} denotes the Planck scale)

$$L_a(\xi) = 0 \quad \text{if } |\xi^0| + |\vec{\xi}| \gtrsim \ell_{\min}$$

- The number N of fields scales like

$$N \simeq \frac{\ell_{\min}}{\varepsilon} \quad \text{with} \quad \varepsilon \lesssim \ell_{\text{Planck}}$$

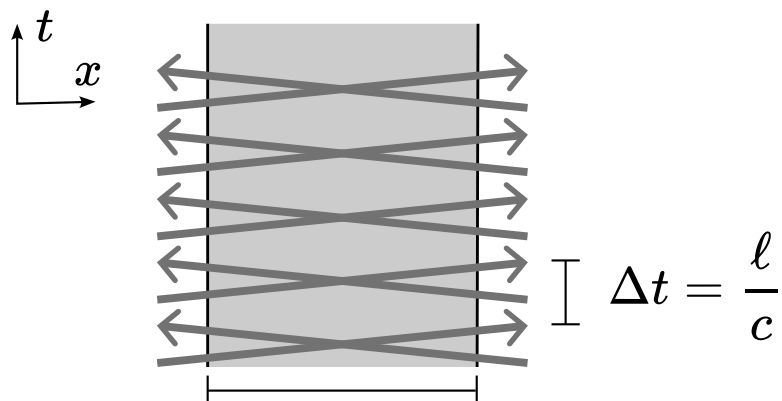
Conserved Scalar Product

- Noether-like theorem*: conservation law for scalar product

$$\begin{aligned} \langle \psi | \phi \rangle_t &:= \int \prec \psi | \gamma^0 \phi \succ_{(t, \vec{x})} d^3 x \\ &\quad - i \int_{x^0 < t} d^4 x \int_{y^0 > t} d^4 y \prec \psi(x) | \mathcal{B}(x, y) \phi(y) \succ_x \\ &\quad + i \int_{x^0 > t} d^4 x \int_{y^0 < t} d^4 y \prec \psi(x) | \mathcal{B}(x, y) \phi(y) \succ_x \end{aligned}$$

- Has structure of **surface layer integral**
- Generalizes probability integral, gives probabilistic interpretation.
- Note: *Scalar product depends on stochastic potentials!*

The non-relativistic limit

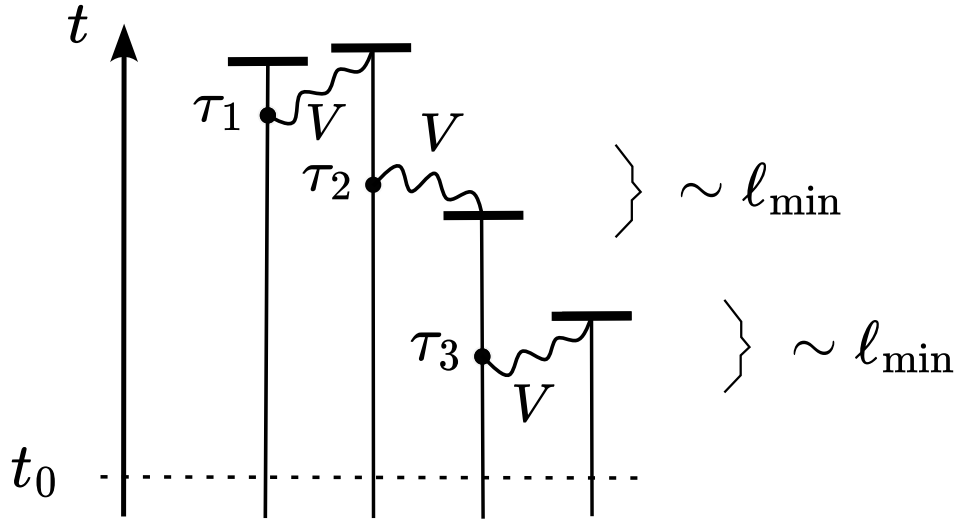


Assume potentials are *Gaussian* and *Markovian*:

$$\begin{aligned}\langle\langle A_a^j(x) \rangle\rangle &= 0 \\ \langle\langle A_a^j(x) A_b^k(x) \rangle\rangle &= \delta(x^0 - y^0) \delta_{ab} C^{jk}(\vec{y} - \vec{x})\end{aligned}$$

The non-relativistic limit

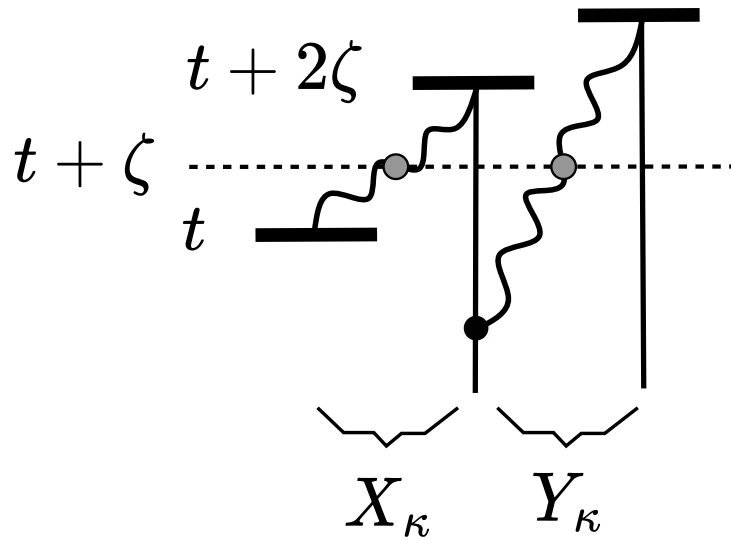
- However, the *nonlocality* of the potential must be taken into account. (Otherwise, no collapse occurs.)
- Thus the usual **Dyson series** looks as follows:



Gaussian pairings

$$\begin{aligned}\mathcal{B}(x, y) &= \sum_{a=1}^N \gamma_j A_a^j \left(\frac{x+y}{2} \right) L_a(y-x) \\ \langle\langle A_a^j(x) A_b^k(x) \rangle\rangle &= \delta(x^0 - y^0) \delta_{ab} C^{jk}(\vec{y} - \vec{x})\end{aligned}$$

- Nonlocality in time described by ζ . Can be treated as an additional parameter of the effective collapse model.



- One also gets pairings between bra and ket in statistical operator.

Main results of analysis

- *Statistical operator* σ_t has time evolution of *Lindblad form*

$$\frac{d\sigma_t}{dt} = -i[H, \sigma_t] - \frac{1}{2} \sum_{\kappa} [K_{\kappa}, [K_{\kappa}, \sigma_t]] \left(1 + \mathcal{O}(\ell_{\min} \|V\|)\right)$$

- There is *dynamical state reduction*, in agreement with *Born's rule*.
- Similar to CSL model, but not the same, due to nonlocality in time.
- The *energy is conserved* in the statistical mean

$$E = \langle\langle \psi | H_0 \psi \rangle\rangle$$

$$\frac{d}{dt} E(t) = \mathcal{O}(\ell_{\min}^2 \|\mathcal{B}\|^3) E(t)$$

Thus our model predicts *no spontaneous radiation* due to a heating of the probe!

The nature of the collapse

- It is *not* the *gravitational field* which triggers the collapse (in contrast to Diosi-Penrose)
- Instead, it is a *multitude of bosonic fields*, specific to the causal action principle.
- Remark:
 - This multitude of fields can be described effectively by a second-quantized electromagnetic field
 - Therefore: collapse is closely related to the electromagnetic interaction in QFT
- But: *length scale of nonlocality* comes into play. Is in turn related to Planck scale. Also gives connection to strength of gravitational field.

Effective description in Fock spaces

- System can be described at any time t by a

$$\text{Quantum state} \quad \omega^t : \mathcal{A} \rightarrow \mathbb{C},$$

where $\mathcal{A}$ is the algebra of observables.

- Can be represented on Fock space $\mathcal{F}$ (fermionic and bosonic)

$$\omega^t(A) = \text{Tr}_{\mathcal{F}}(\sigma^t A) \stackrel{\text{if pure state}}{=} \langle \Psi | A | \Psi \rangle$$

(joint work with *Claudio Dappiaggi*, *Niky Kamran* and *Moritz Reintjes*)

Effective description in Fock spaces

- For our purpose, it suffices to consider Hartree-Fock state

$$\Psi = \psi_1 \wedge \cdots \wedge \psi_q$$

- Dynamics again described by the nonlocal Dirac equation
 - Collapse happens as soon as one one-particle wave function collapses.
 - Thus collapse phenomena are predominant for mesoscopic and macroscopic systems
-

Outlook: The Relativistic Model

- ongoing work
 - Dirac equation is already relativistic.
 - Stochastic background fields break Lorentz invariance
 - Concept: Stochastic background fields originate from the early universe and/or are generated by the matter on earth and of the surrounding stars and galaxies.
 - Replace Markov property by *propagation with speed of light*. Also gives rise to "smearing in time."
-

Summary

- Consider *causal fermion systems in Minkowski space*
- Described by *family of fermionic wave functions*, encoded in wave evaluation operator Ψ
- Causal action principle gives rise to *plethora of fields*

- Coupling of these fields to the Dirac equation is nonlocal on a scale $\ell_{\min} \ll m^{-1}$.
- Similar to CSL model, we obtain a **stochastic** and a **nonlinear term**.
- But: has a different mathematical structure, due to nonlocality in time and an adapted formulation of the conservation laws.
- In particular, our model predicts **no heating** of the probe.
- Further consequences are work in progress ...

Thank you for your attention

www.causal-fermion-system.com

Derivation of Lindblad dynamics

- Note: Standard scalar product

$$(\psi|\phi)_t := \int \prec \psi | \gamma^0 \phi \succ_{(t, \vec{x})} d^3x$$

is **not** conserved in time.

- Only the **modified scalar product is conserved**,

$$\begin{aligned} \langle \psi | \phi \rangle_t &= (\psi | \phi)_t \\ &\quad - i \int_{x^0 < t} d^4x \int_{y^0 > t} d^4y \prec \psi(x) | \mathcal{B}(x, y) \phi(y) \succ_x \\ &\quad + i \int_{x^0 > t} d^4x \int_{y^0 < t} d^4y \prec \psi(x) | \mathcal{B}(x, y) \phi(y) \succ_x \end{aligned}$$

Derivation of Lindblad dynamics

- Transform one into the other:

$$\begin{aligned} \langle \psi | \phi \rangle_{t_0} &= (\psi | (1 + \mathcal{S}_{t_0}) \phi)_{t_0} \quad \text{for all } \psi, \phi \in \mathcal{H}_m \\ \psi &\mapsto \tilde{\psi} := \sqrt{1 + \mathcal{S}_{t_0}} \psi \\ \langle \psi | \phi \rangle_{t_0} &= (\tilde{\psi} | \tilde{\phi})_{t_0} \end{aligned}$$

- Working with $\tilde{\psi}$, one can use the standard scalar product.
- The **statistical operator** is

$$\sigma_t := \ll |\psi\rangle \langle \psi| \gg = \ll |\tilde{\psi}\rangle \langle \tilde{\psi}| \gg$$

- Now compute

$$\frac{d}{dt}\sigma_t = \ll \frac{d}{dt}(|\tilde{\psi})(\tilde{\psi}|) \gg = \dots$$

to leading order in $\ell_{\min} \|\mathcal{B}\|$.

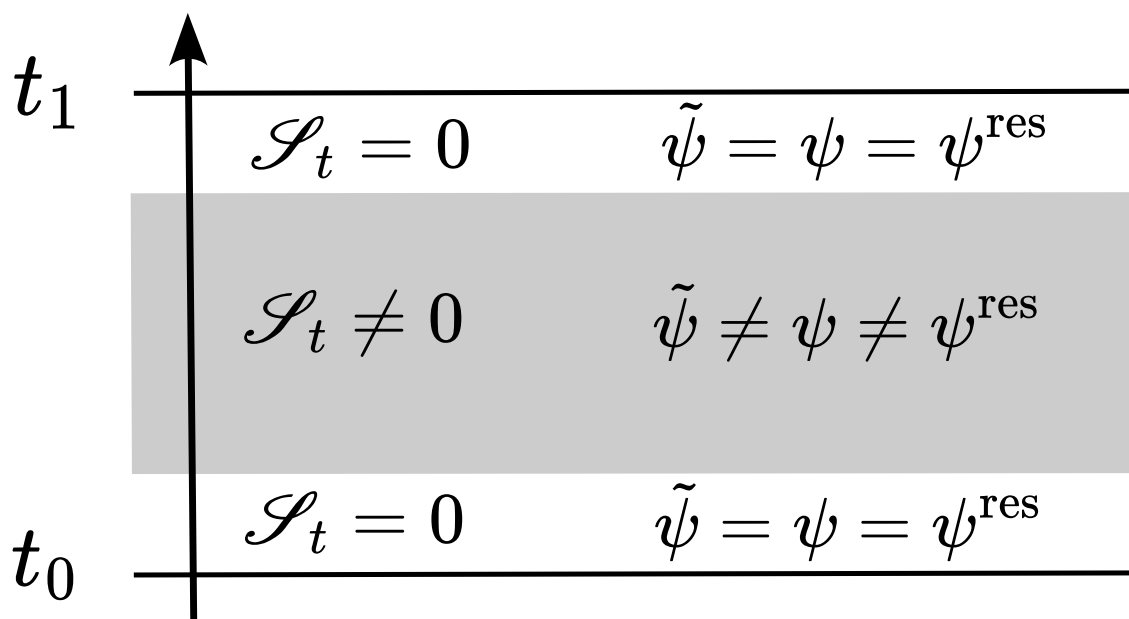
Reduction of the state vector

- Try to use standard assumptions:
 - Observable $\mathcal{O}$ commutes with Hamiltonian
 - Typical example: Position measurement, use locality of time evolution.
 - Problem: Operator $\mathcal{S}_t$ is **nonlocal!**. Therefore:
 - Work with the original (untilded) wave functions.
 - Makes it necessary to also work with the time-dependent scalar product $\langle \cdot | \cdot \rangle_t$.
-

Reduction of the state vector

- Consider situation similar to a *scattering process* and *rescale* the wave functions,

$$\psi^{\text{res}}(t) := c(t) \psi(t) \quad \text{with} \quad c(t) := \frac{1}{\sqrt{\ll (\psi(t)|\psi(t)) \gg}}$$



Reduction of the state vector

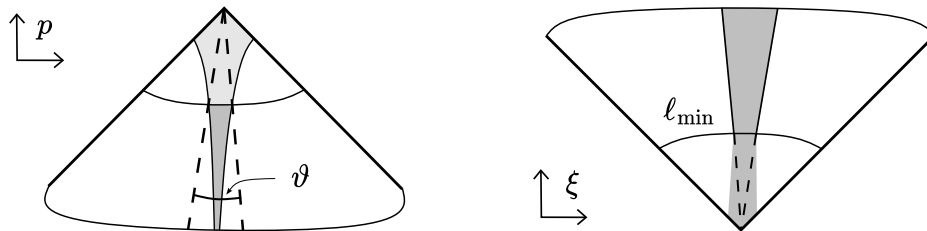
$$\begin{aligned}\frac{d}{dt} \ll (\psi^{\text{res}}(t) | \psi^{\text{res}}(t)) \gg &= 0 \\ \frac{d}{dt} \ll (\psi^{\text{res}} | \mathcal{O} \psi^{\text{res}}) \gg &= 0 \\ \frac{d}{dt} \ll (\psi^{\text{res}} | \mathcal{O}^2 \psi^{\text{res}}) - (\psi^{\text{res}} | \mathcal{O} \psi^{\text{res}})^2 \gg &\leq 0\end{aligned}$$

and strictly negative unless ψ^{res} is an eigenstate.

- Shows **collapse**
- Proves **Born rule**

Alte Folien

Linearized fields in Minkowski space



$$v = \frac{1}{\sqrt{\ell_{\min} \omega_{\min}}} \quad \text{with} \quad v_{\min} = \sqrt{\frac{\ell_{\text{Planck}}}{\ell_{\min}}}.$$

- Different wave functions "feel" different potentials
- The low-energy wave functions (i.e. $|\omega| \lesssim \ell_{\text{Planck}}^{-1}$) "feel all the potentials at the same time".

$$(i\partial + \mathcal{B} - m)\psi = 0$$

- *Hamiltonian formulation:*

$$\begin{aligned}i\partial_t \psi &= (H_0 + V)\psi \\ H_0 &= -i\gamma^0 \vec{\gamma} \vec{\nabla} \\ (V\psi)(t) &= \int_{-\infty}^{\infty} V(t, t') \psi(t') dt' \\ (V(t, t')\psi)(\vec{x}) &= \int_{\mathbb{R}^3} \left(-\gamma^0 \mathcal{B}((t, \vec{x}), (t', \vec{y})) \right) \psi(\vec{y}) d^3 y\end{aligned}$$

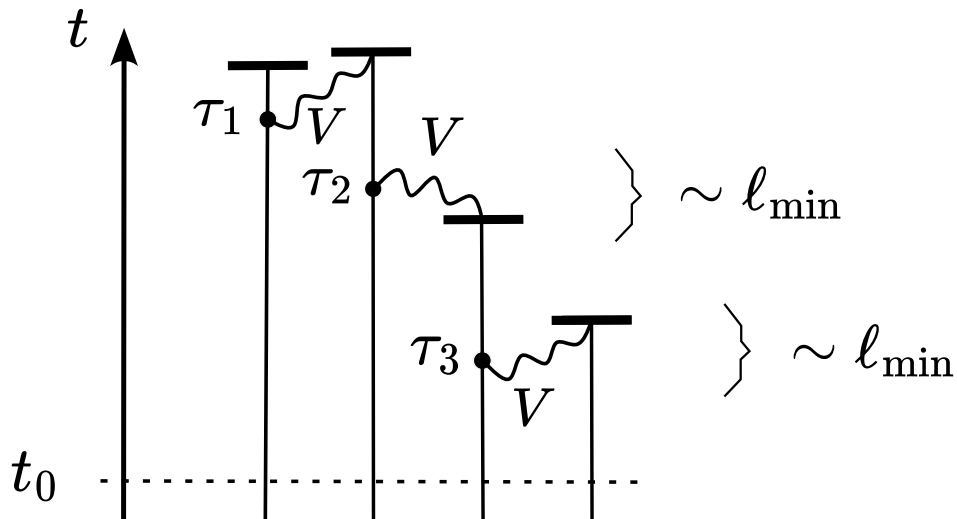
The non-local Dyson series

$$i\partial_t\psi = (H_0 + V)\psi$$

- Can be solved with *nonlocal Dyson series*

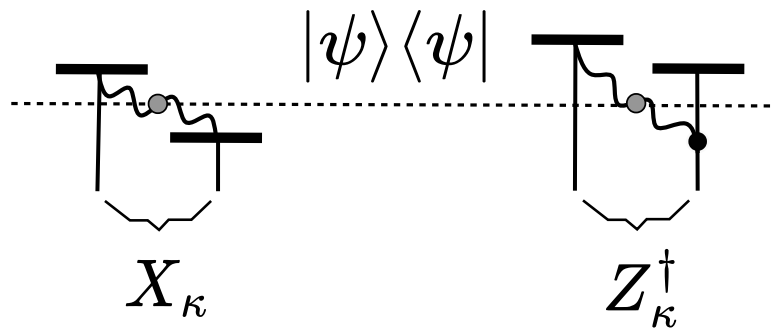
$$\begin{aligned}\psi(t) &= \psi(t_0) + \int_{t_0}^t \dot{\psi}(\tau) d\tau = \psi(t_0) + i \int_{t_0}^t (V\psi)(\tau) d\tau \\ &= \dots = \text{(apply iteratively)} \\ &= \psi(t_0) + \int_{t_0}^t d\tau \int_{-\infty}^{\infty} d\zeta (-iV(\tau, \zeta)) \psi(t_0) \\ &\quad + \int_{t_0}^t d\tau_1 \int_{-\infty}^{\infty} d\zeta_1 (-iV(\tau_1, \zeta_1)) \\ &\quad \times \int_{t_0}^{\zeta_1} d\tau_2 \int_{-\infty}^{\infty} d\zeta_2 (-iV(\tau_2, \zeta_2)) \psi(t_0) \\ &\quad + \dots\end{aligned}$$

The non-local Dyson series

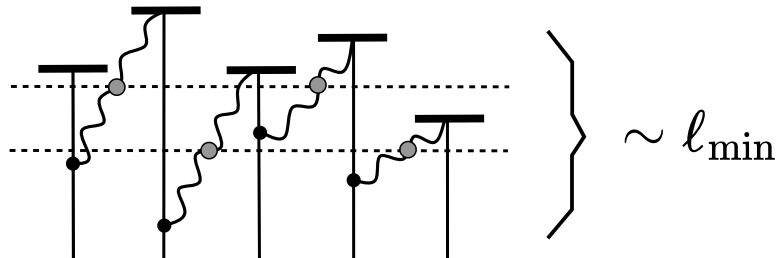


Gaussian pairings

- Take into account \Cg{pairings between bra and ket



- Example in higher order:



Effective description in Fock spaces

- System can be described at any time t by a

Quantum state $\omega^t : \mathcal{A} \rightarrow \mathbb{C},$

where $\mathcal{A}$ is the algebra of observables.

- Can be represented on Fock space $\mathcal{F}$ (fermionic and bosonic)

$$\omega^t(A) = \text{Tr}_{\mathcal{F}}(\sigma^t A) \stackrel{\text{if pure state}}{=} \langle \Psi | A | \Psi \rangle$$

(joint work with *Claudio Dappiaggi*, *Niky Kamran* and *Moritz Reintjes*)

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- For our purpose, it suffices to consider Hartree-Fock state

$$\Psi = \psi_1 \wedge \cdots \wedge \psi_q$$

- Dynamics again described by the nonlocal Dirac equation
- Collapse happens as soon as one one-particle wave function collapses.
- Thus collapse phenomena are predominant for mesoscopic and macroscopic systems

Parameters of the model

- The length scale $\ell_{\min}$. Equivalently, the number N of fields related to $\ell_{\min}$ by

$$N \simeq \frac{\ell_{\min}}{\varepsilon}$$

- The strength of the stochastic field as described by the *covariance*,

$$\ll A_a^j(x) A_b^k(x) \gg = \delta(x^0 - y^0) \delta_{ab} C^{jk}(\vec{y} - \vec{x})$$

and the *nonlocal kernels* $L_a(y - x)$

The nonlinear term

- Interestingly, it *does not need to be specified*. Description on various levels possible:
 - Take into account *nonlinear coupling*,

$$-\square(A_a)^k = e^2 J_a^k$$

$$J_a^k(z) = \int_M \prec \psi(x) | \gamma^k L_a(x, y) \psi(y) \succ \big|_{x=z-\xi/2, y=z+\xi/2} d^4 \xi$$

- Take the *many-particle perspective*:

$$\text{Hartree-Fock state} \quad \Psi = \psi_1 \wedge \cdots \wedge \psi_q$$

In the causal fermion system description, the potential $\mathcal{B}$ is *encoded in* this family of *wave functions*. Therefore, Dirac equation for Ψ is nonlinear.

Abstract of the talk

Causal fermion systems as a dynamical collapse theory

Talk in Castiglioncello, October 2026

The theory of causal fermion systems is an approach to fundamental physics. It gives quantum mechanics, general relativity and quantum field theory as limiting cases and is therefore a candidate for a unified physical theory. The dynamics of causal fermion systems is described by a variational principle called the causal action principle. After a short general introduction, I will explain how, in the non-relativistic limit, the causal action principle gives rise to an effective collapse theory. The effective model has similarities with the continuous spontaneous localization model, but differs from it by a conservation law for the probability integral as well as a non-locality in time on a microscopic scale. Moreover, our collapse model does not give rise to a heating of the probe, in the sense that its energy is conserved in the statistical mean.