Internal quantum reference frames and external frame independence

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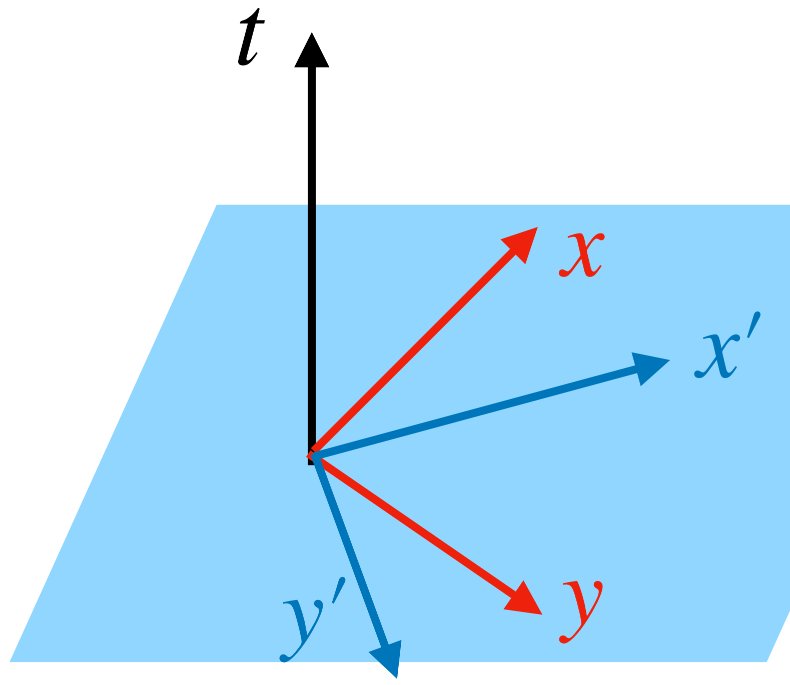
roughly based on:

PH, Mele, Kotecha 2308.09131
de la Hamette, Galley, PH, Loveridge, Müller 2110.13824
Araújo-Regado, PH, Sartini 2506.23459

Relativities

Galilean

"All the laws of mechanics the same in every inertial frame"



frame-dependent decomposition of space

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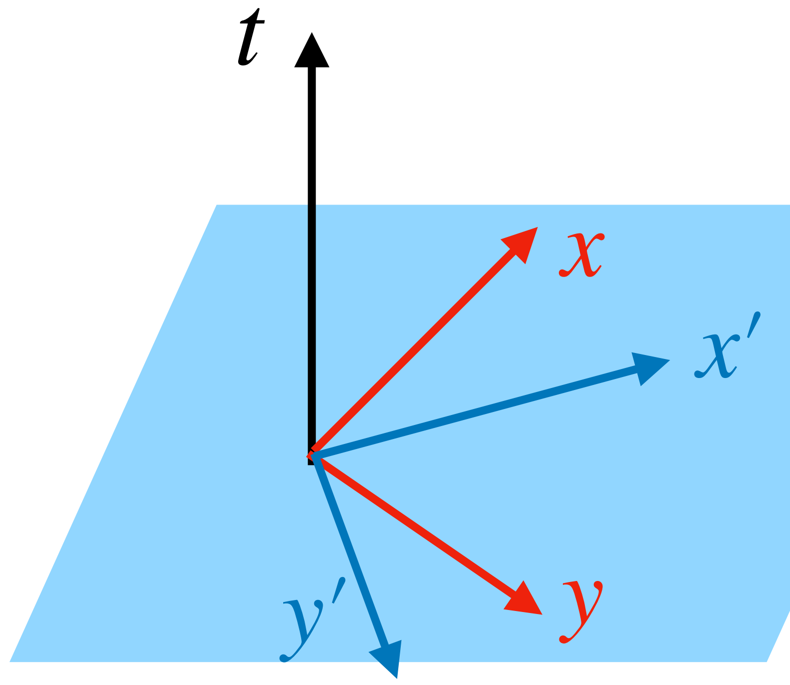
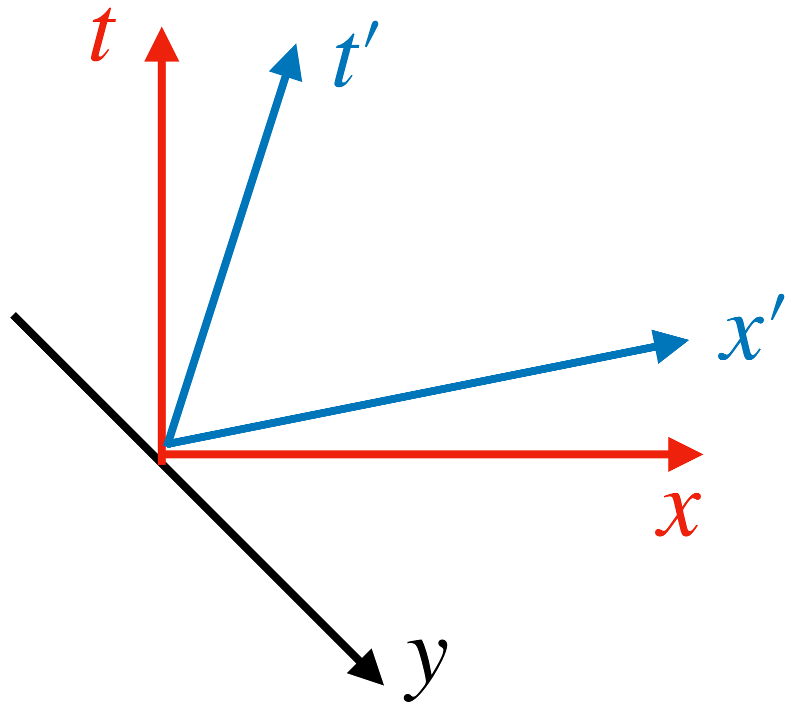


frame-dependent decomposition of space

$$\frac{1}{c} \neq 0$$

Special

“All the laws of physics (exc. Newt. gravity) the same in every inertial frame”



frame-dependent decomposition of spacetime into space and time

Relativities

$$\frac{1}{c} \neq 0 \longrightarrow$$

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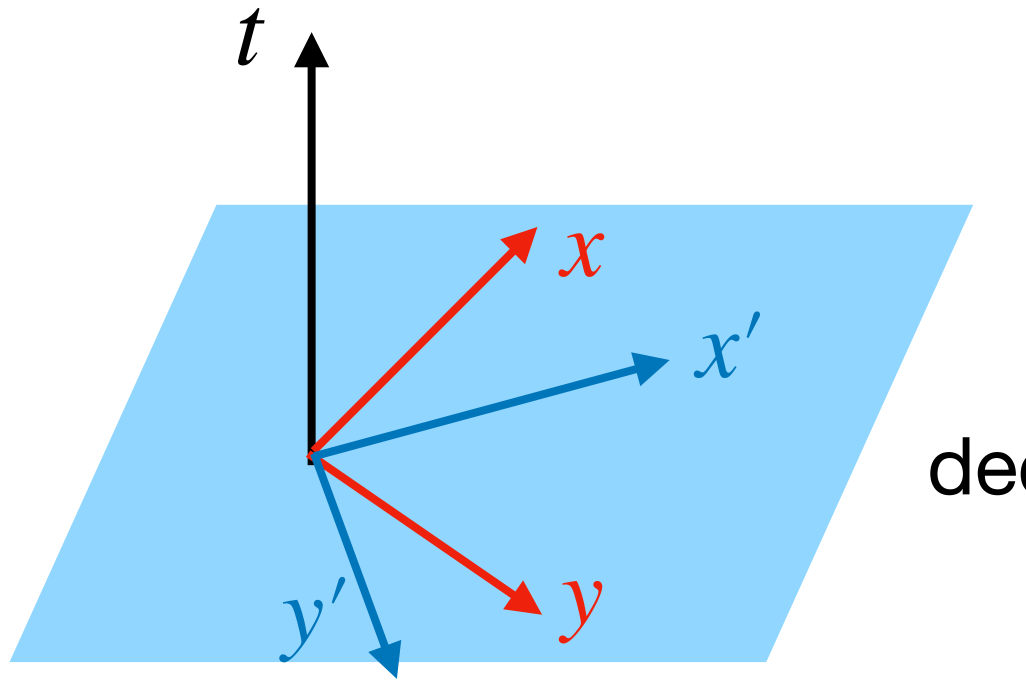

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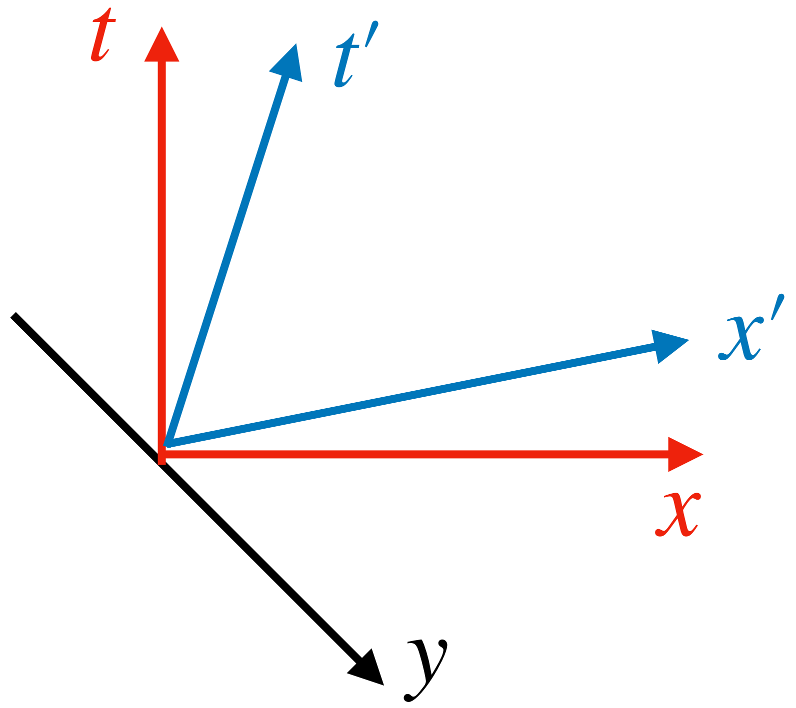
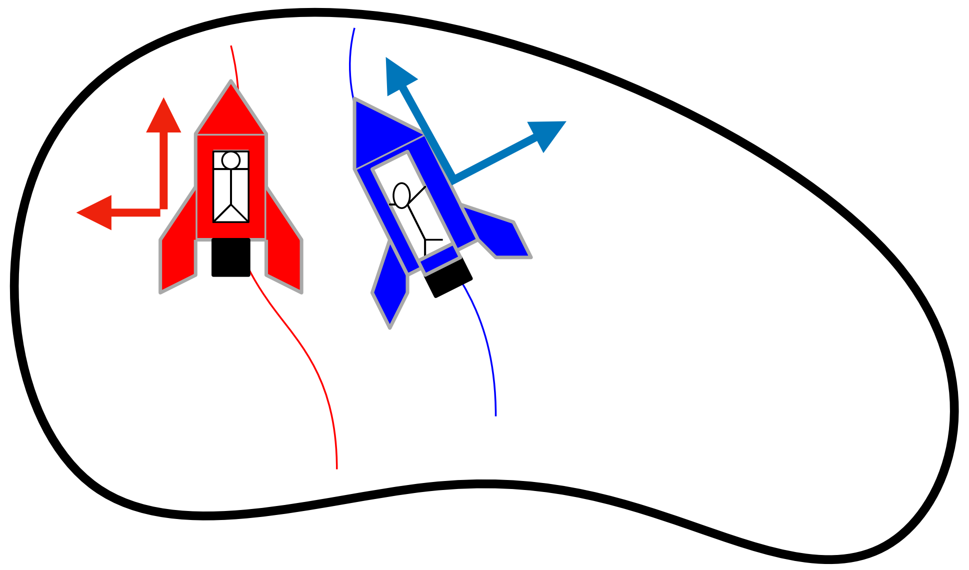


frame-dependent decomposition of spacetime into space and time

$$\Big\downarrow\; G \neq 0$$

General

“All the laws of physics the same in every frame”



frame-dependent local decomposition of spacetime into space and time

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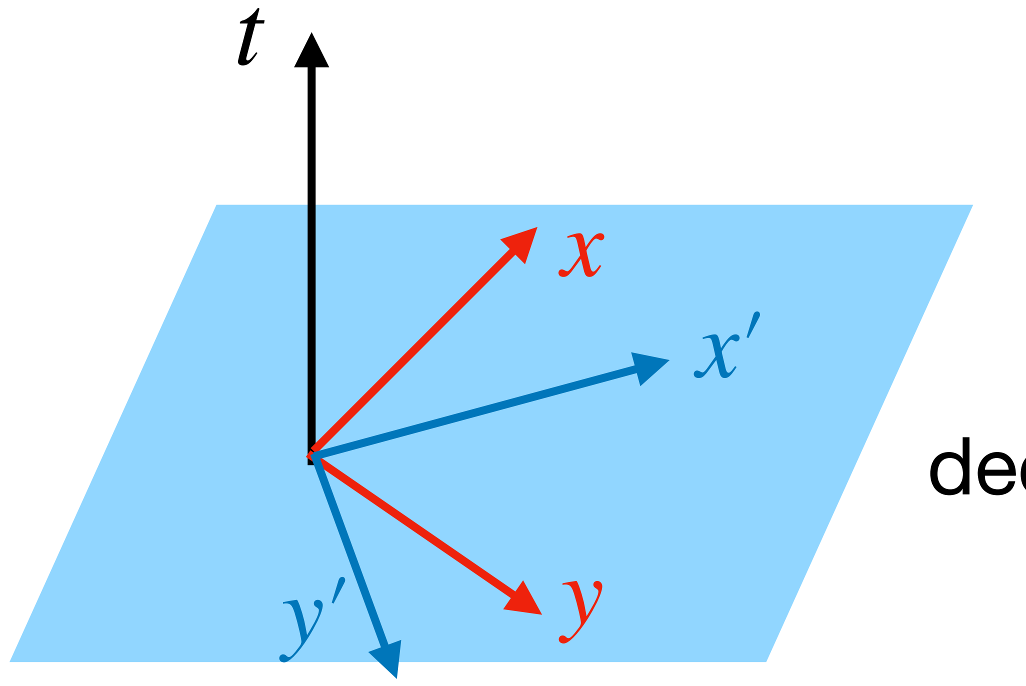

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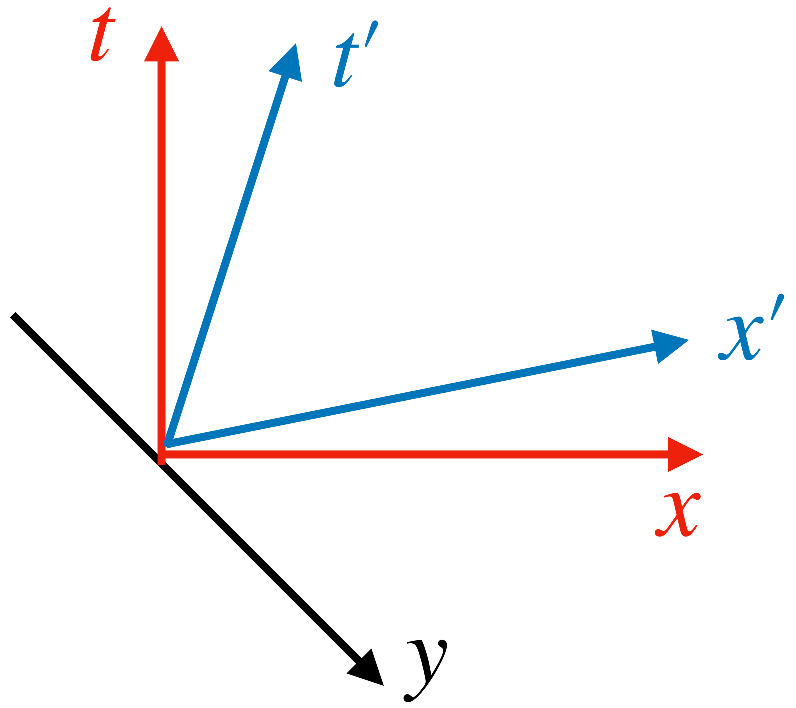


frame-dependent decomposition of spacetime into space and time

$G \neq 0$

$\hbar \neq 0$

Quantum?

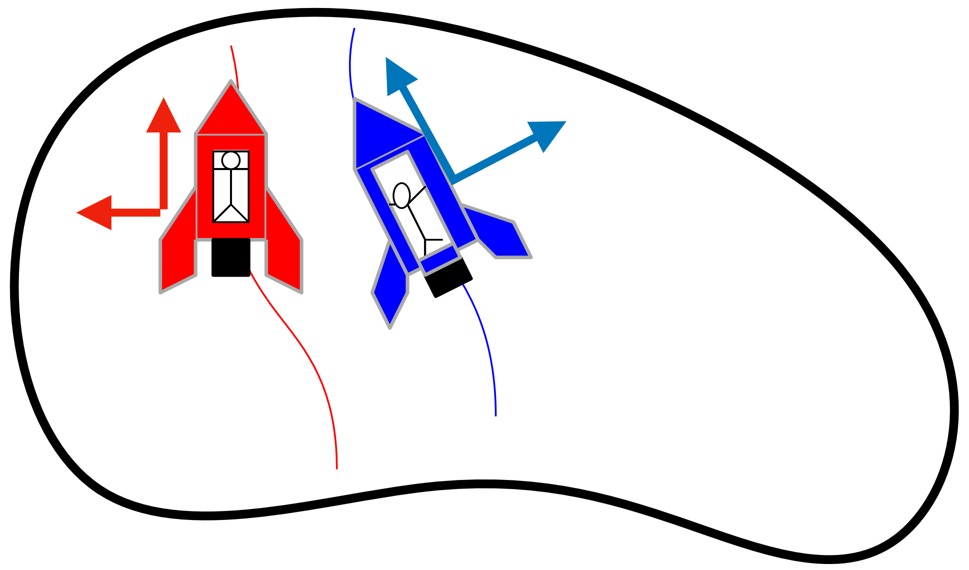
"All the … laws of … the same in every … QRF"

?

QRF-dependent decomposition of ….

General

"All the laws of physics the same in every frame"



frame-dependent local decomposition of spacetime into space and time

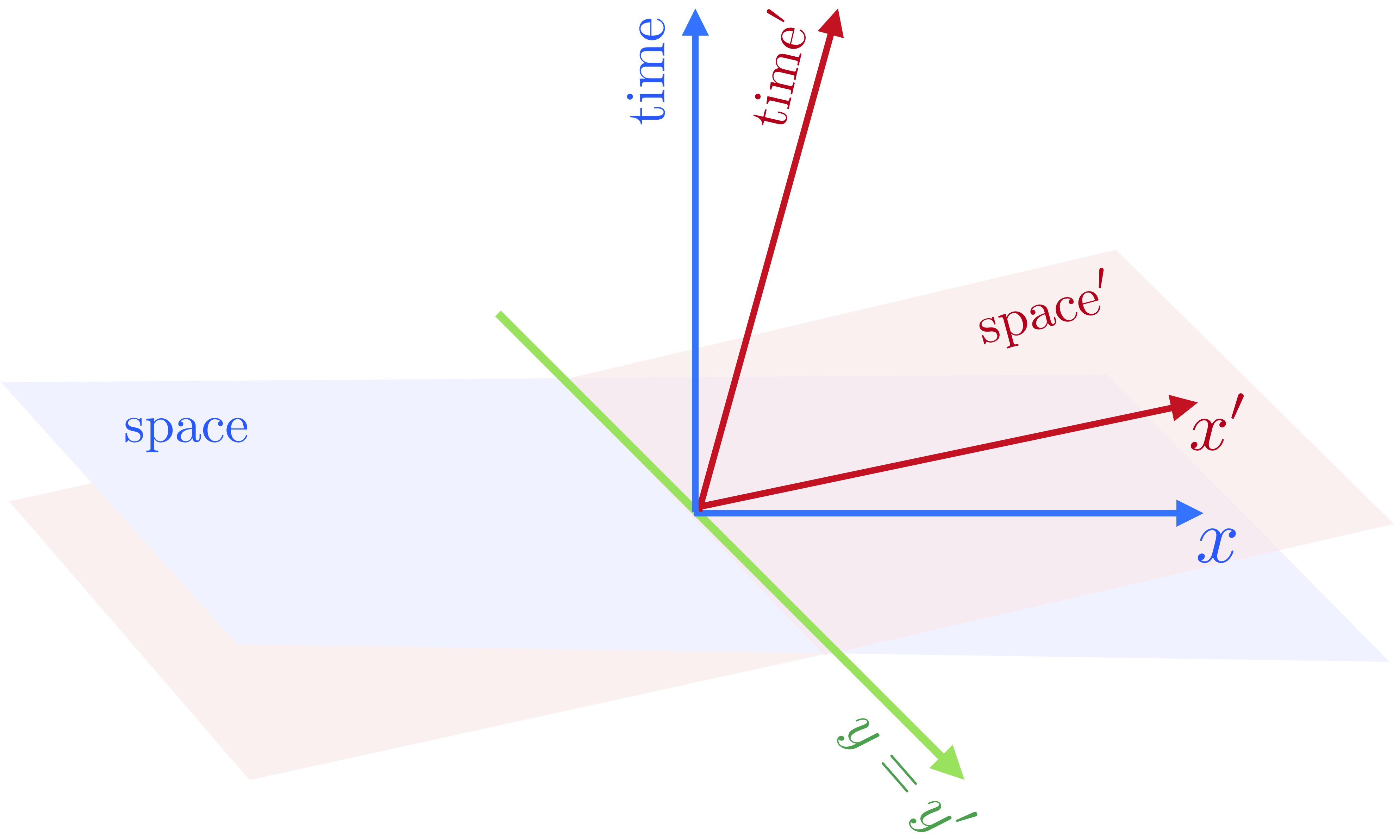
Quantum reference frames are internal subsystems and a universal tool for…

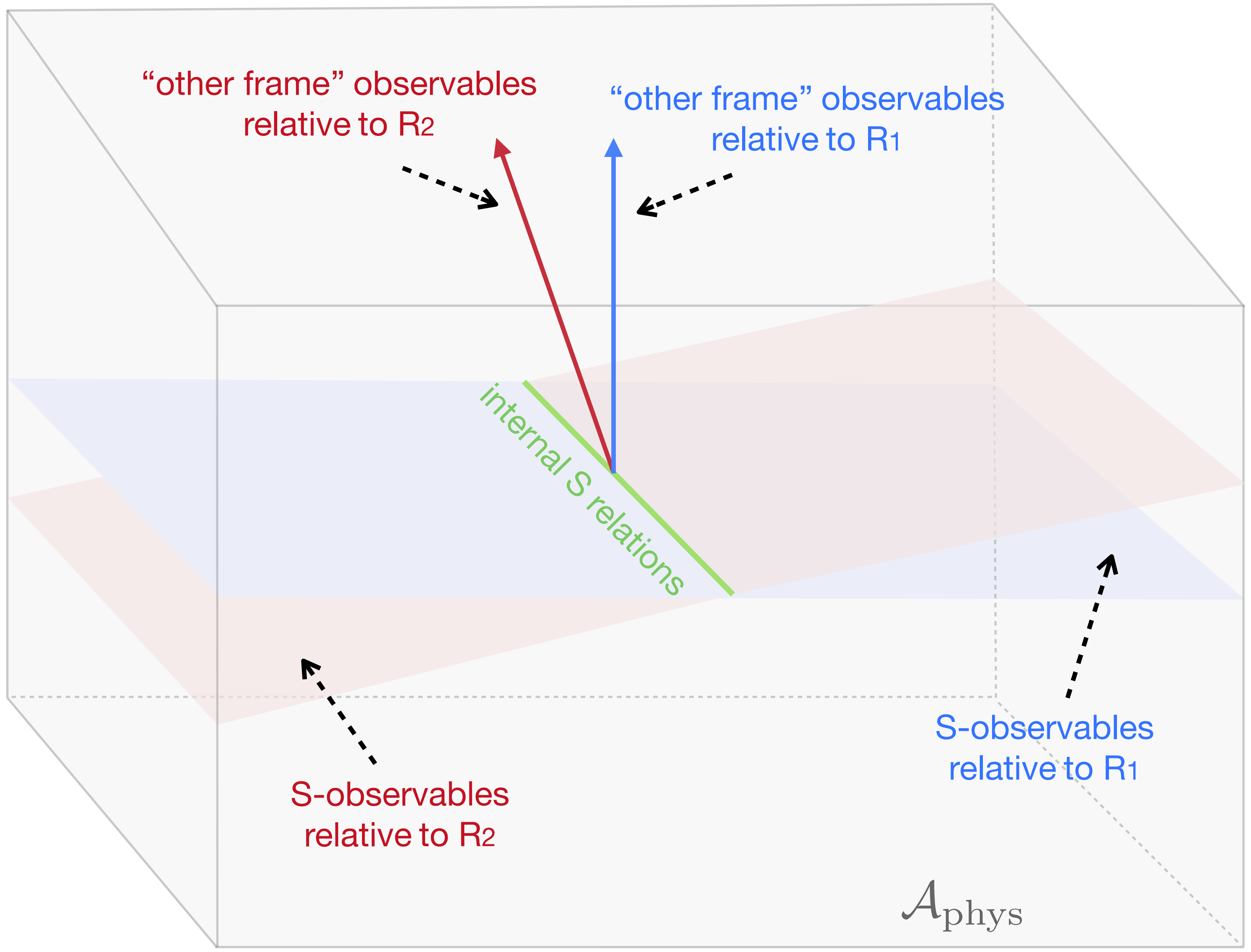
- **dealing with symmetries in quantum systems**
- **describing a quantum system "from the inside"**

  (describing subsystems relative to one another)
- **defining subsystem partitions in gauge systems**
- **completing the relational paradigm**

applicable whenever an external reference isn't available/desired

Relativity of simultaneity vs. subsystem relativity





Why quantum reference frames?

- **<u>foundational interest:</u>**

  — all frames in experiments physical
  — no external frames (background indep.)
  — quantum frame relations ⇔ quantum spacetime structure?

- **<u>practical interest:</u>**

  — needed for gauge-inv. description (or gauge fixing)
  — act as regulator for QFT algebras and entropies
  — definition of locality
  — applications in QI
  — …

Menu

- Introduction to QRFs and notions of external frame independence
- subsystem relativity
- Physical and gauge QRFs in lattice gauge theories
  + entanglement entropies (maybe)

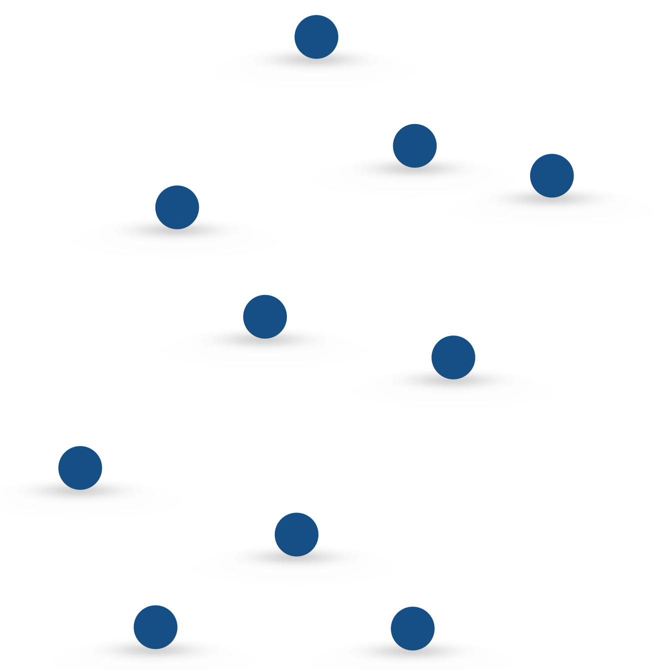
Internal QRFs and external frame independence

[de la Hamette, Galley, PH, Loveridge, Müller 2110.13824; PH, Kotecha, Mele 2308.09131; PH, Smith, Lock 1912.00033; Vanrietvelde, PH, Giacomini, Castro-Ruiz 1809.00556,….]

RFs and symmetries

Premise:

System $\tilde{S} = R_1 R_2 S$ subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when $\tilde{S}$ considered in isolation



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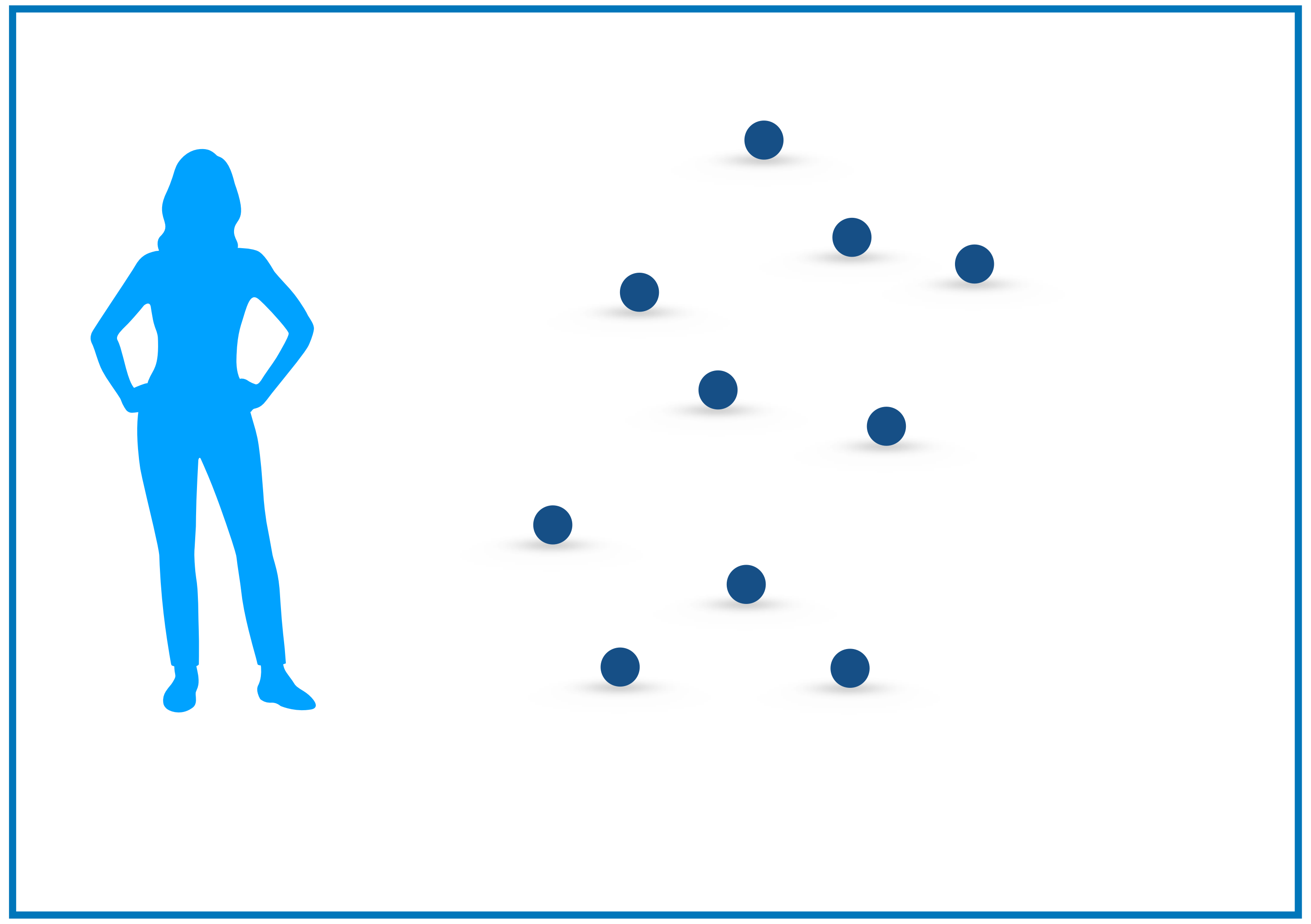

external frame

quantum information/foundations:
lab frame

gauge theory & gravity:
fictitious (or bdry)

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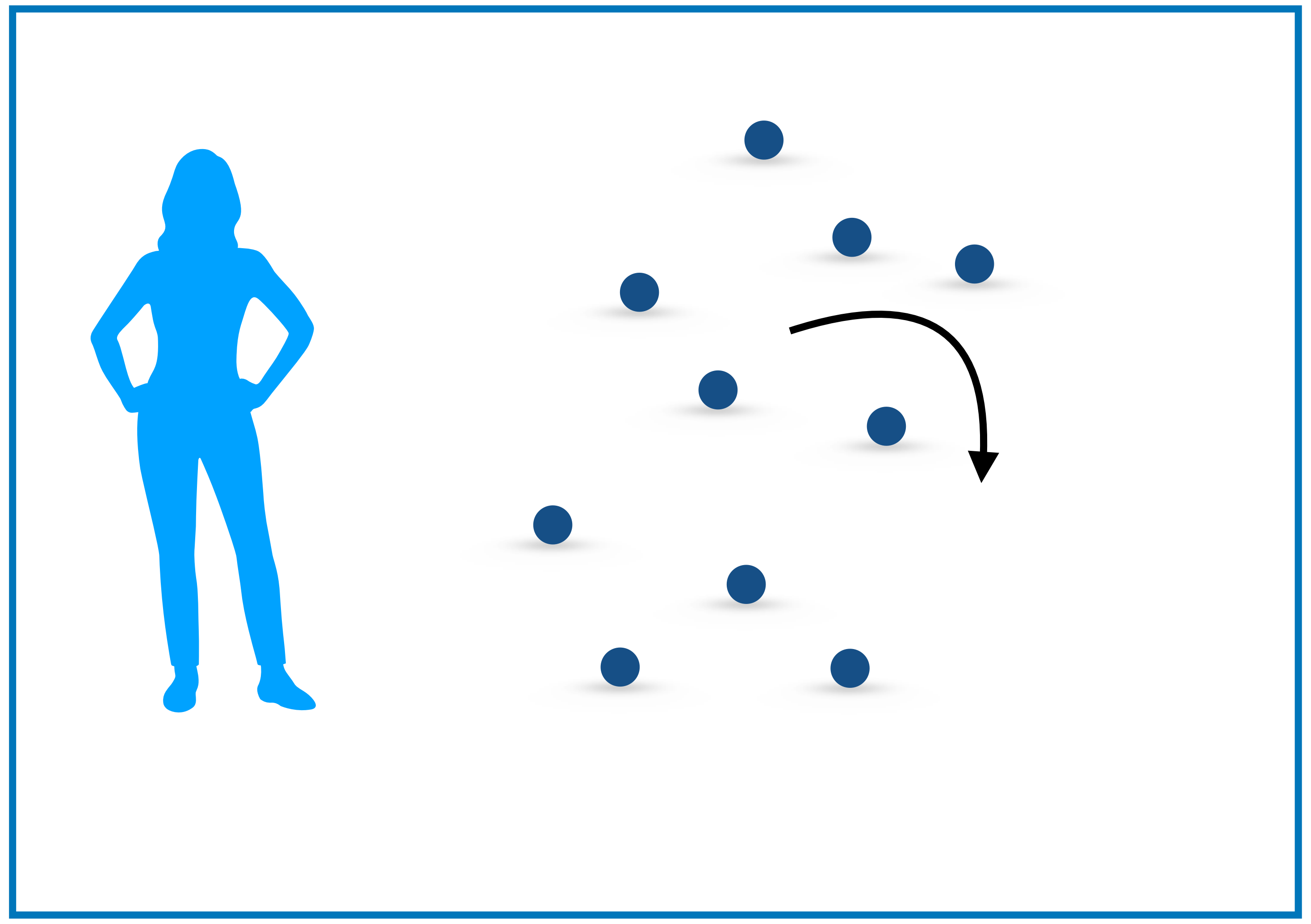

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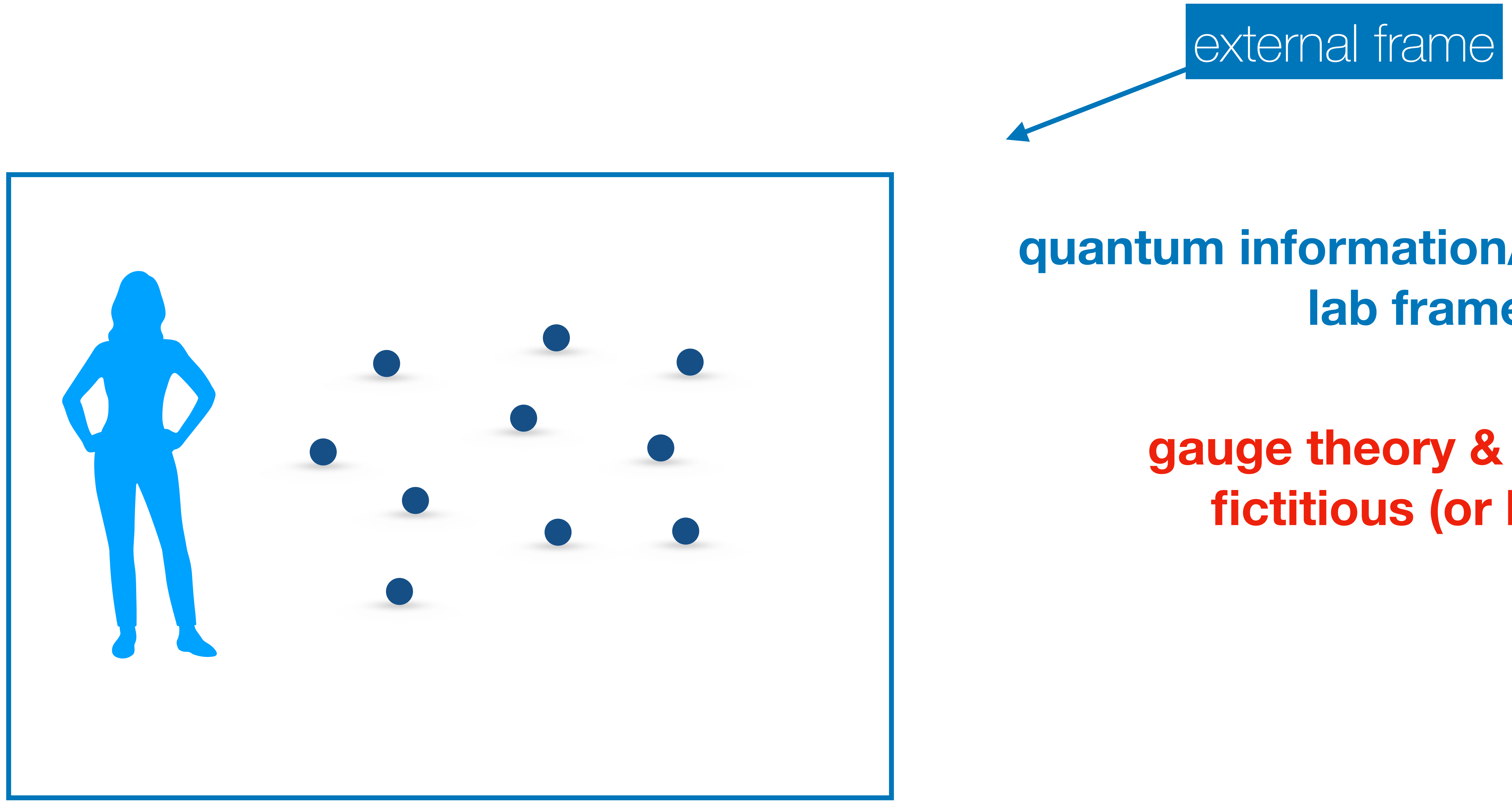

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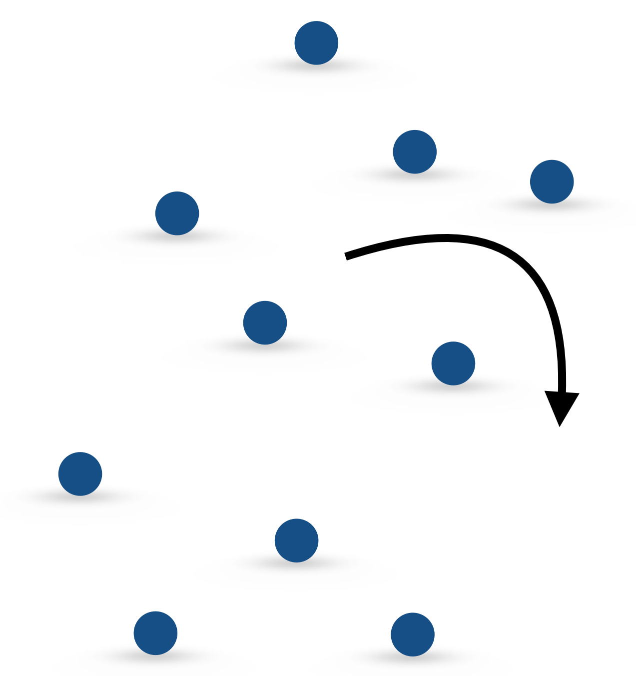

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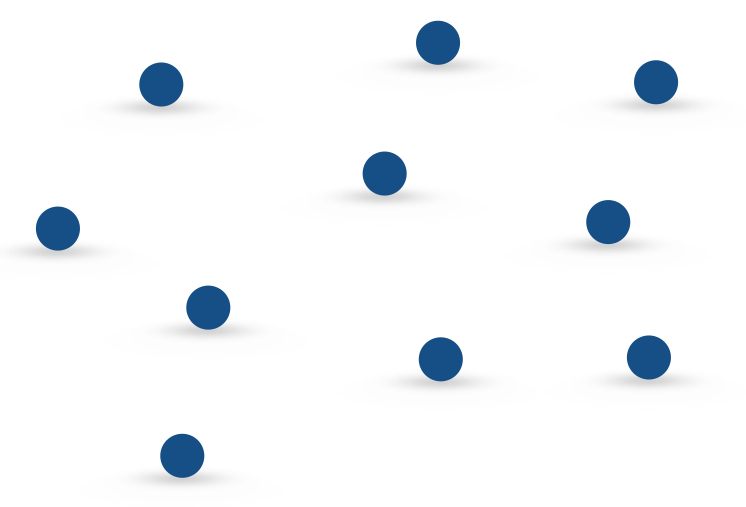

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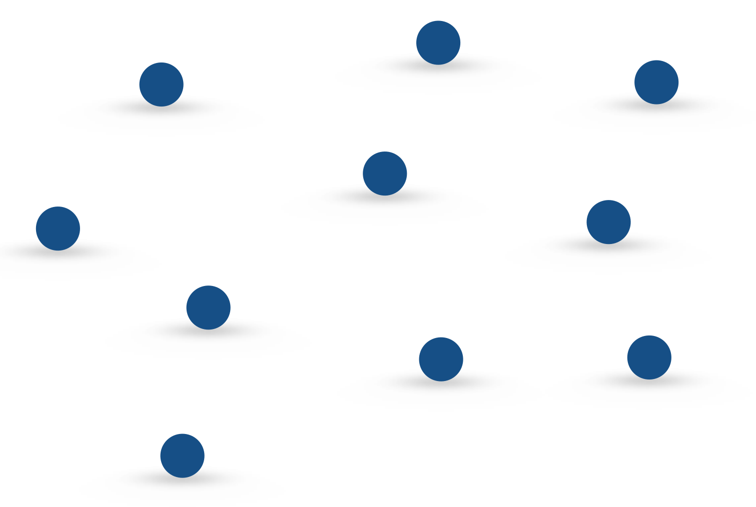

internally indistinguishable

G-transformations = ext. frame transf.

$\Rightarrow$ want external frame indepepence, i.e. G-invariance

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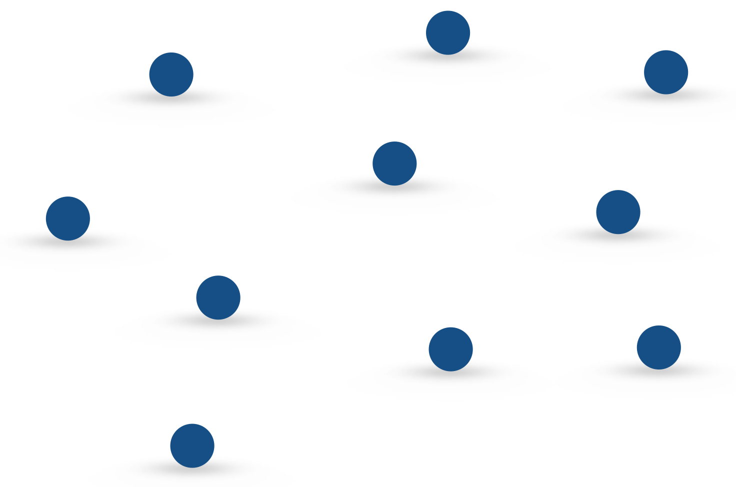

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quantum information/foundations:
change of ext. lab frame

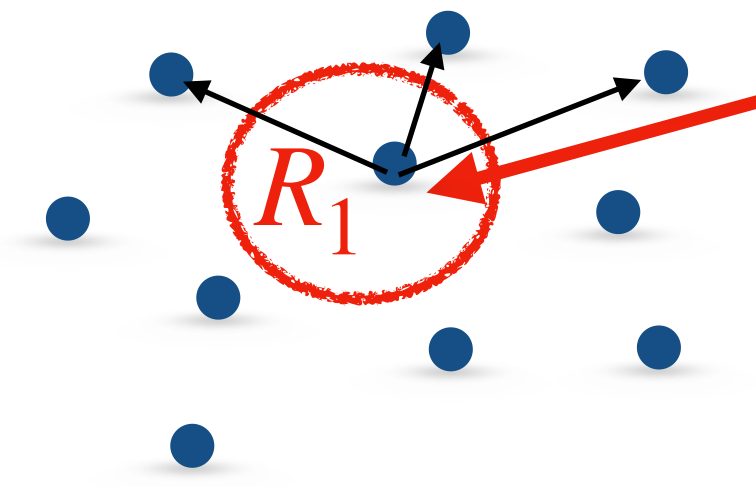
gravity:
e.g. change of background coordinates (diffeo)
$\Rightarrow$ change of fictituous background frame

G-transformations = ext. frame transf.

$\Rightarrow$ want external frame indepepependence, i.e.
G-invariance

RFs and symmetries

Describe $\tilde{S}$ relative to internal reference subsystem R_1



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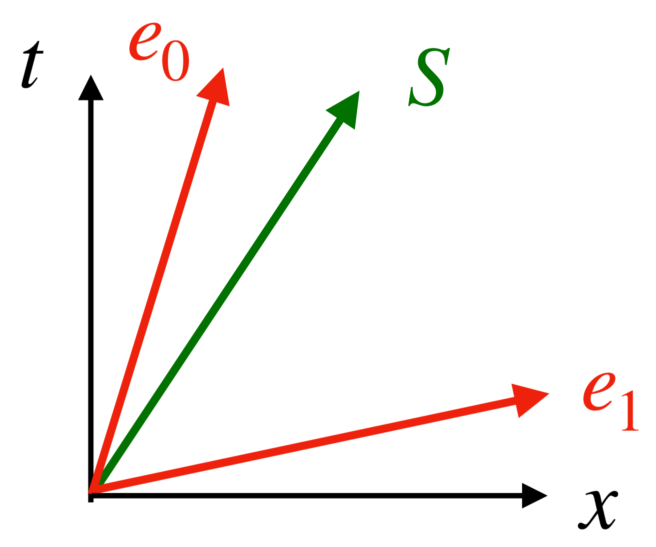
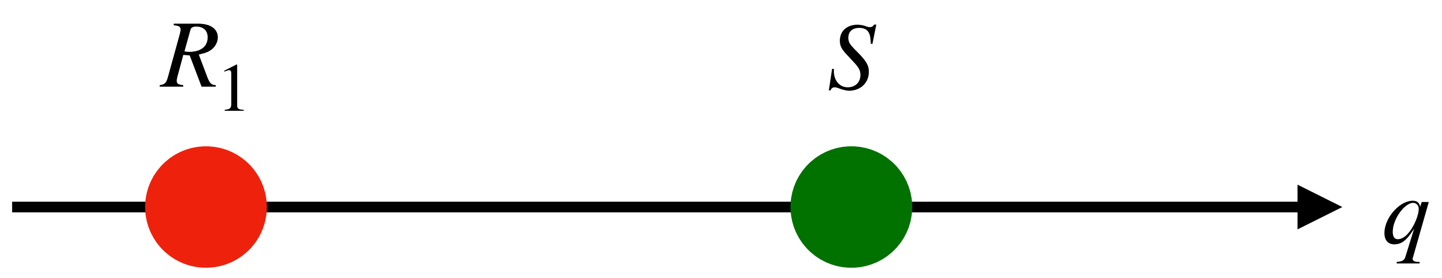
internal frames in SR vs. QT



external Lorentz frame



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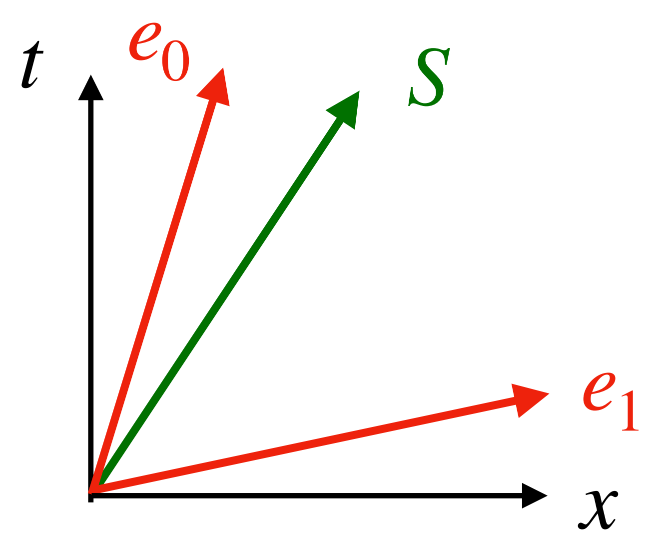

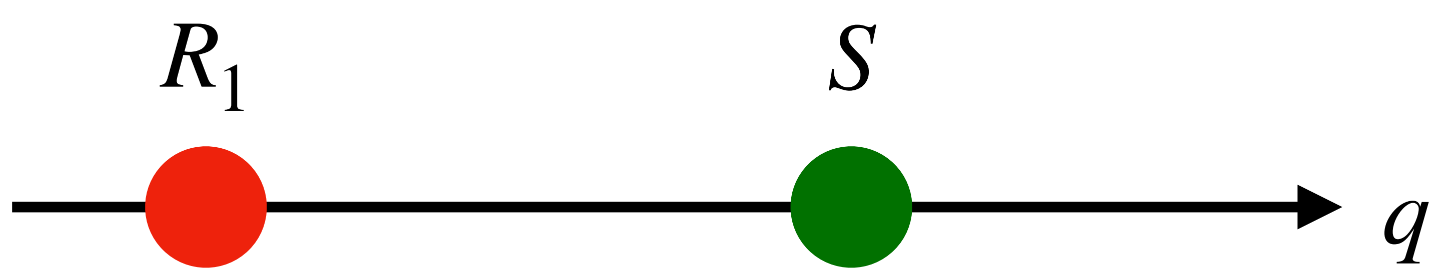

internal frame orientations	**tetrad** $e_a^\mu \in \mathrm{SO}_+(3,1)$ (because $\eta_{\mu\nu}\, e_a^\mu\, e_b^\nu = \eta_{ab}$)

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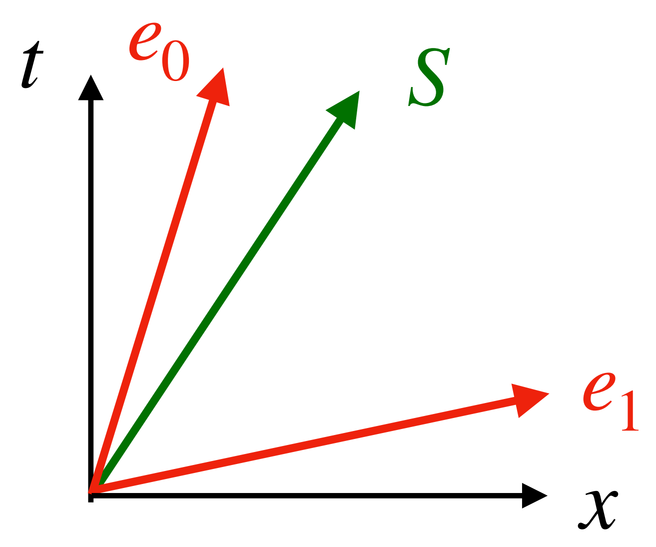
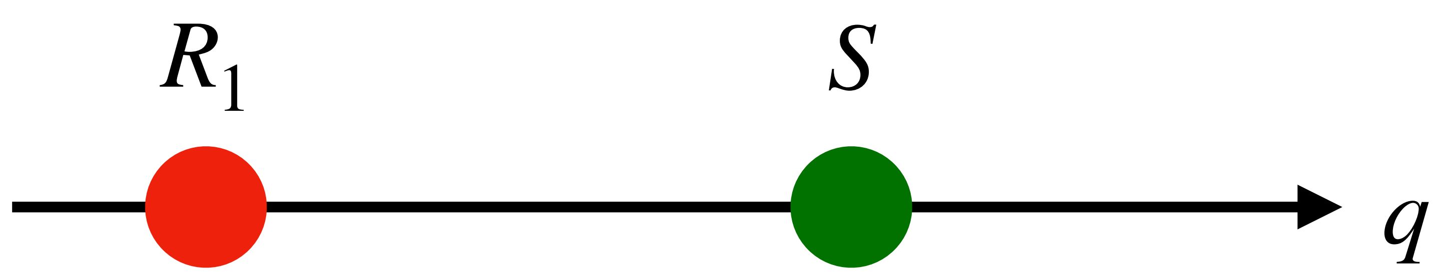




internal frame orientations	**tetrad** $e_a^\mu \in \mathrm{SO}_+(3,1)$ (because $\eta_{\mu\nu}\, e_a^\mu\, e_b^\nu = \eta_{ab}$)	**covariant POVM** $E(g\,.\,X) = U_R^g\, E(X \subset G)\,(U_R^g)^\dagger$ **in practice, coh. states** $\lvert g\rangle$, $g \in G$

Example: quantum clocks

[PH, Smith, Lock '19]

clock states for non-deg. continuous spectrum clock Hamiltonian H_C **(gen. of** $\mathbb{R}$**)**

$$|t\rangle = \frac{1}{\sqrt{2\pi}} \int_{\mathrm{Spec(H_C)}} d\epsilon \, e^{-it\epsilon} |\epsilon\rangle$$

clock orientations $t \in \mathbb{R}$ (group-valued)

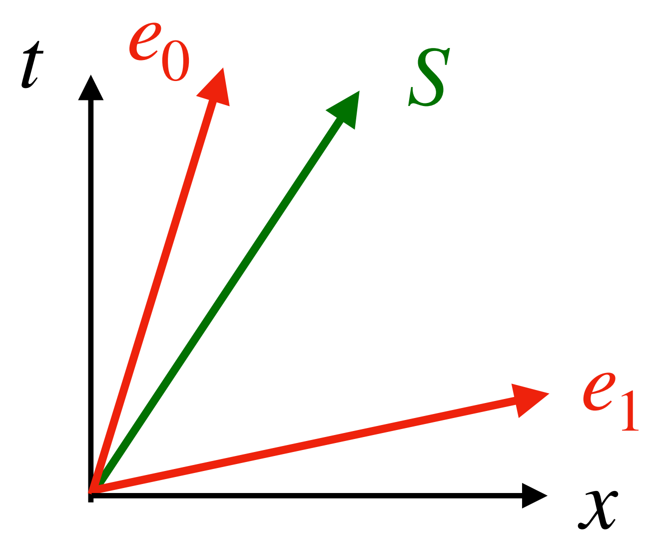
covariant: $U_C(t)|t'\rangle = |t + t'\rangle$ $\qquad U_C(t) = e^{-itH_C}$

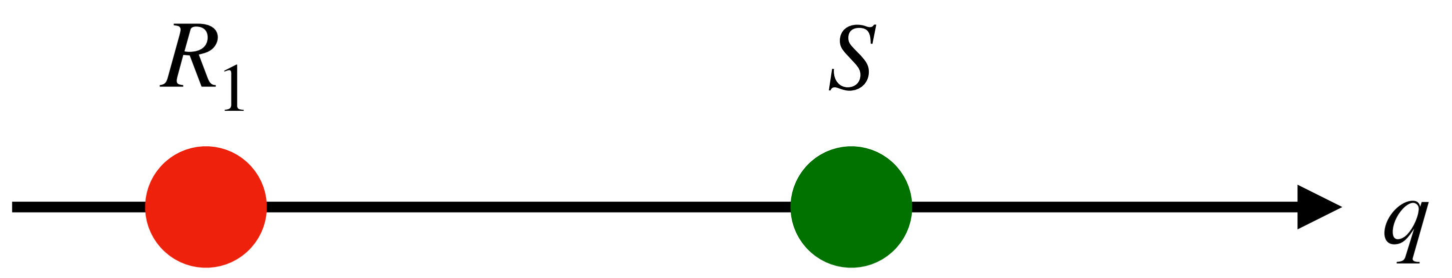
POVM elements $E_C(X) = \int_{X \subset \mathbb{R}} dt \, |t\rangle\langle t|$

normalized $E_C(\mathbb{R}) = I$ (well-defined probabilities)

fuzzy clock states $\langle t | t' \rangle \nsim \delta(t - t')$ **(unless ideal clock with** $\mathrm{Spec(H_C)} = \mathbb{R}$**)**

internal frames in SR vs. QT





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externally distinguishable configurations	$\{e_a^\mu, S^\mu\}$	

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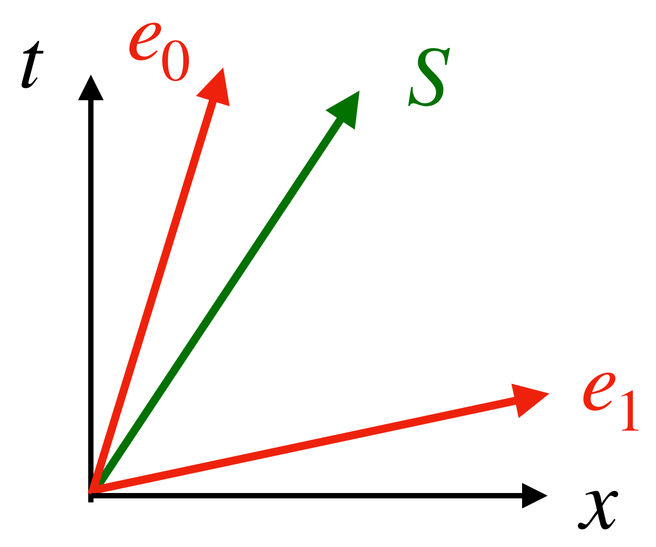

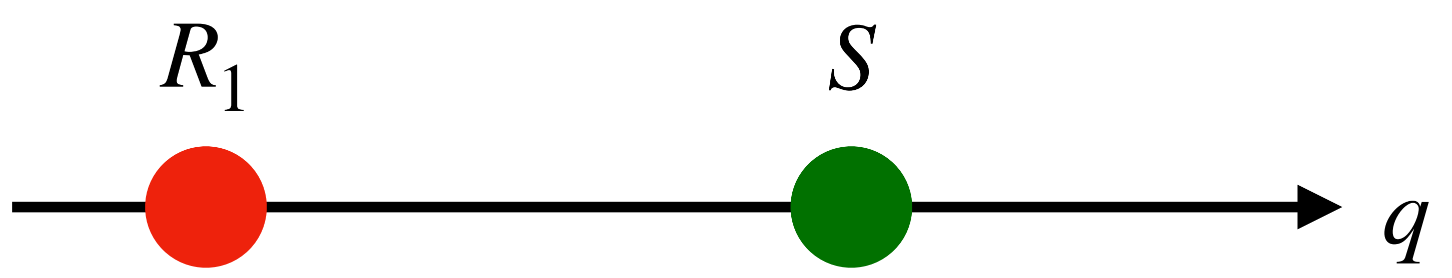

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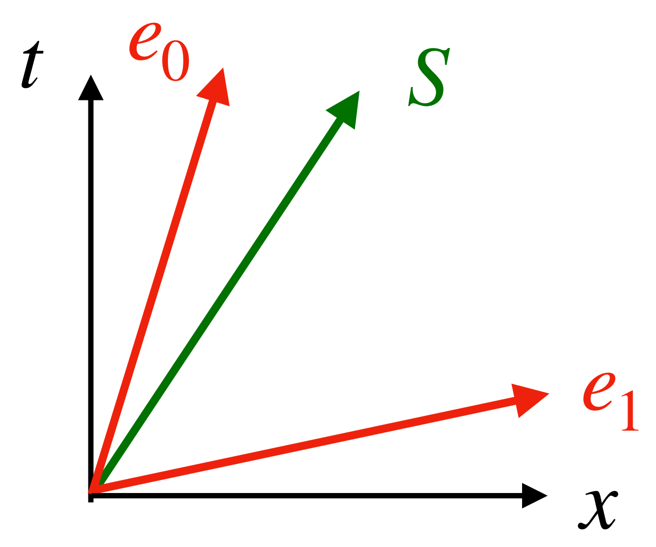
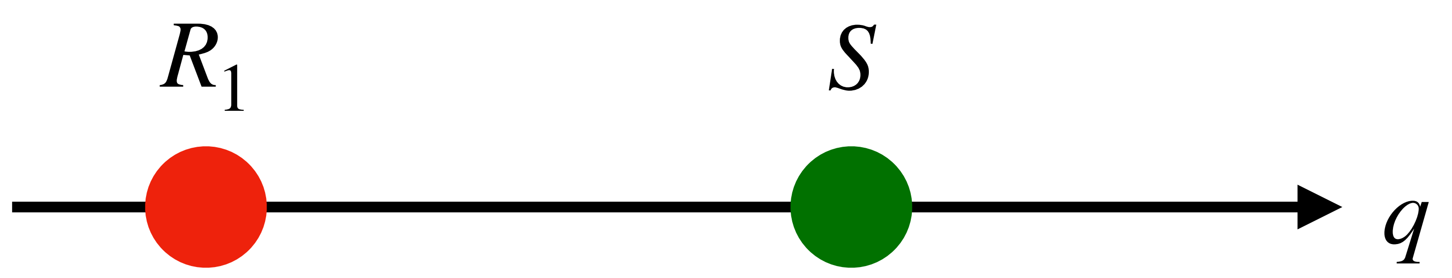
internal frames in SR vs. QT





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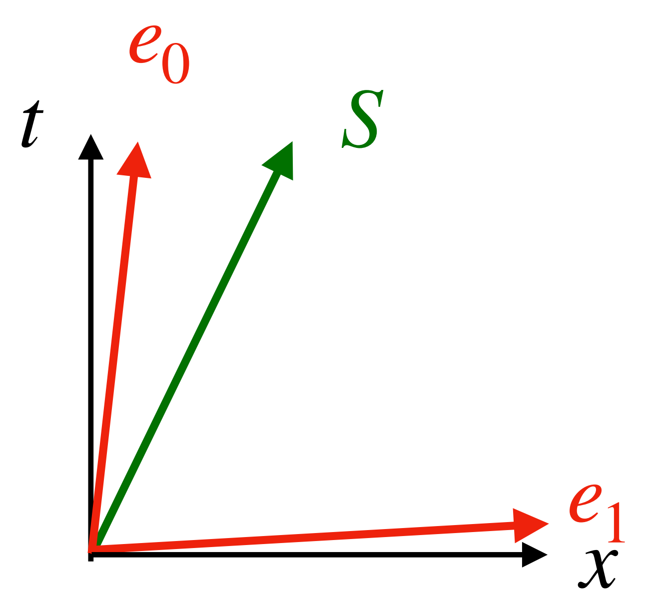

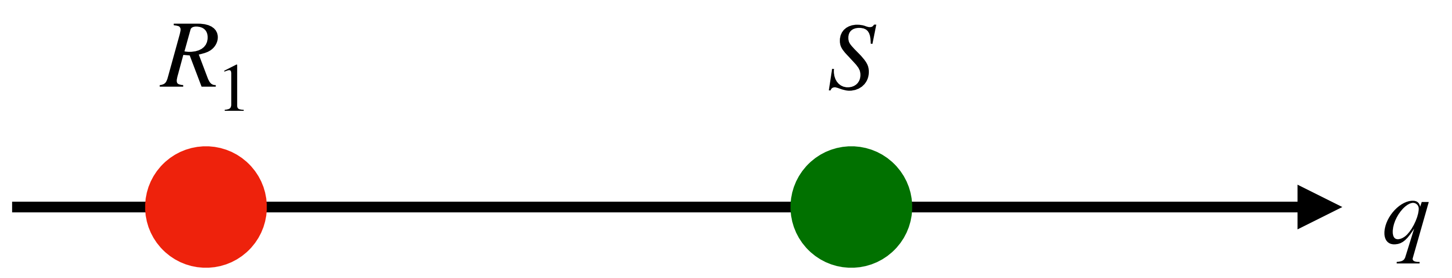

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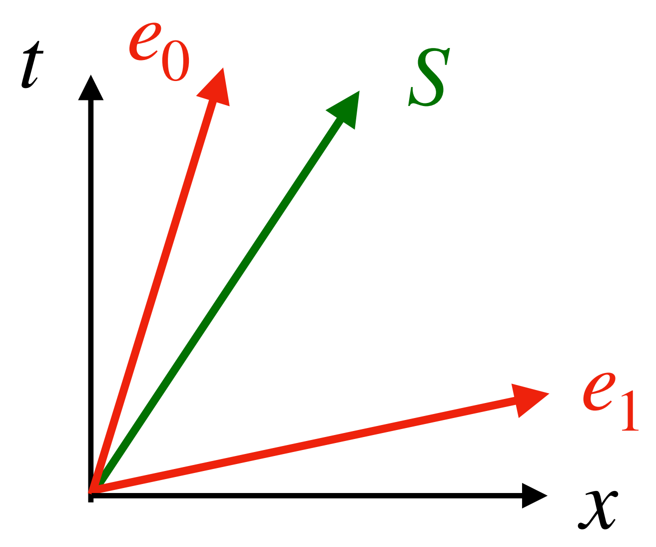

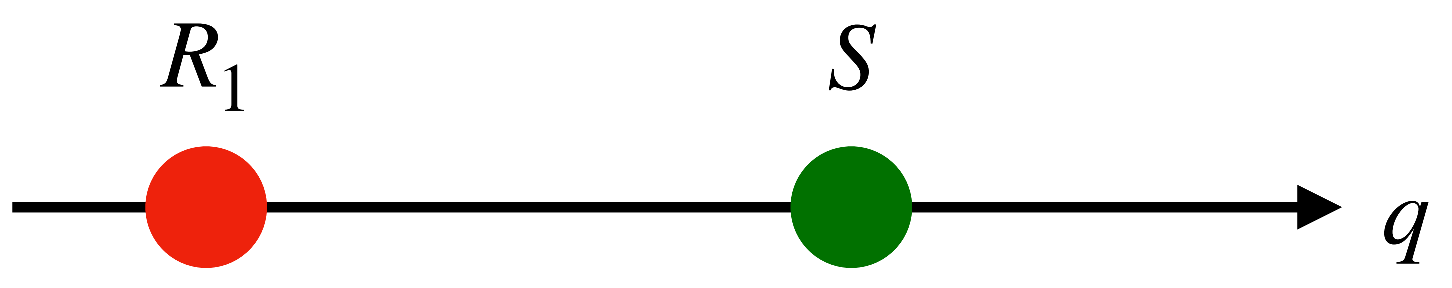

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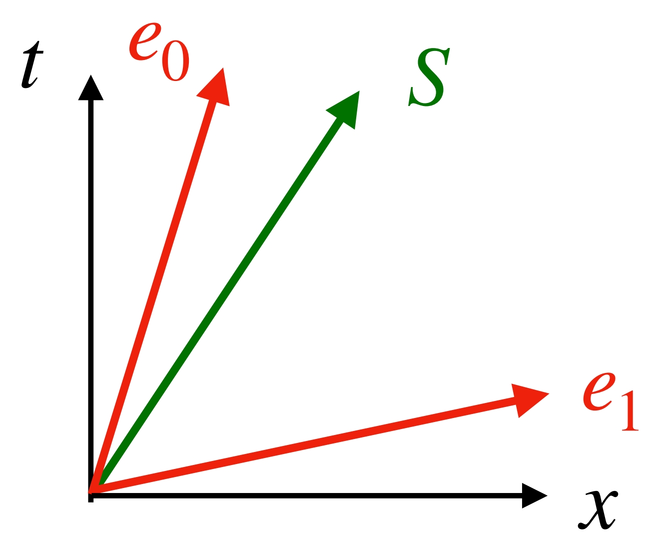

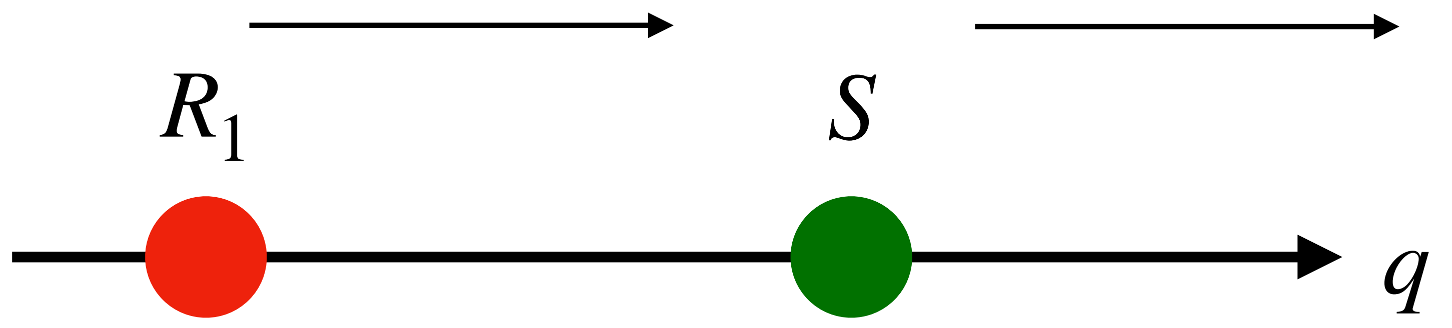

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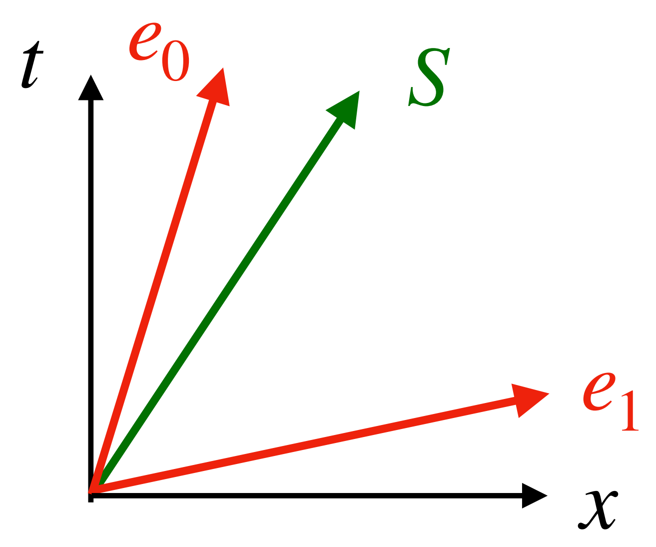

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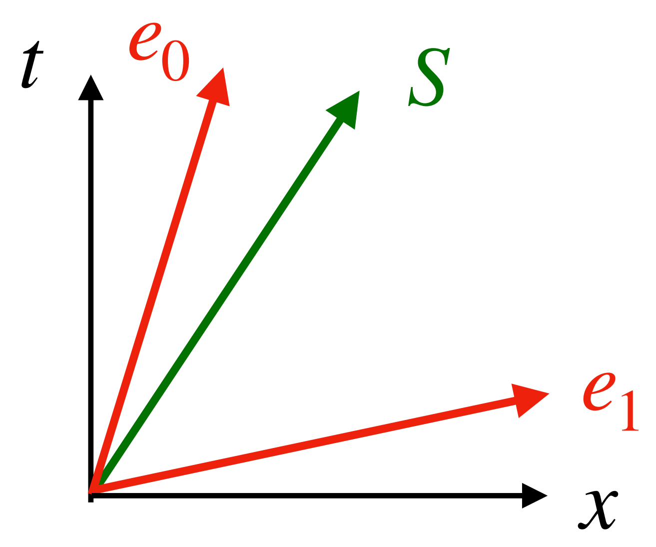

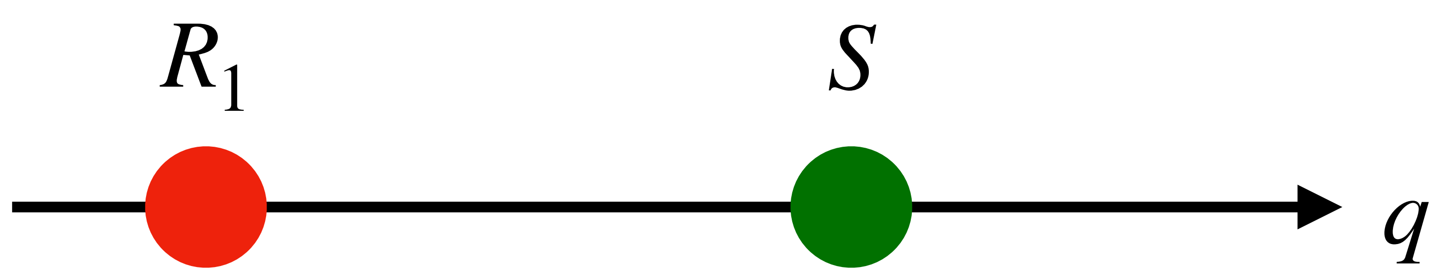

internal frame orientations	**tetrad** $e^\mu_a \in \mathrm{SO}_+(3,1)$ (because $\eta_{\mu\nu}\, e^\mu_a\, e^\nu_b = \eta_{ab}$)	**covariant POVM** $E(g\,.\,X) = U^g_R\, E(X \subset G)\, (U^g_R)^\dagger$ **in practice, coh. states** $\lvert g\rangle$, $g \in G$
externally distinguishable configurations	$\{e^\mu_a, S^\mu\}$	**kin. Hilbert space** $\mathscr{H}_{\mathrm{kin}} = \mathscr{H}_R \otimes \mathscr{H}_S$ **kin. algebras** $\mathscr{A}_{\mathrm{kin}} = \mathscr{A}_R \otimes \mathscr{A}_S$
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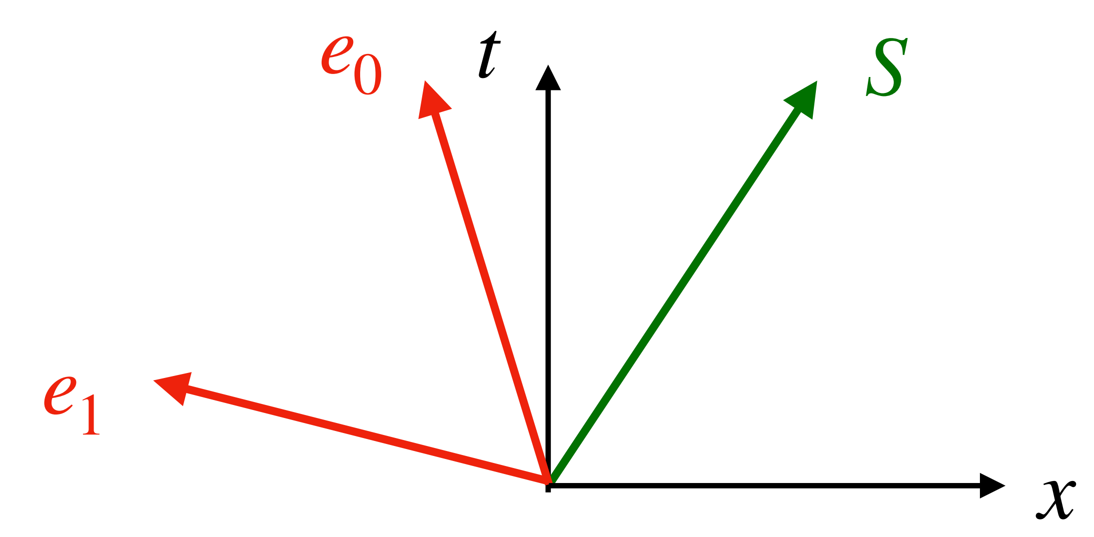

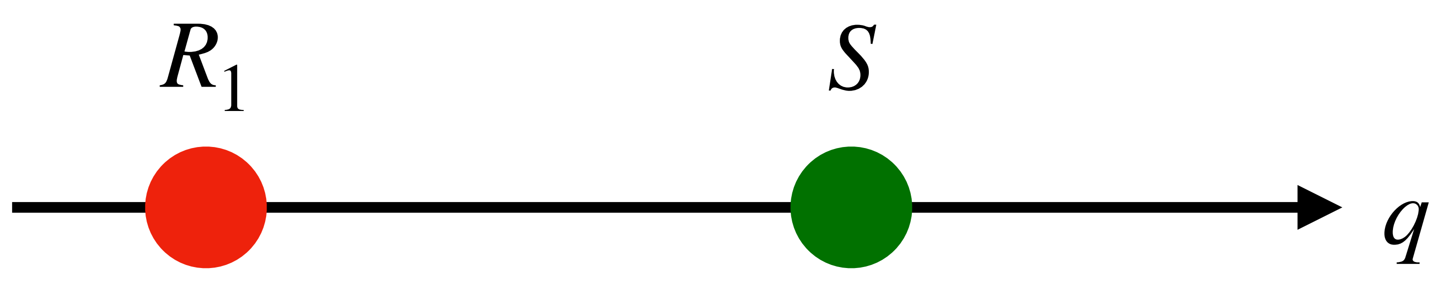

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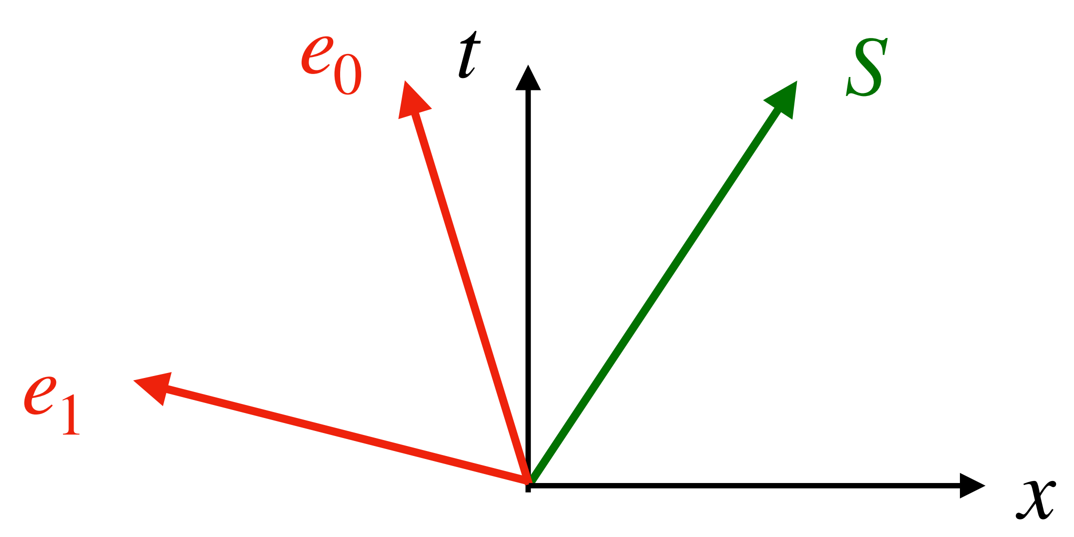

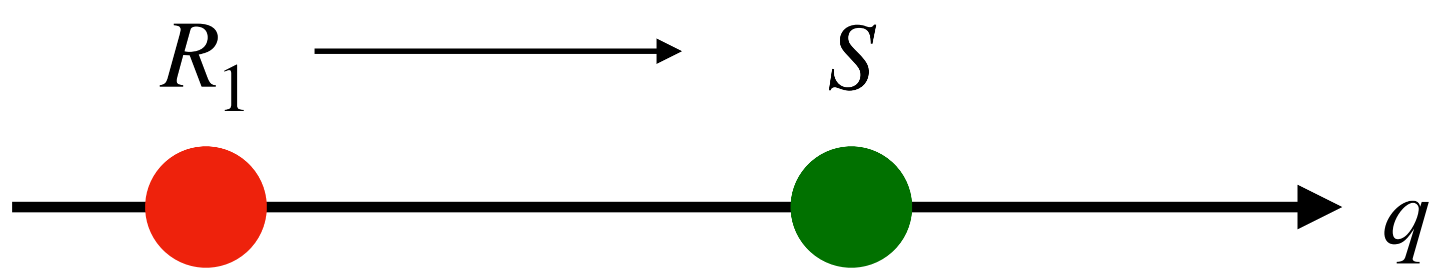

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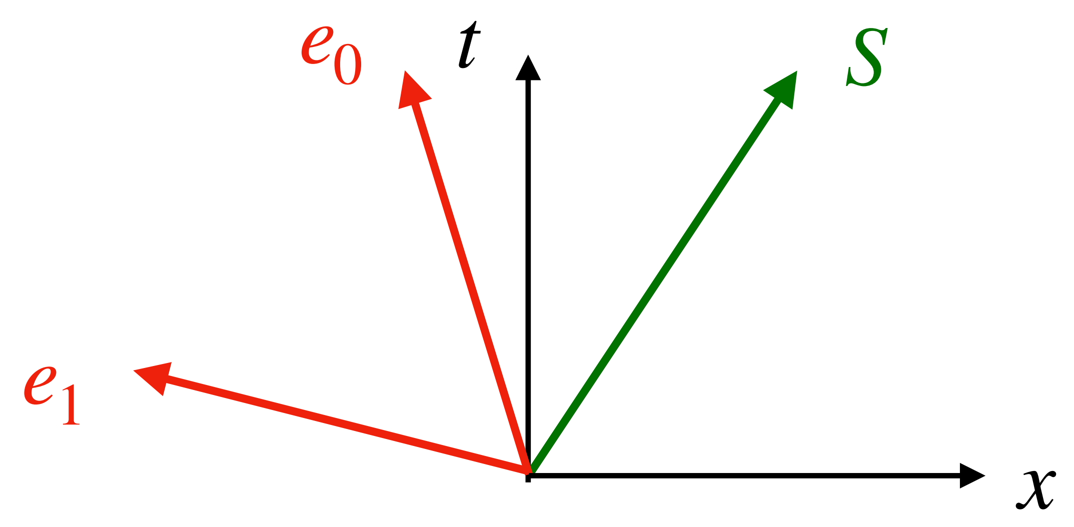

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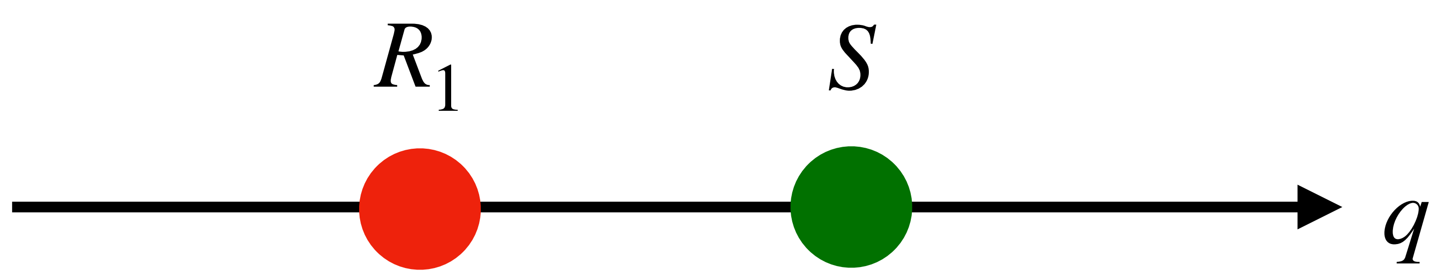

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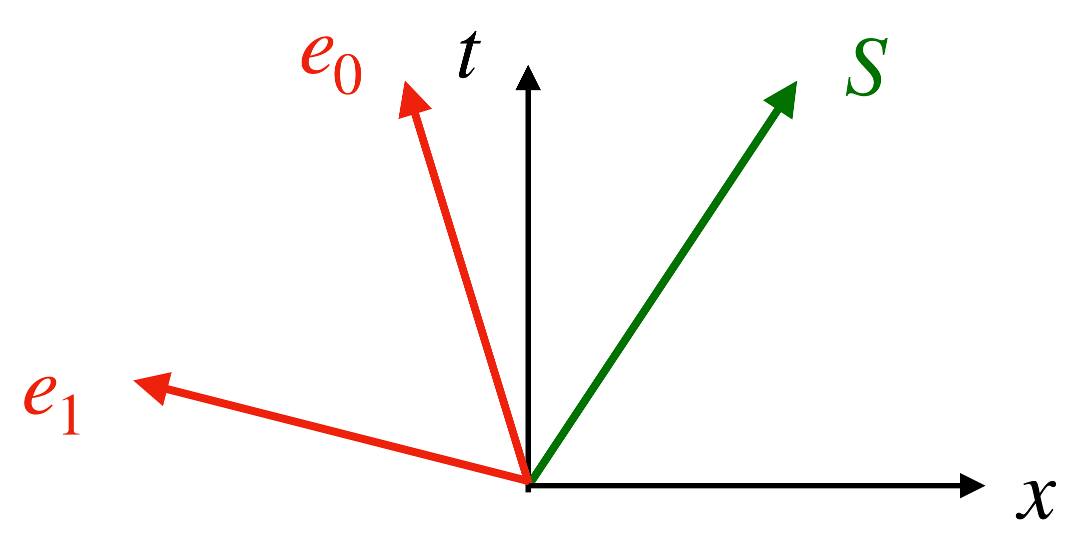

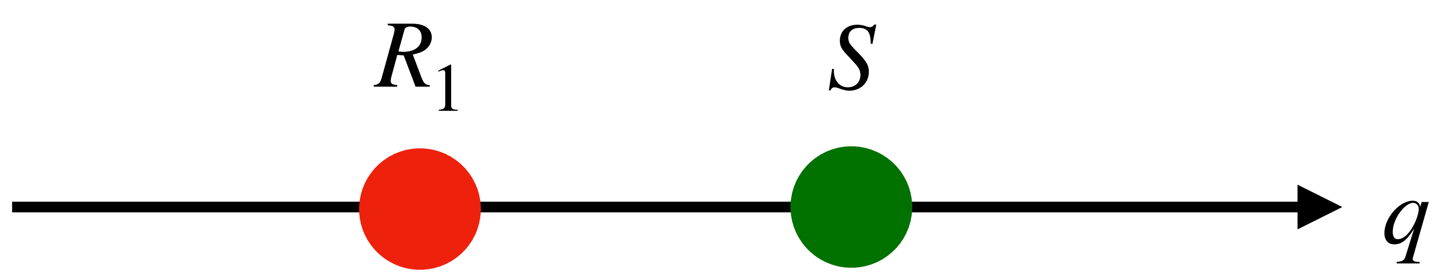

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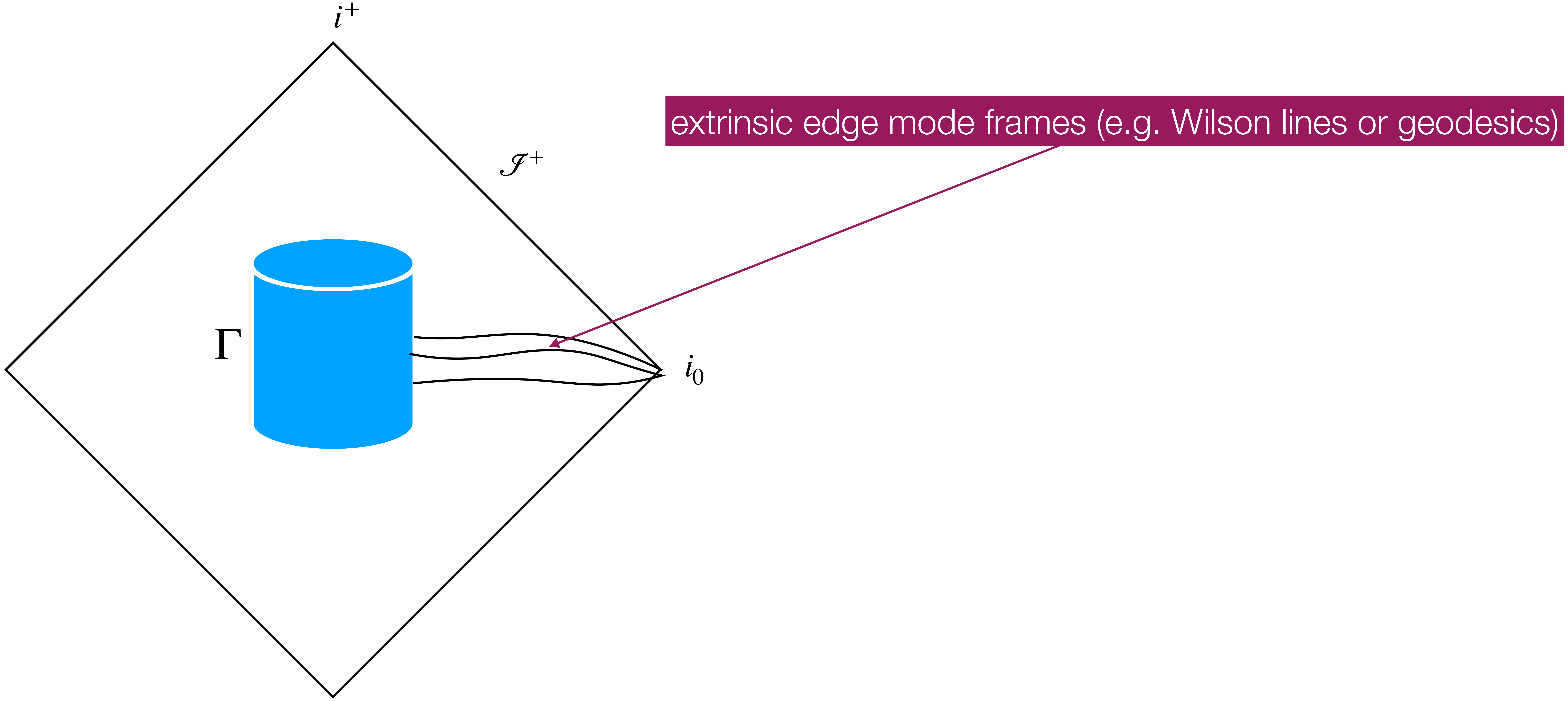
Reorientations are physical

- **relational time evolution** [PH, Smith, Lock ’19; De Vuyst, Eccles, PH, Kirklin ’24]
- **corner symmetries in gauge theory and gravity**

[Carrozza, PH ’21; Carrozza, Eccles, PH ’22; Goncalo-Araujo, PH, Sartini, Tomova ’24]



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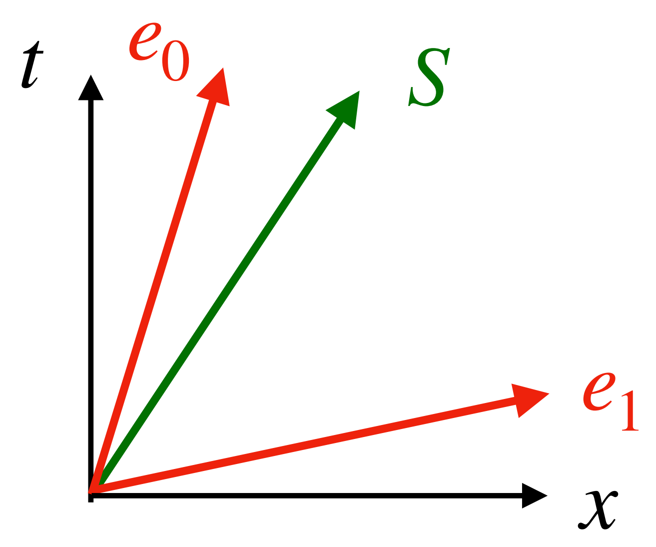
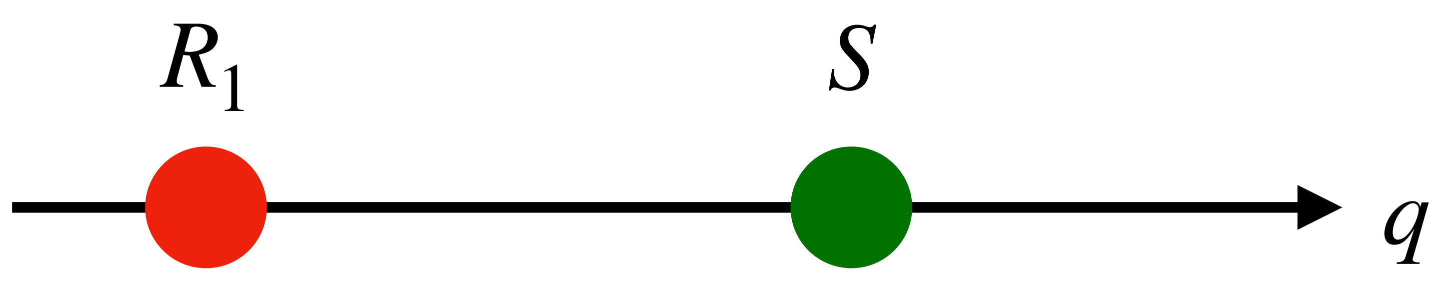
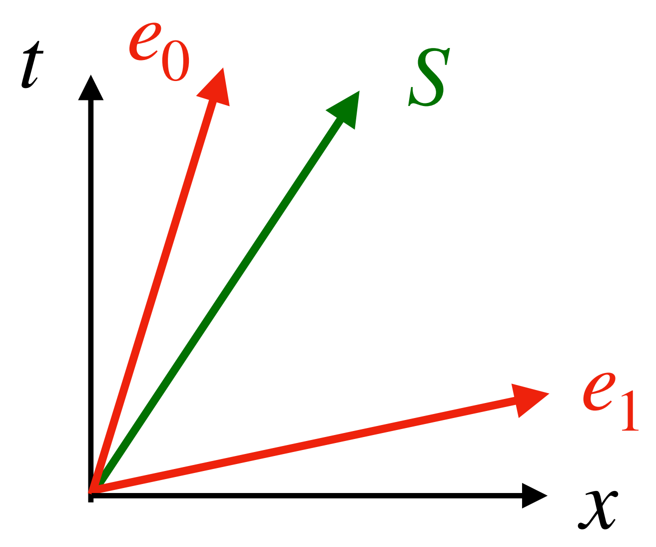
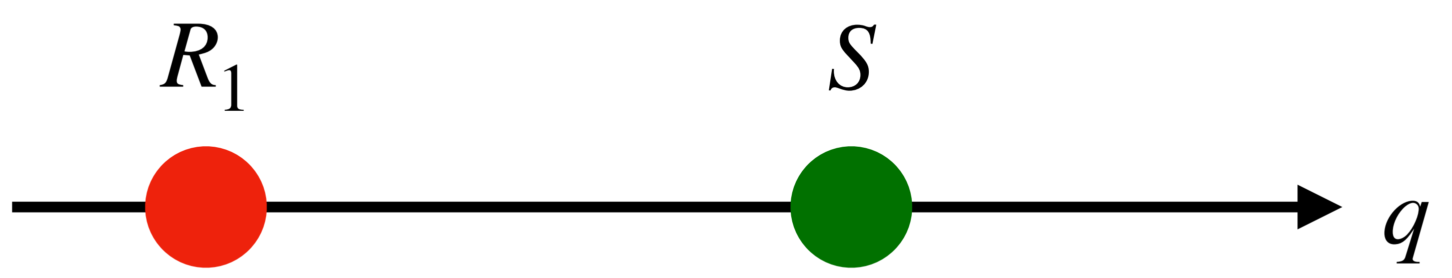




internally distinguishable configurations (extern frame indep.)	$\{S_\mu S^\mu,\ \ S_\mu e^\mu_a,\ \ e^\mu_a e^b_\mu\}$ **relational/dressed observables**

internal frames in SR vs. QT





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relational/dressed observables

external frame indep. observables and states?

External frame independent observables

minimal condition for observables: $O \in \mathscr{A}_{\text{kin}}$ **s.t.** $[O, U^g_{RS}] = 0, \forall g \in G$

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Examples: relational observables

1. relational dynamics in gravity: some Hamiltonian constraint with clock $R = C$ $\qquad C = H_C + H_S$

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measures f_S when clock reads τ

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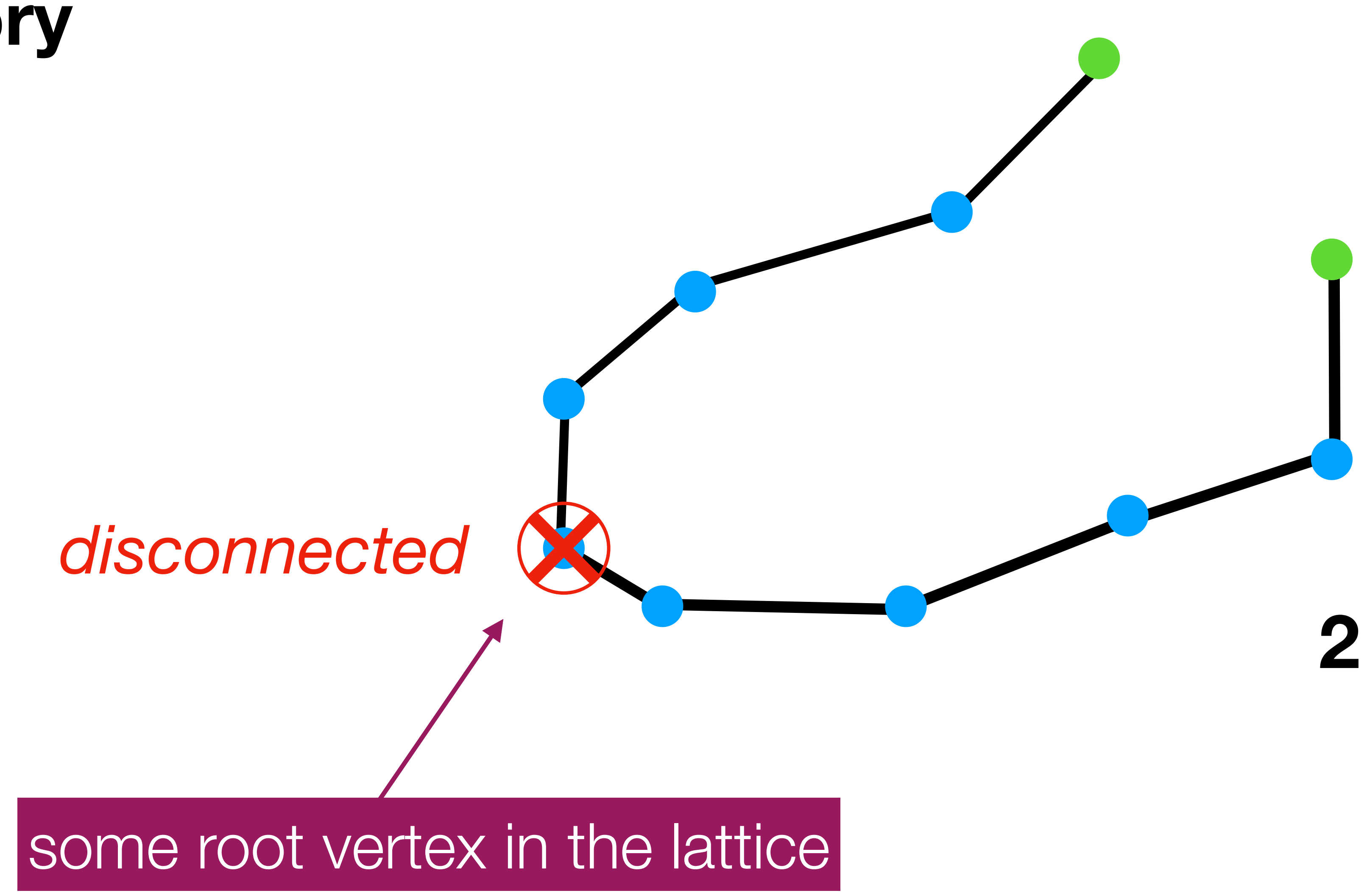

2 Wilson lines define G-valued QRF R at green nodes

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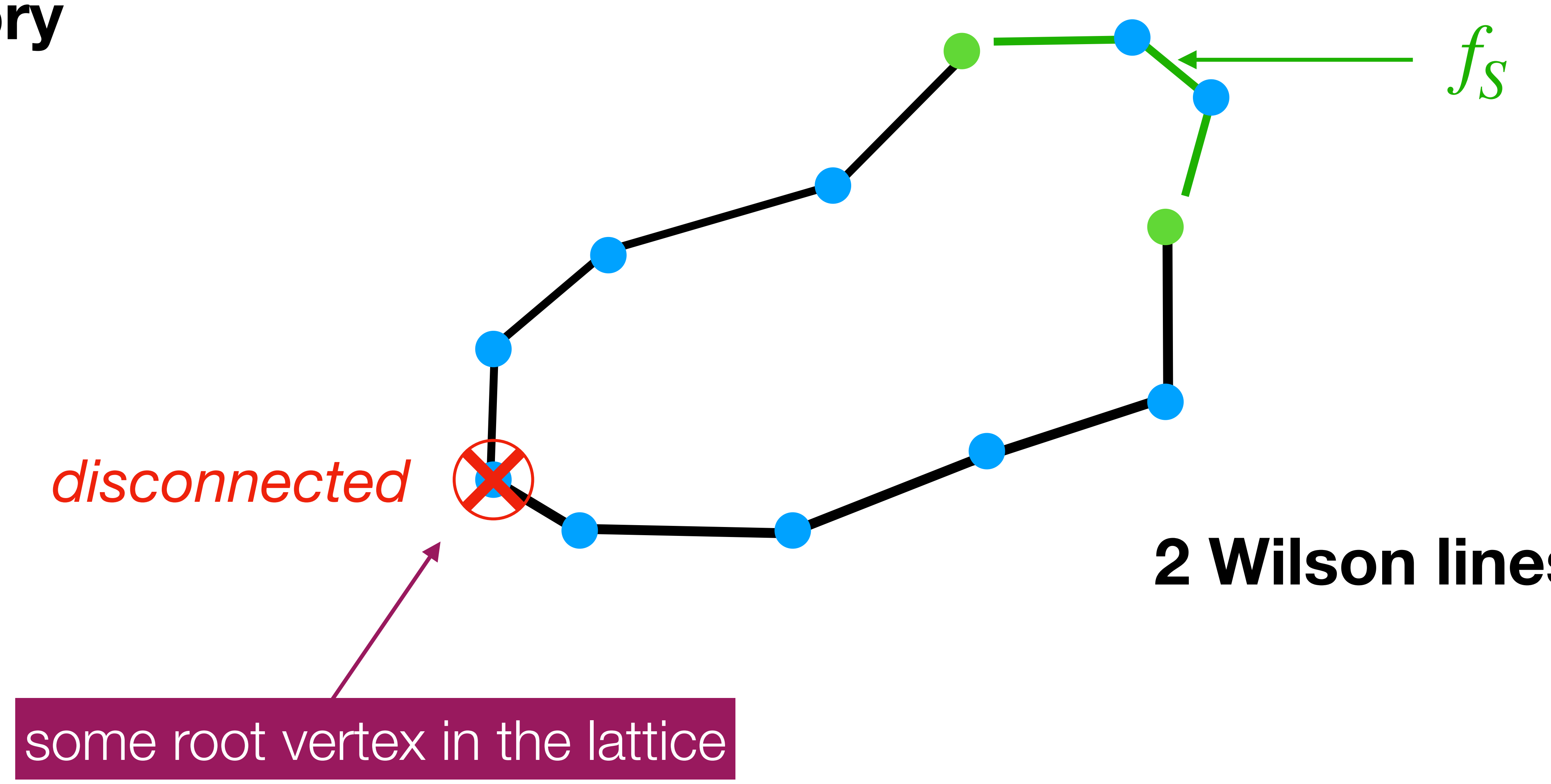

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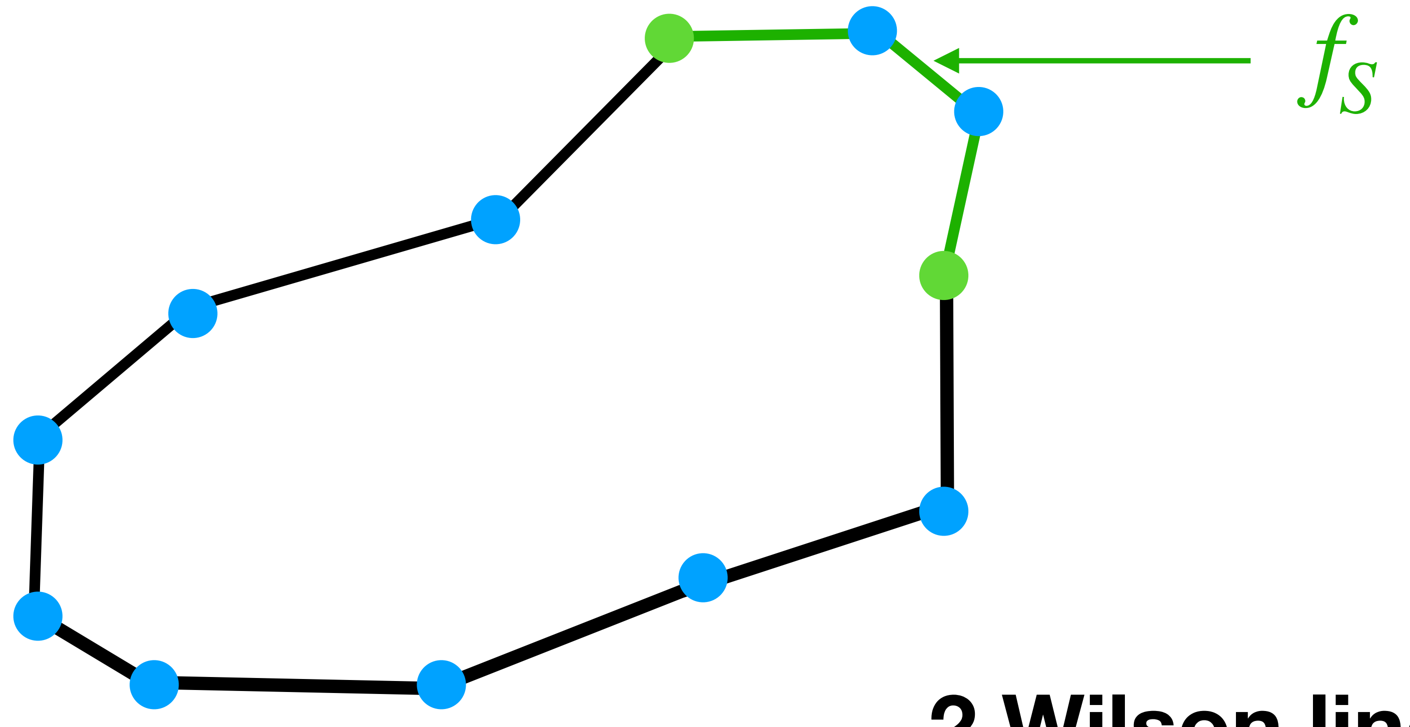


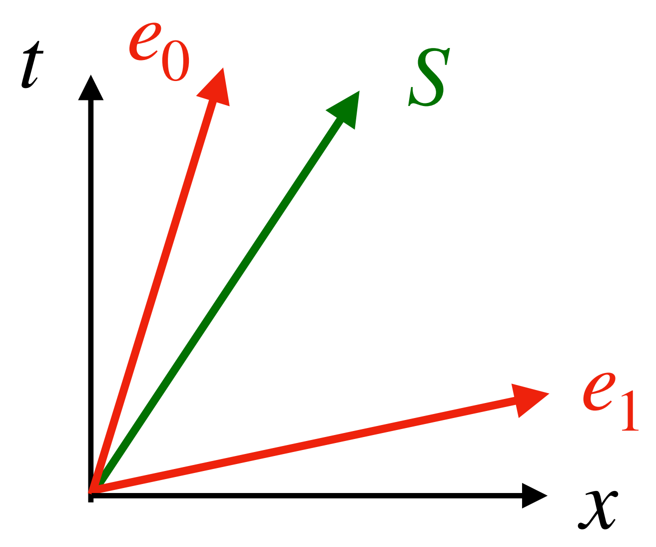
connected

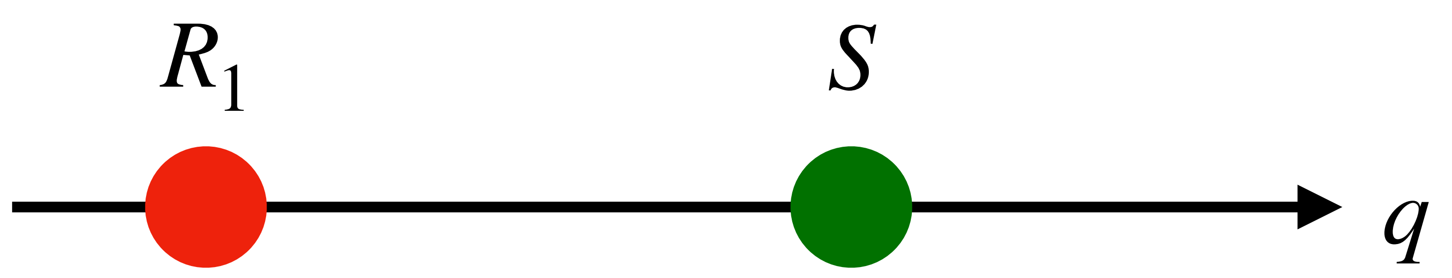
some root vertex in the lattice

2 Wilson lines define G-valued QRF R at green nodes

$\Rightarrow$ **rel. obs. $O^{e}_{|R}(f_S)$ = complete Wilson loop**

internal frames in SR vs. QT





observables: $O \in \mathscr{A}_{\text{kin}}$ **s.t.** $[O, U^g_{RS}] = 0, \forall g \in G$

internally distinguishable configurations
(extern frame indep.)

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What states do they act on?

Two notions of external frame independence

states can be G-invariant in two different ways:

weak: $[\rho_{\text{inv}}, U^g_{RS}] = 0, \forall\, g \in G$

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image of the incoherent group average

$$\rho_{\text{inv}} = \mathcal{G}(\rho), \quad \rho \in \mathcal{S}(\mathcal{H}_{\text{kin}})$$

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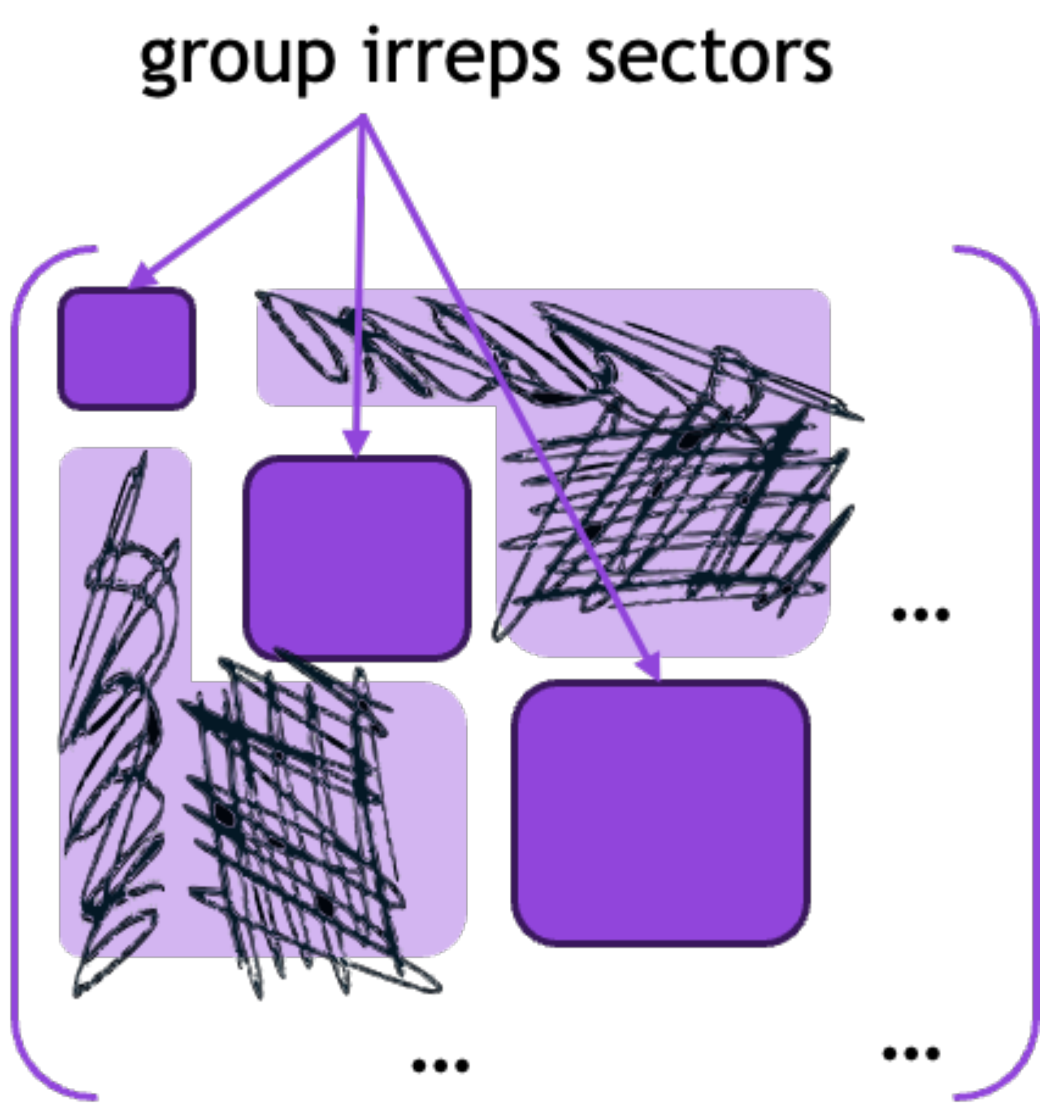

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image of the **coherent group average**

$$|\psi) = \Pi_{\text{pn}}\,|\psi\rangle, \quad |\psi\rangle \in \mathcal{H}_{\text{kin}} \qquad \Pi_{\text{pn}} = \int_G dg\, U^g_{RS}$$

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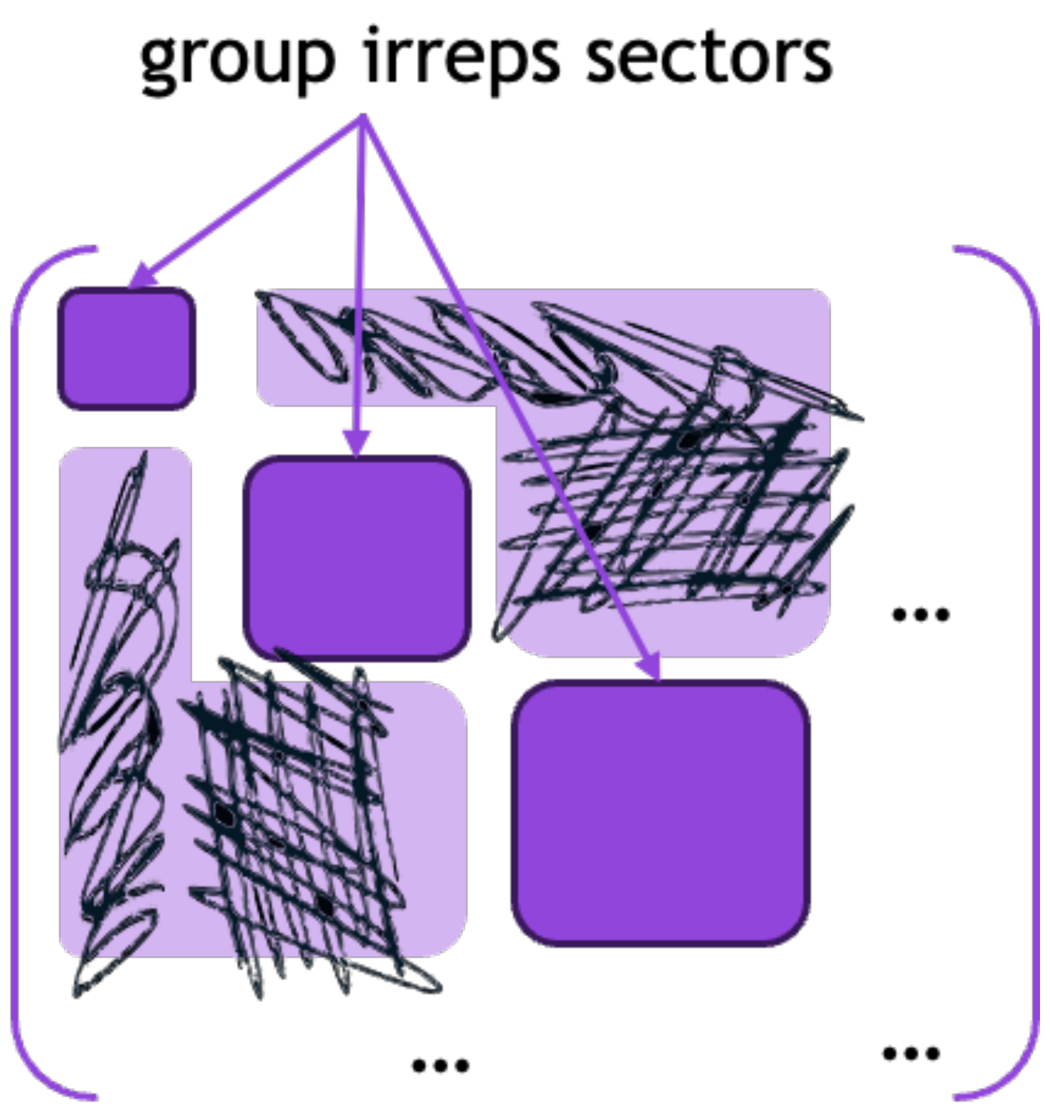

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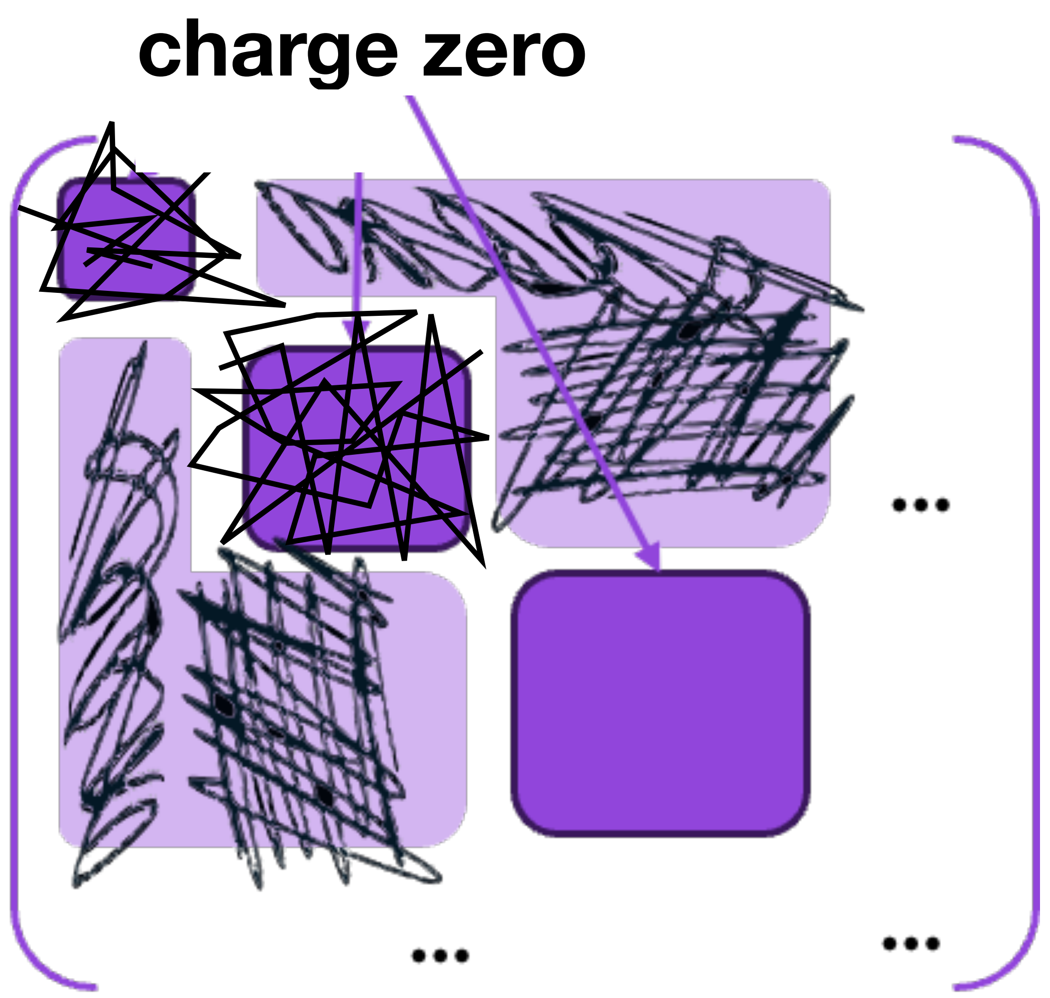

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(“observer can’t distinguish between external frame transformed states”)

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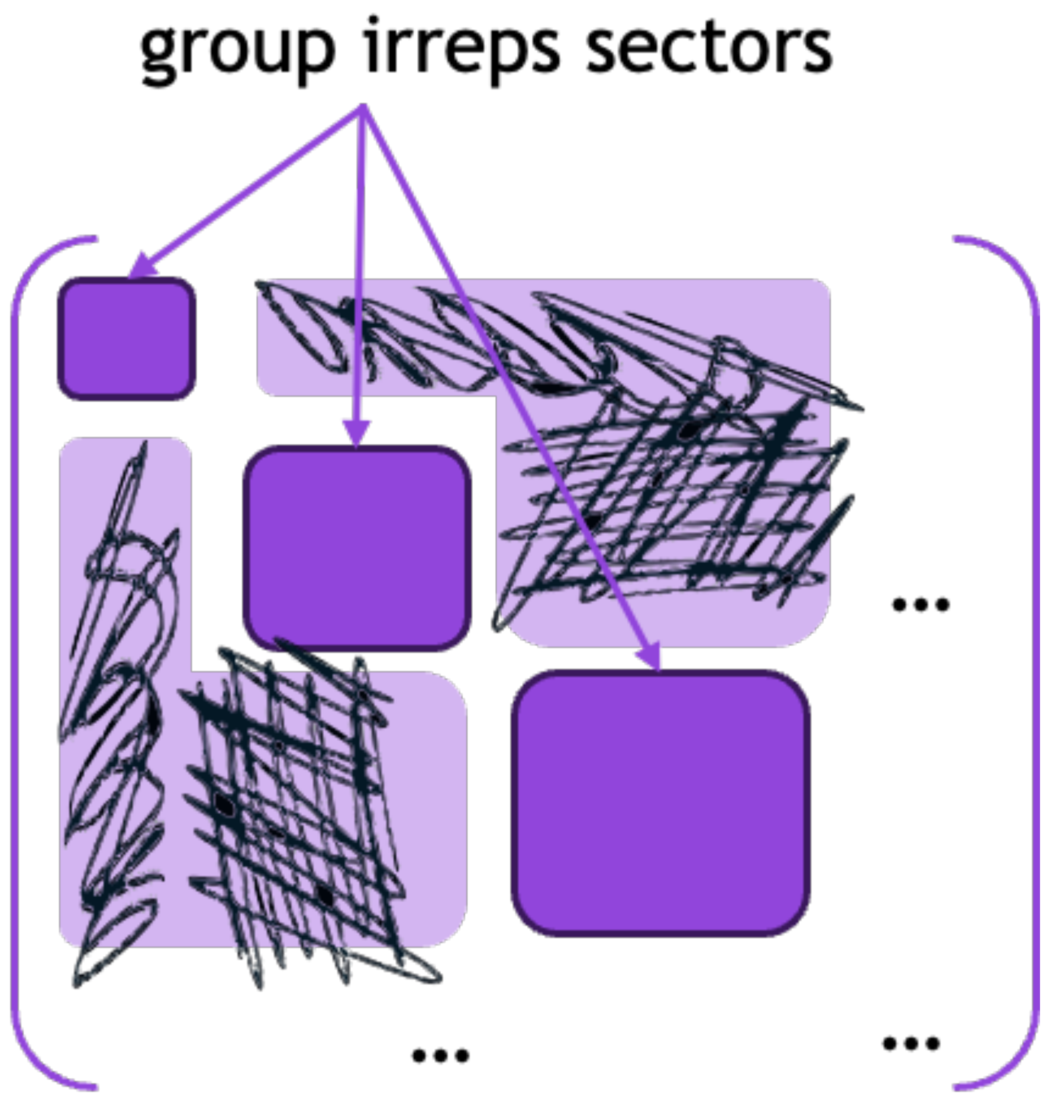

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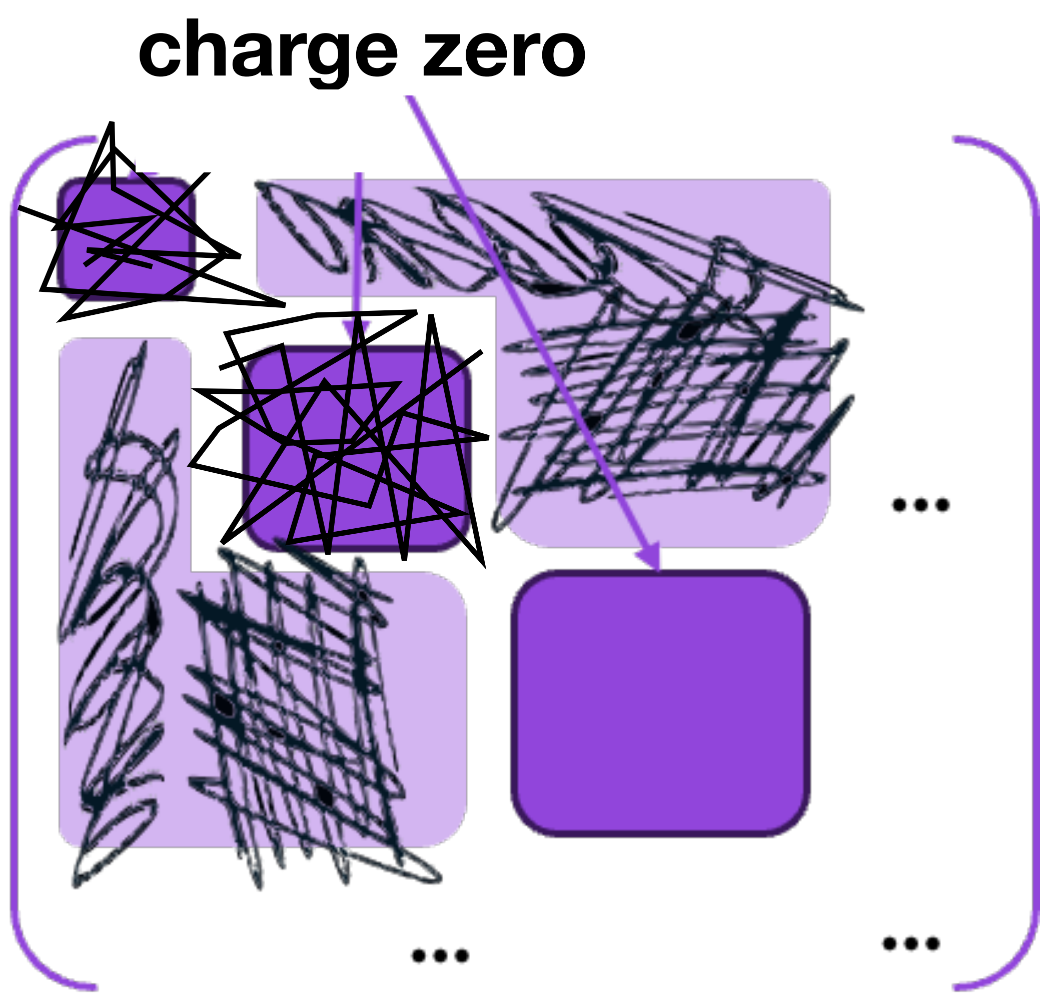

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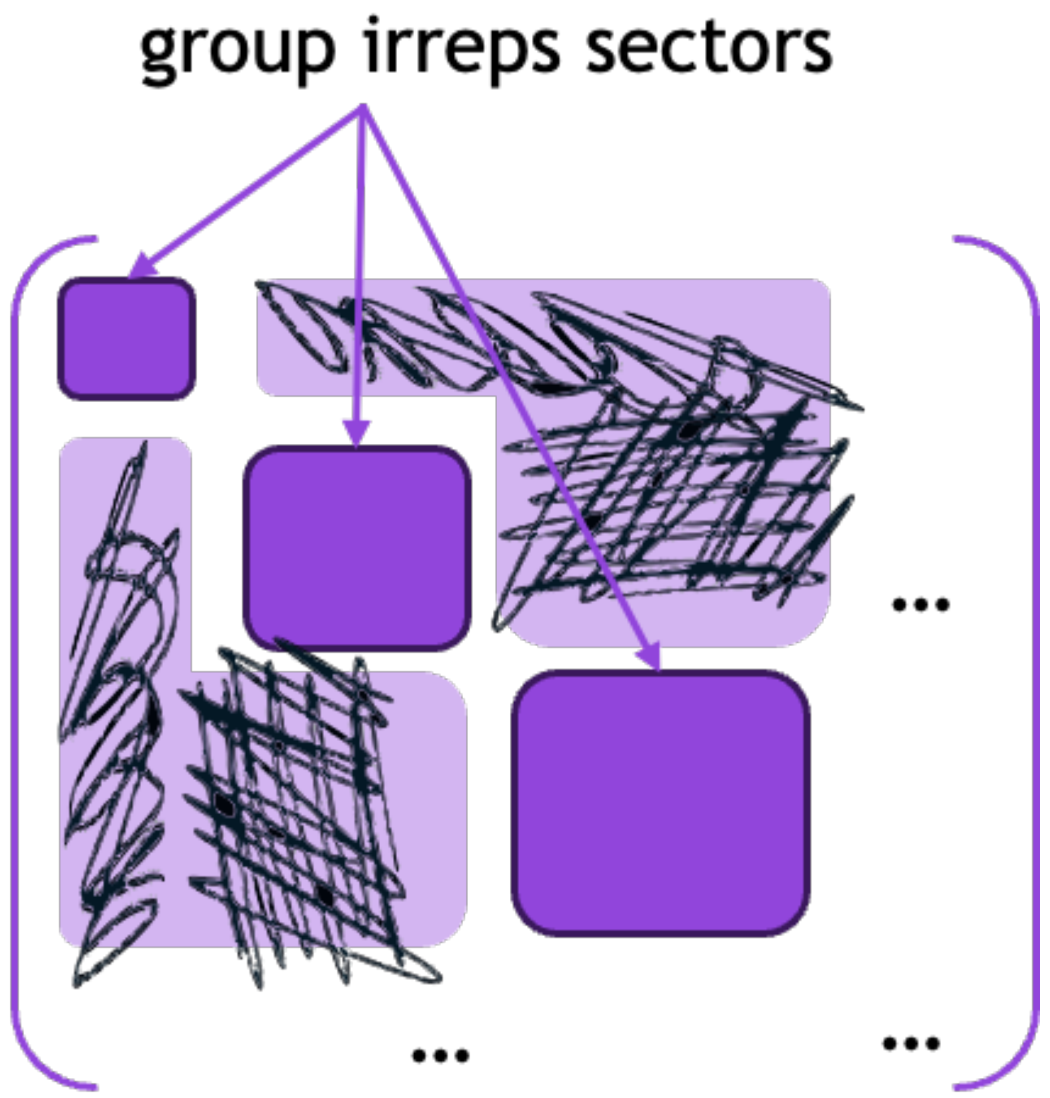

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$$(*) \quad + \quad |\psi\rangle - U^g_{RS}|\psi\rangle \mathrel{\hat{\approx}} 0, \ \ g \in G$$

("observer can also not distinguish their difference from zero")

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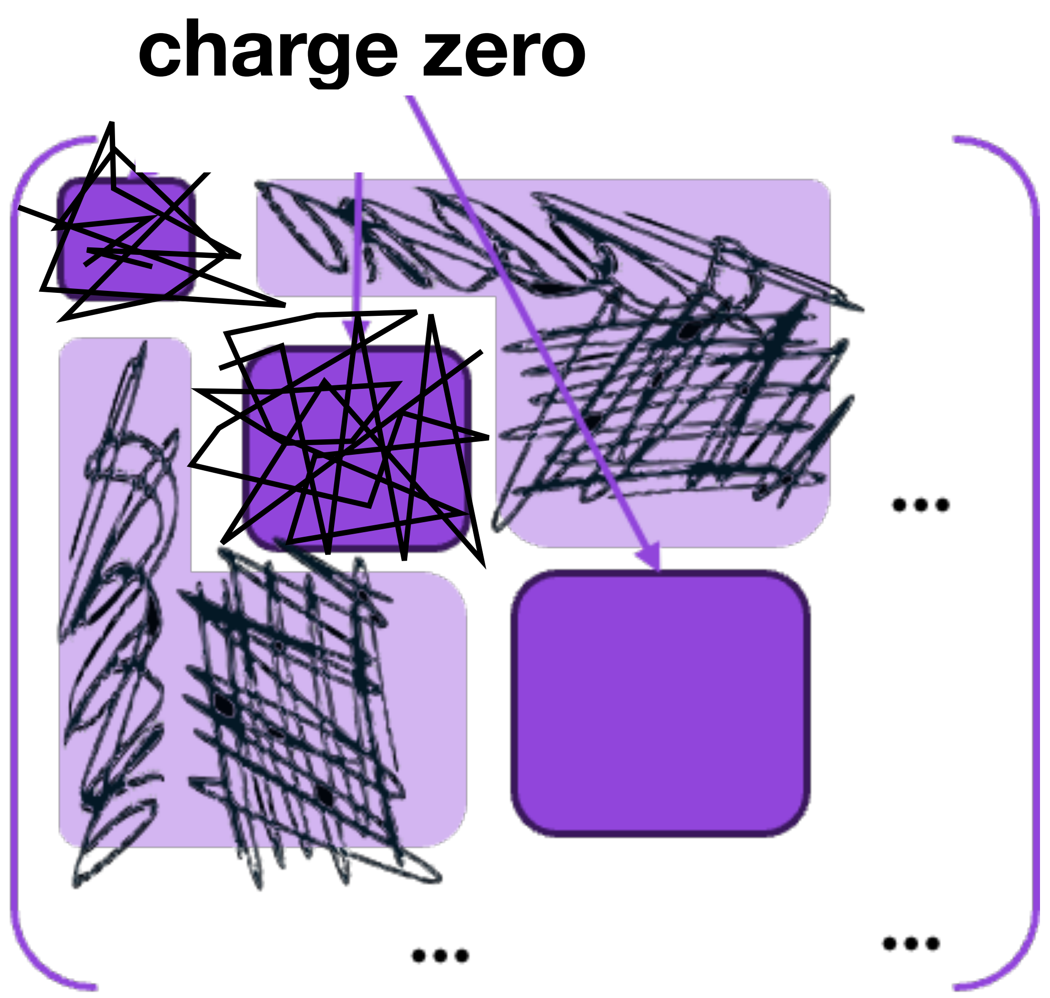

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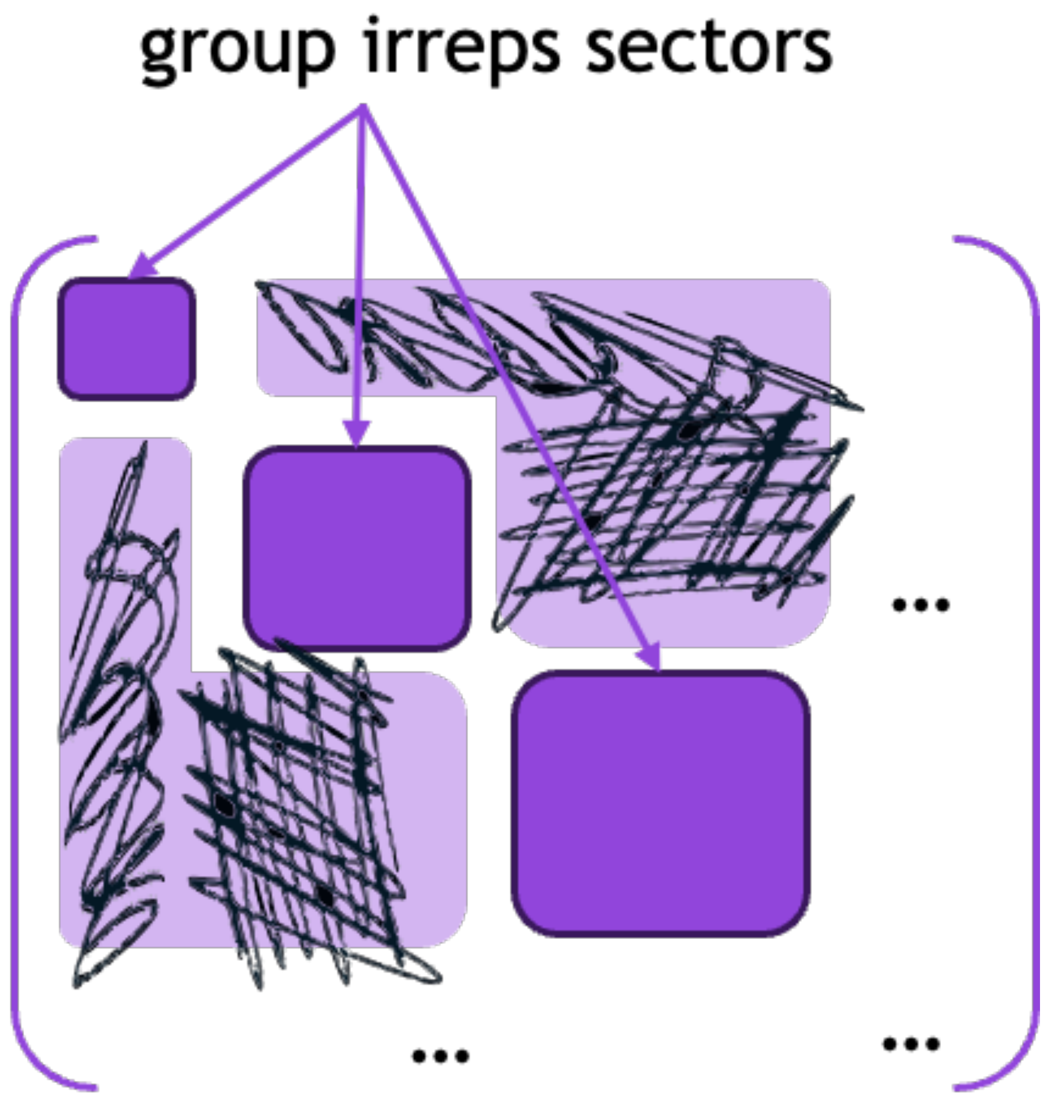
operational consequence:

CAN distinguish superpositions of external frame transformed states from their corresp. weighted mixtures



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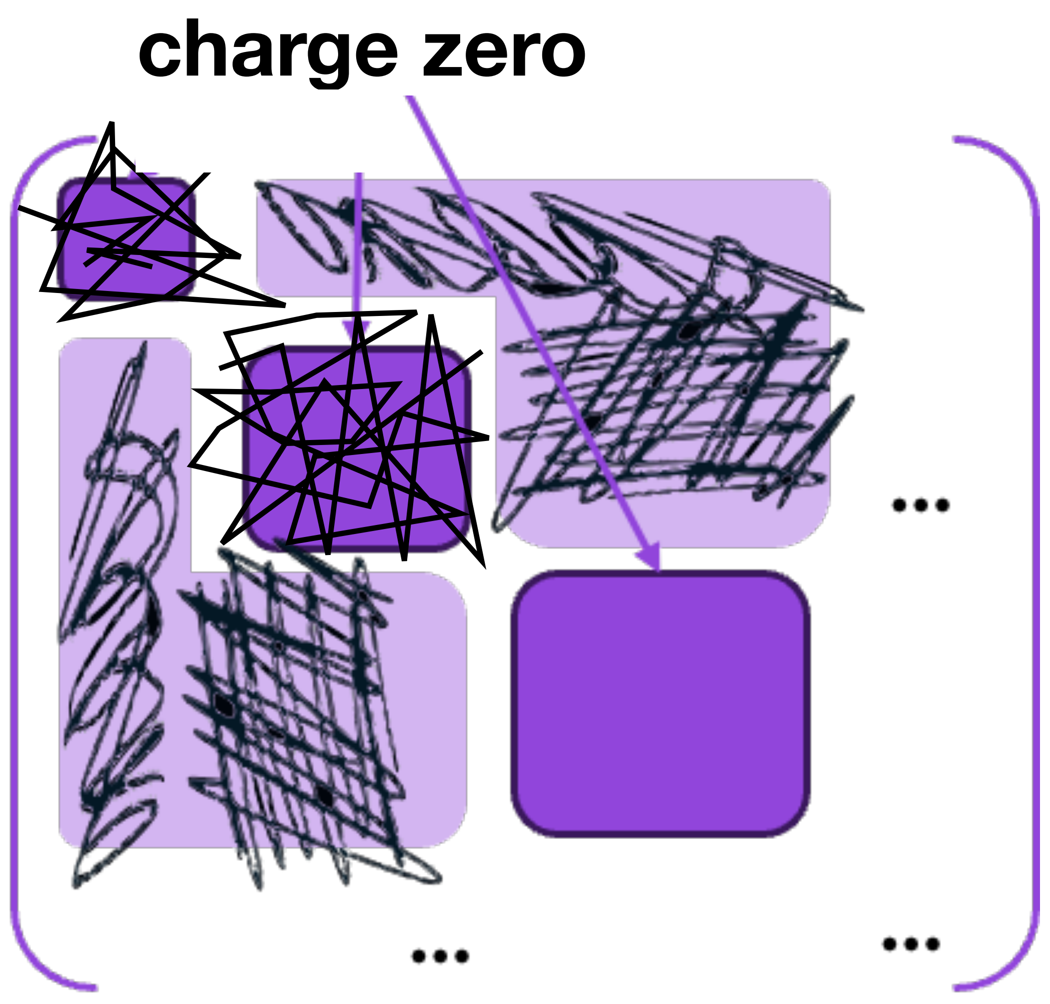

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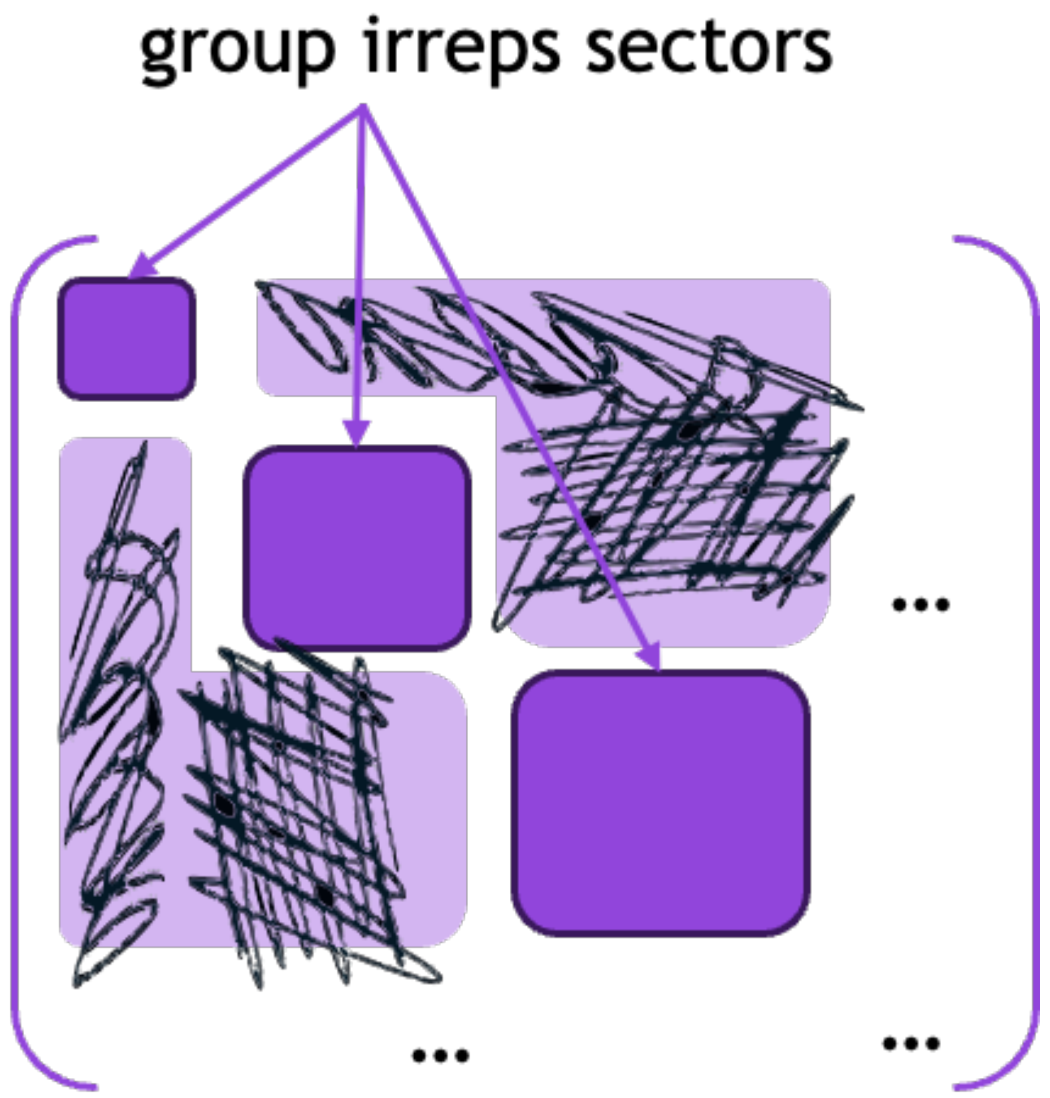
("observer can also not distinguish their difference from zero")

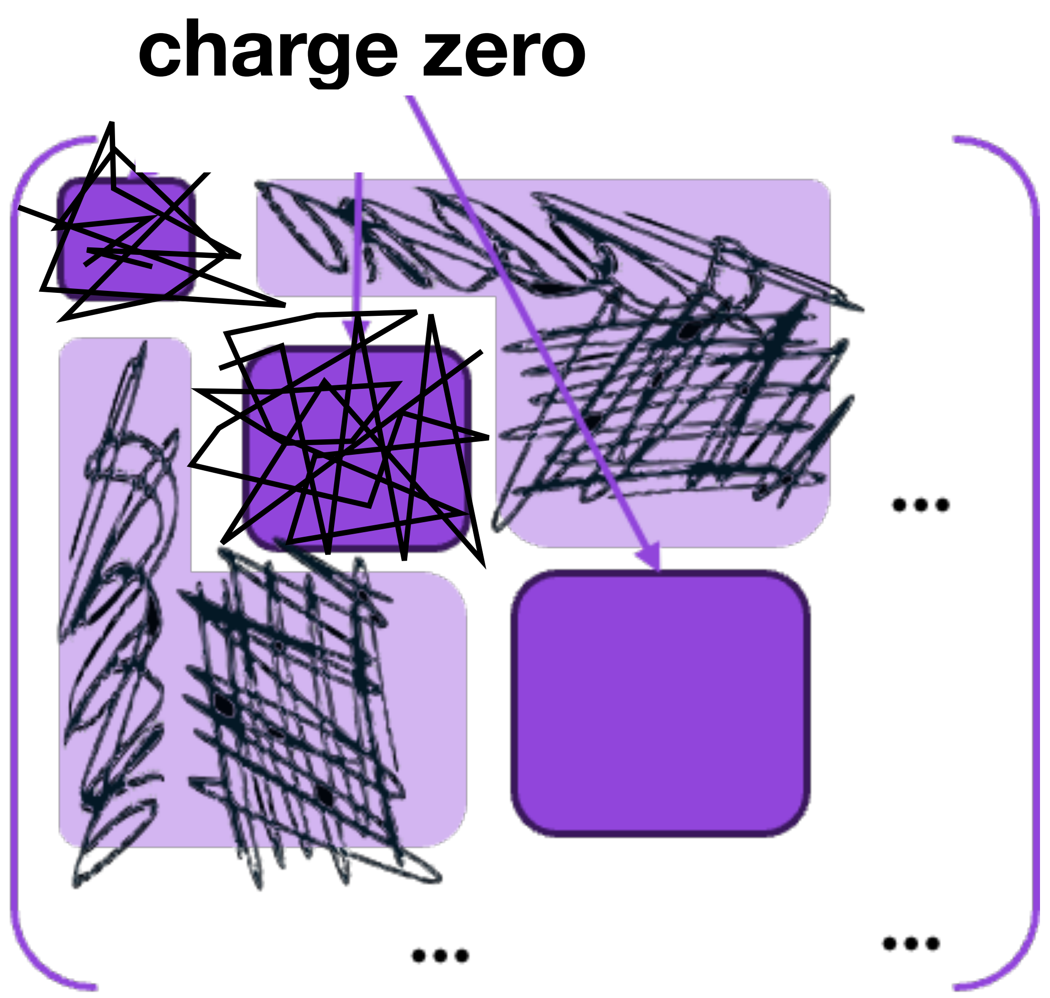
operational consequence:

CAN distinguish superpositions of external frame transformed states from their corresp. weighted mixtures



operational consequence:

CAN'T distinguish superpositions of external frame transformed states from their corresp. weighted mixtures



Two notions of external frame independence

states can be G-invariant in two different ways:

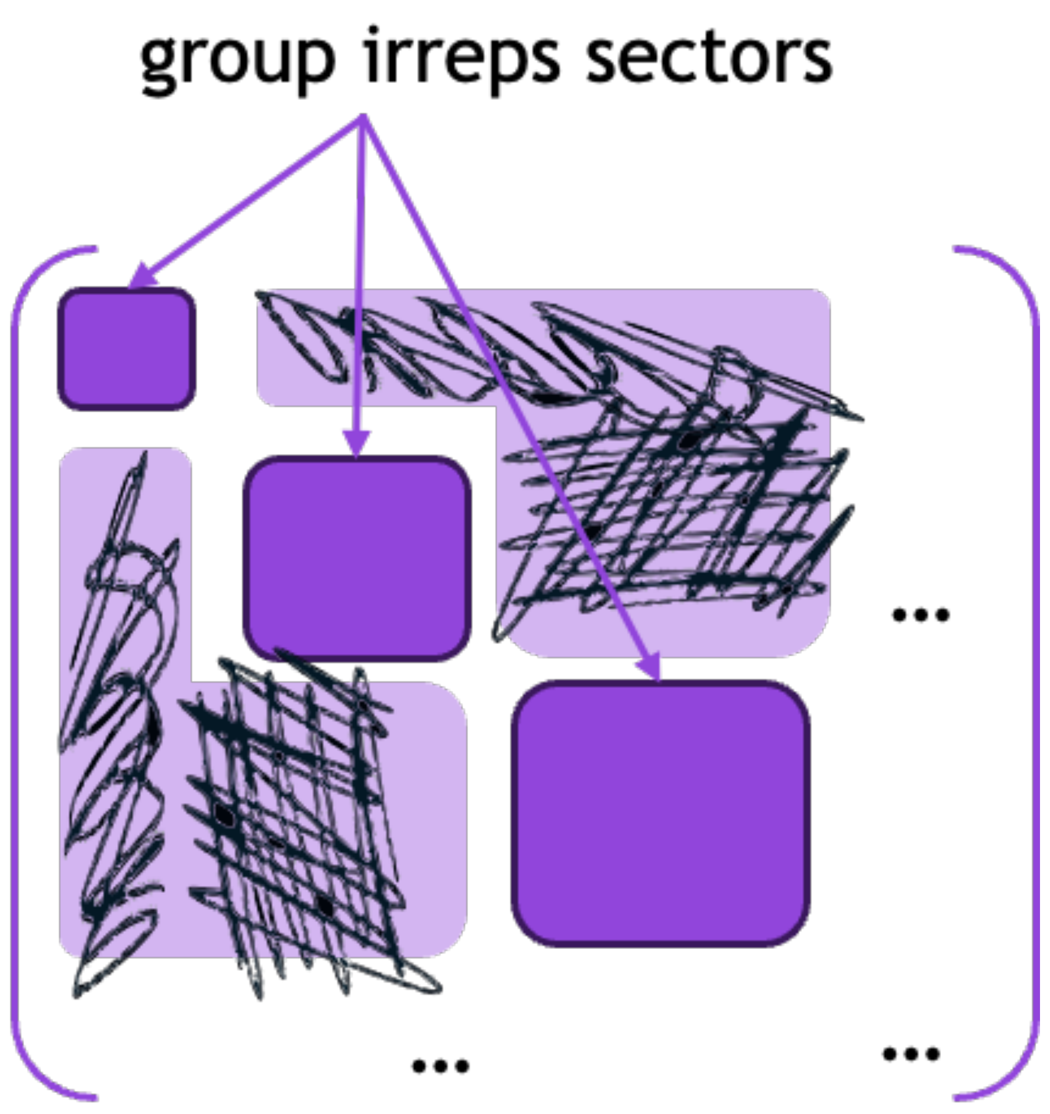
weak: $[\rho_{\text{inv}}, U^g_{RS}] = 0, \; \forall\, g \in G$

result of one equivalence relation:

$|\psi\rangle \sim |\psi'\rangle \quad \Leftrightarrow \quad |\psi'\rangle = U^g_{RS}|\psi\rangle$ **for some** $g \in G$ **(*)**

("observer can't distinguish between external frame transformed states")

QRF orientation data removed (conjugate to U^g_{RS}), but charge/irrep. data preserved



strong: $U^g_{RS}|\psi\rangle = |\psi\rangle, \; \forall\, g \in G$

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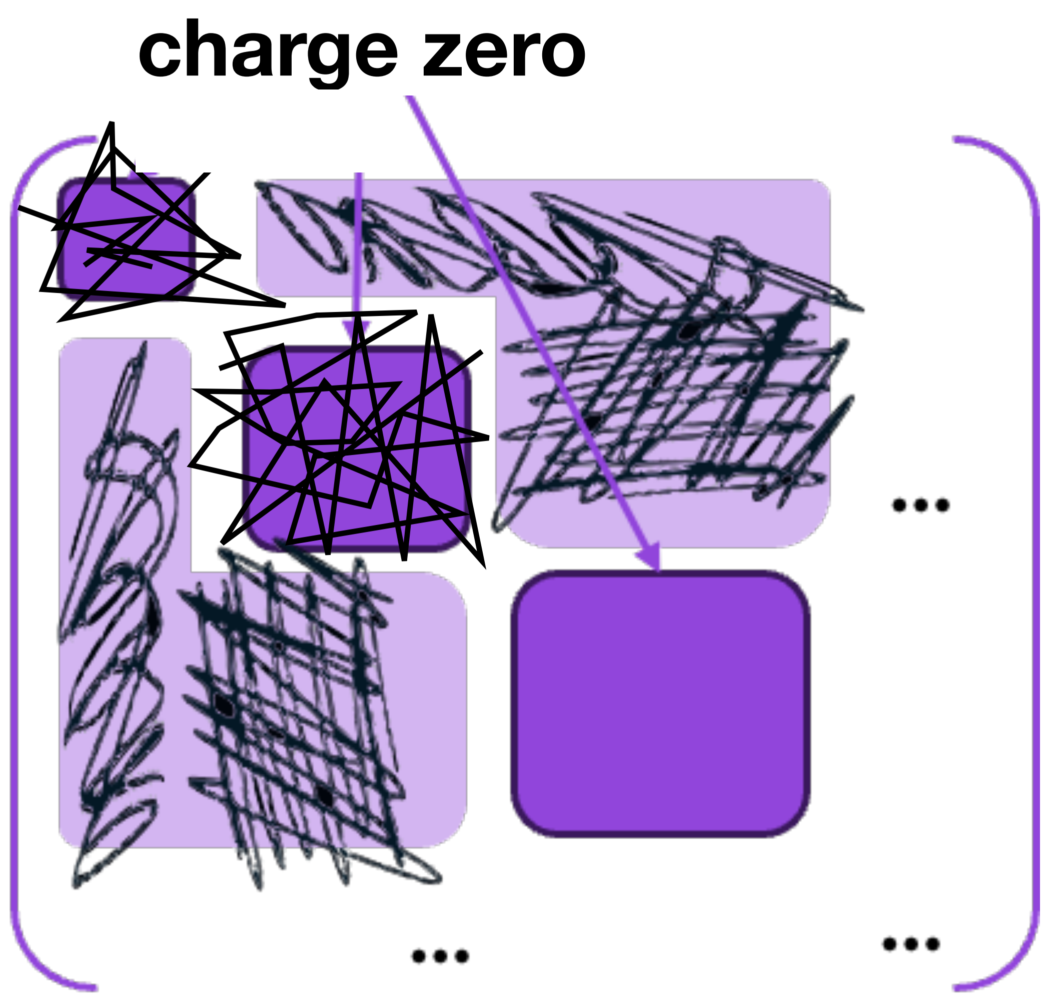

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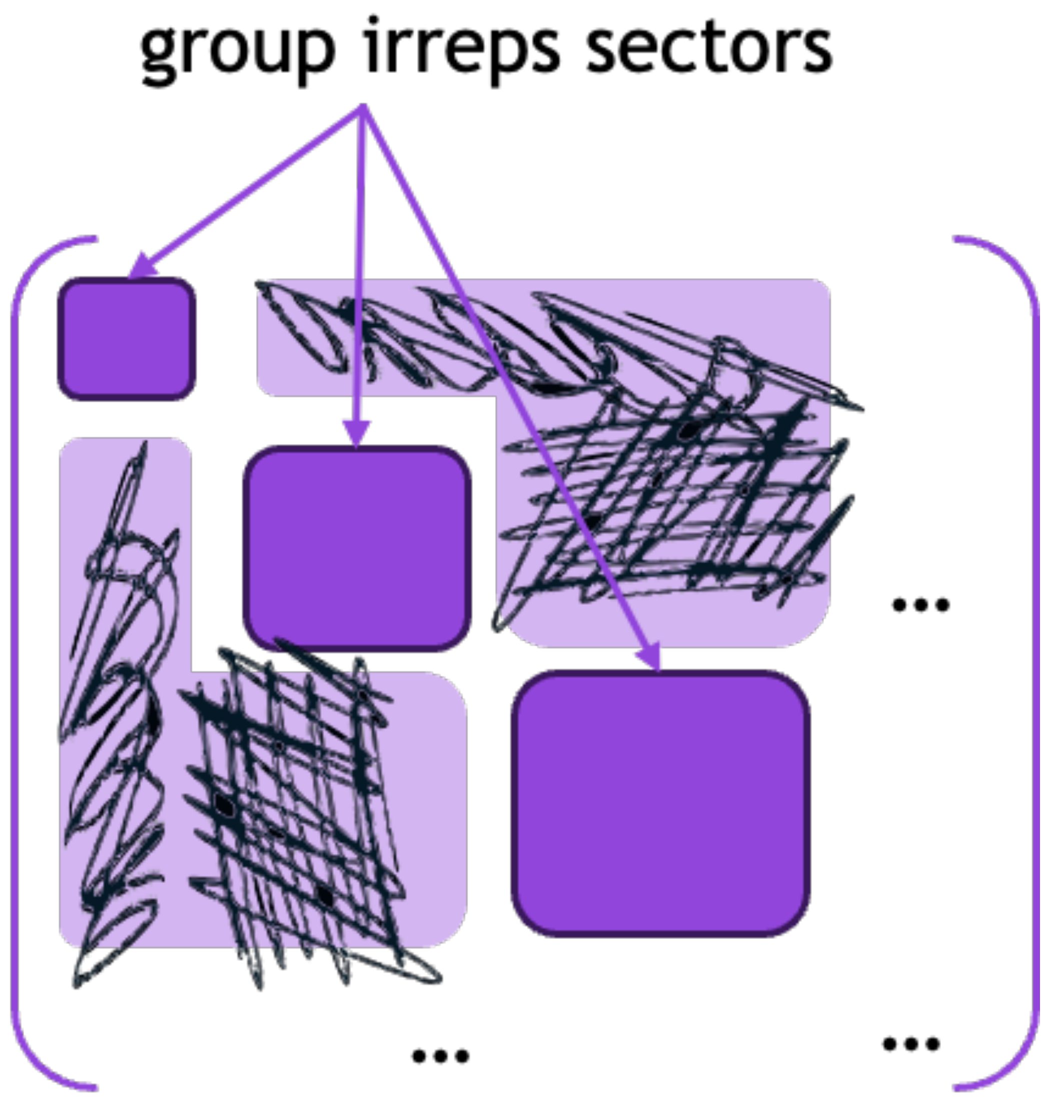
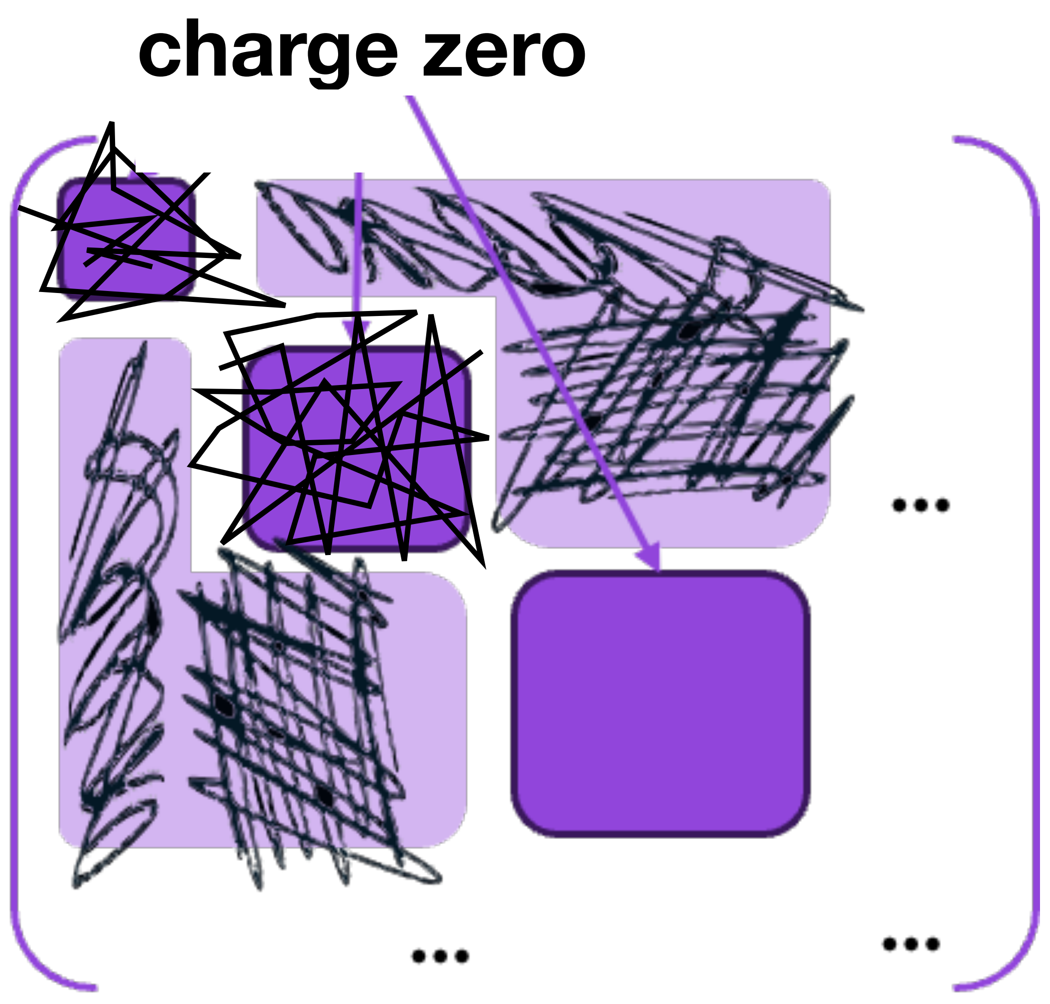
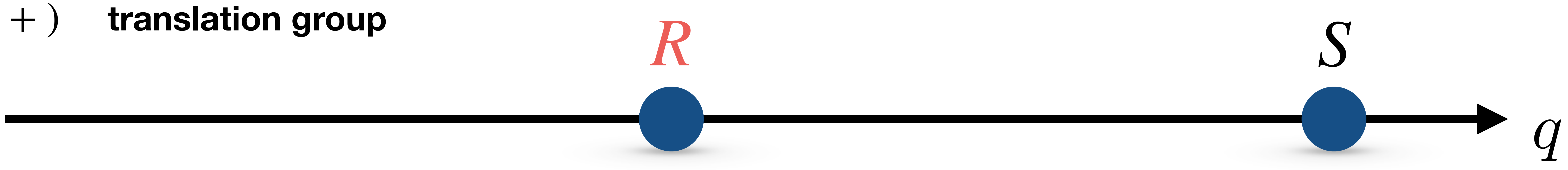


QRF orientation data removed and charge/irrep. data fixed



Example: particles on a line

$G = (\mathbb{R}, +)$ **translation group**



$\mathscr{H}_{\text{kin}} = L^2(\mathbb{R})_R \otimes L^2(\mathbb{R})_S$ **and** $U^g_{RS} \equiv U_{RS}(x) = \exp(-ixP),\ x \in \mathbb{R},\ P = p_R + p_S$

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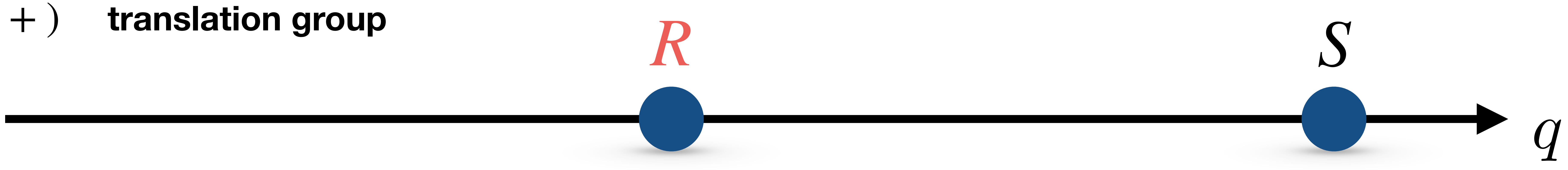

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$\mathscr{A}^G_{\text{kin}} = \langle q_S - q_R, p_S, P \rangle$ $\Rightarrow$ **frame orientation observable q_R eliminated**

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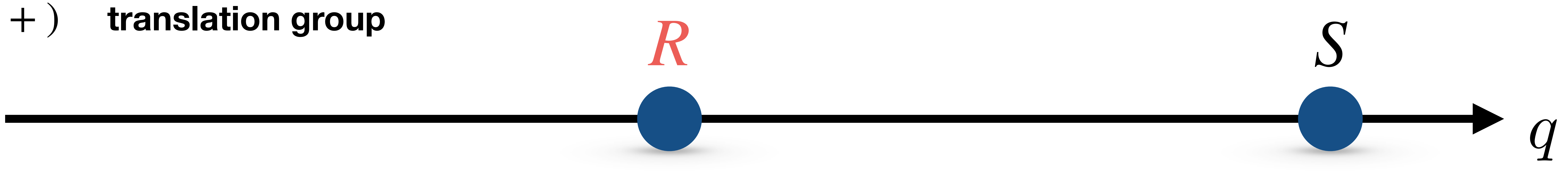

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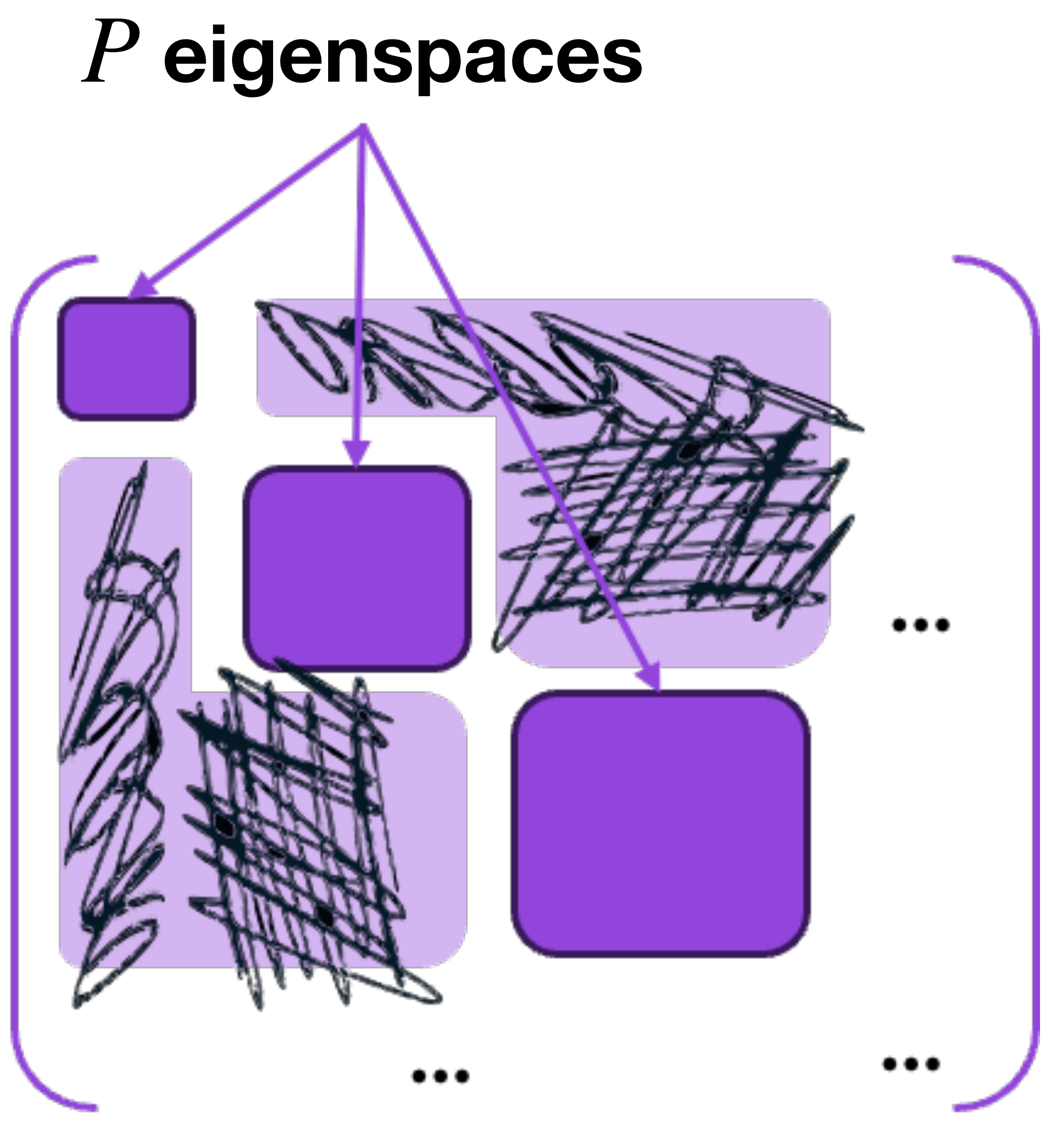

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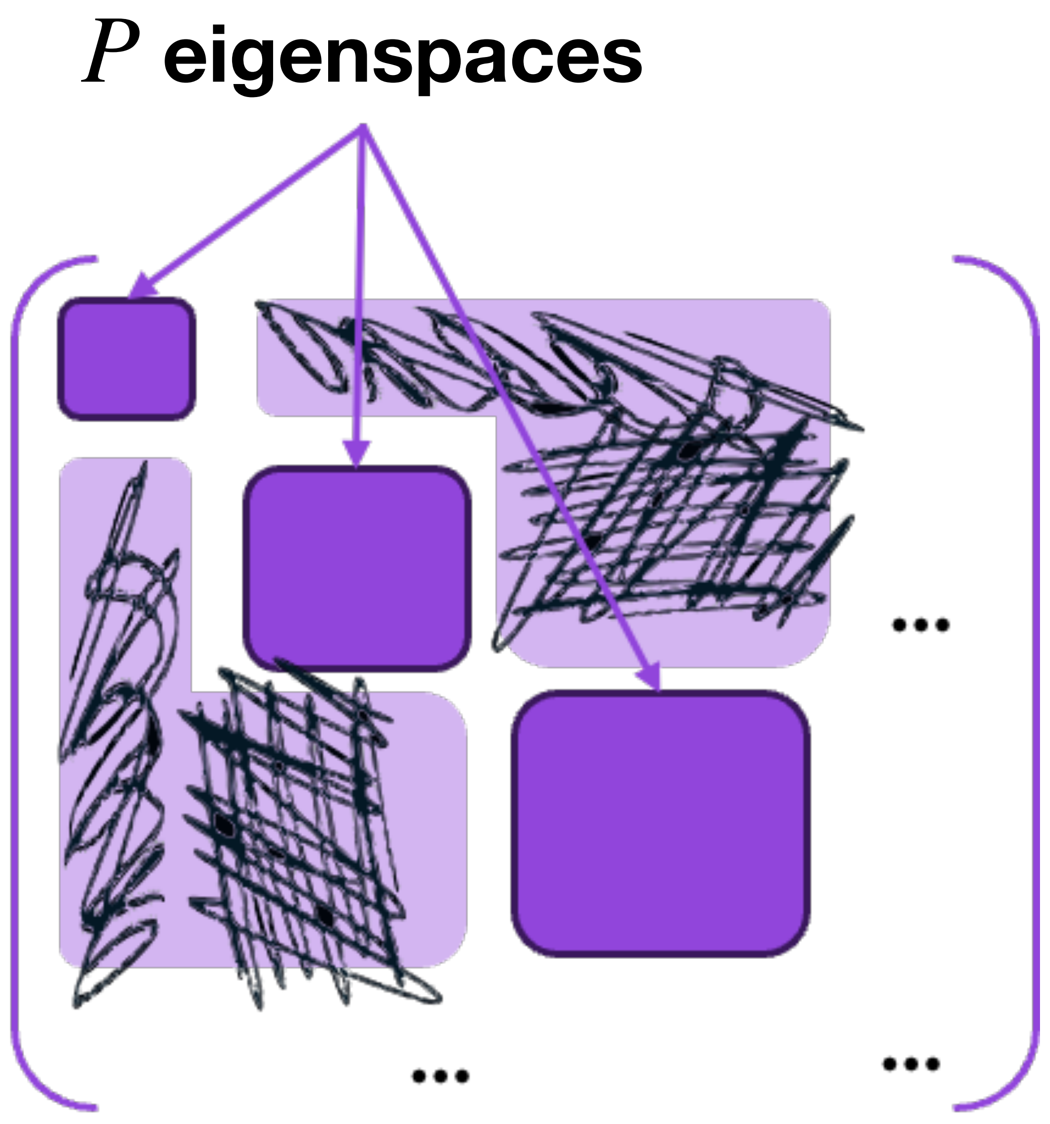

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$\Rightarrow$ **"half of the QRF subsystem" eliminated**

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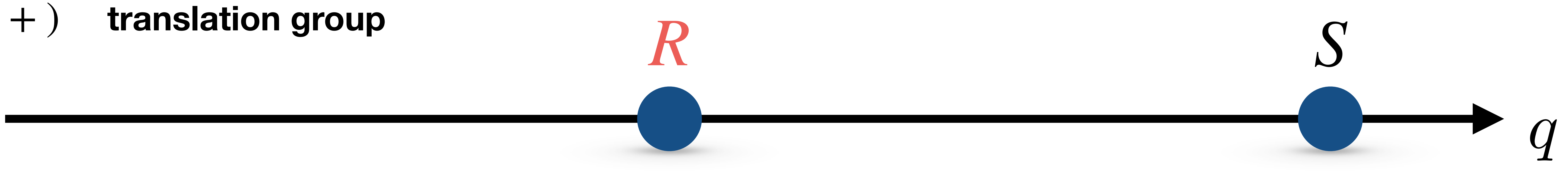

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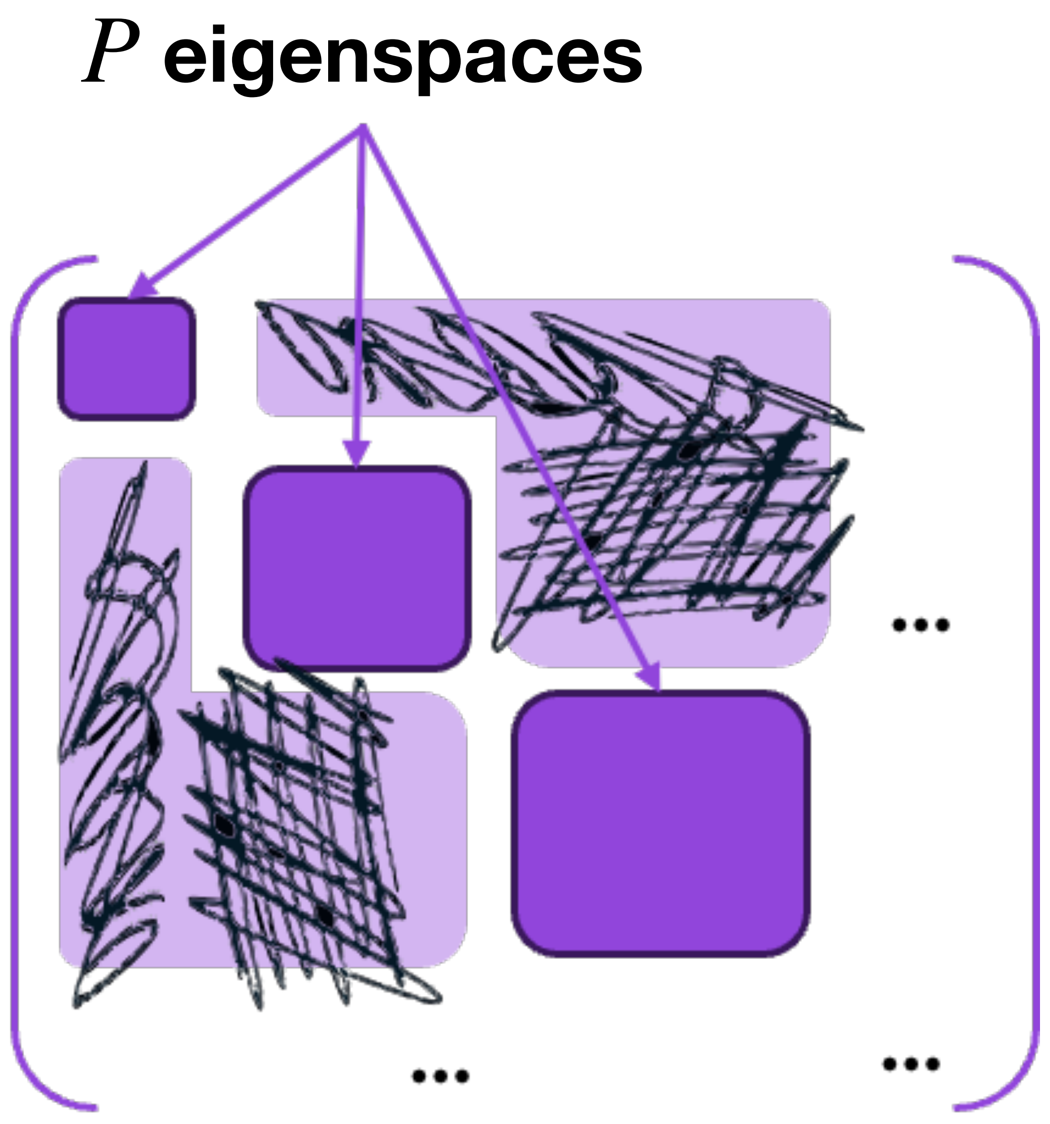
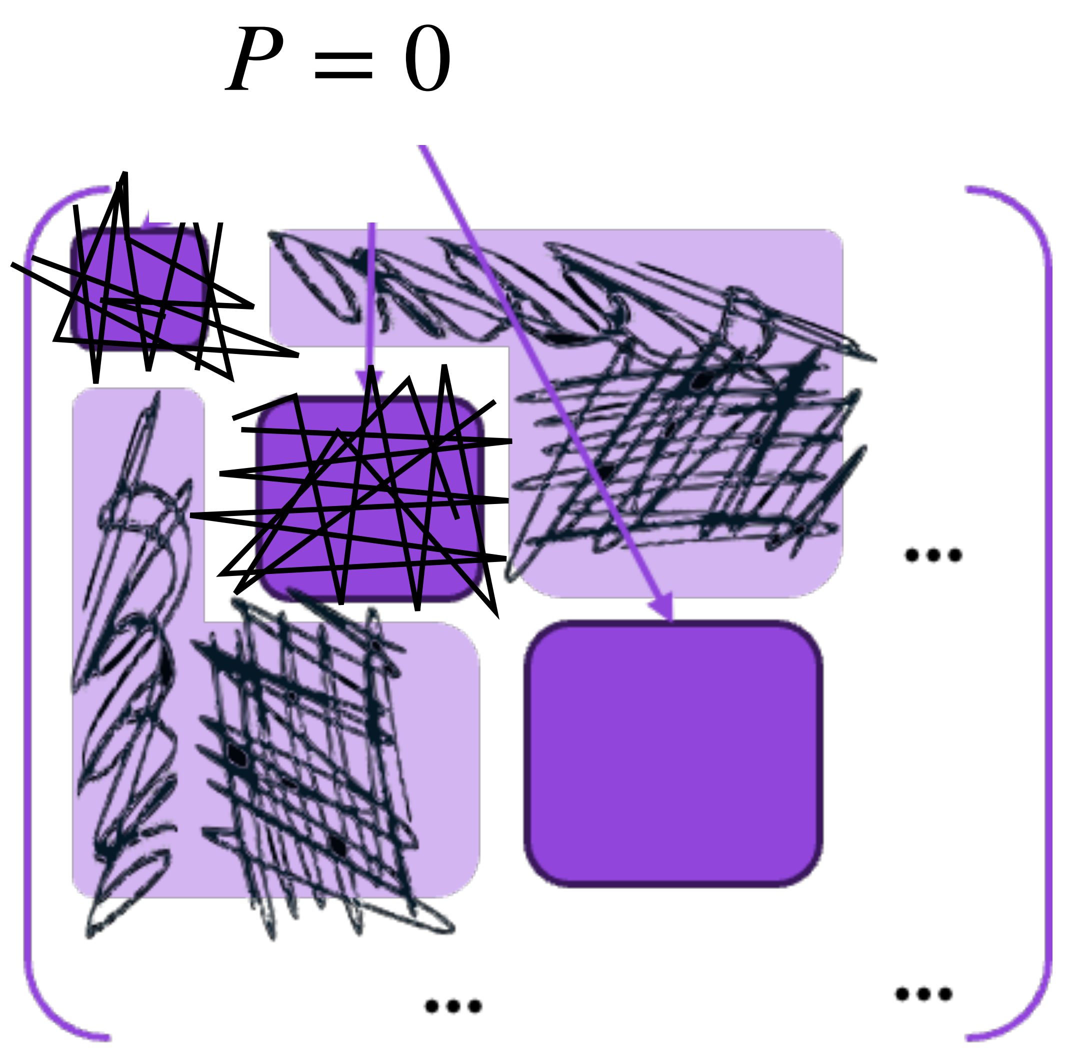


P preserved and conjugate to q_R

$\Rightarrow$ "half of the QRF subsystem" eliminated

strong ext. frame indep.

$$\int_{\mathbb{R}} dx U_{RS}(x)\,\rho \int_{\mathbb{R}} dy U_{RS}(y) =$$



P also eliminated

$\Rightarrow$ "entire QRF subsystem" eliminated $\Rightarrow$ fully redundant

Physical symmetry vs. redundancy

Given: symmetry group G ("external frame transformations")

weak ext. frame indep.: G **physical**

- lab frame transformations
- large gauge transformations in gauge theory and gravity
- corner symmetries in gauge theory and gravity
- …

strong ext. frame indep.: G **param. redundancy**

- small gauge transformations in gauge theory and gravity
- stabilizer symmetry in QEC
- quantum simulations
- operational restrictions
- …

are the orientations of the QRF (in principle) measurable?

Yes

(but requires access to an ext. farme)

No

Potpourri of approaches to QRF covariance

perspectival
[Giacomini, Castro-Ruiz, Brukner ‘17; de la Hamette, Galley ‘20; de la Hamette, Kabel, Castro-Ruiz, Brukner ’22,…]

extra-particle
[Castro-Ruiz, Oreshkov ’21; Garnier, Hausmann, Castro-Ruiz ’25]

effective (semiclassical)
[Bojowald, PH, Tsobanjan ’10; PH, Kubalova, Tsobanjan ’12]

quantum information-theoretic
[Aharonov, Susskind ’67; Bartlett, Rudolph, Spekkens ’06; Krumm, PH, Müller ’20]

algebraic I
[Bojowald, Tsobanjan ’19; ’22; De Vuyst, PH, Tsobanjan *to appear*]

operational
[Fewster et al. ’24; Carette, Glowacki, Loveridge ’23; Glowacki, Loveridge, Waldron ’23]

algebraic II
Ahmad, Klinger, Leigh ‘24

perspective-neutral
[Vanrietvelde, PH, Giacomini, Castro-Ruiz ‘18; de la Hamette, Galley, PH, Loveridge, Müller ‘21; PH, Smith, Lock ‘19; PH, Kotecha, Mele ’23; …]

Potpourri of approaches to QRF covariance

weak ext. frame indep.

perspectival
[Giacomini, Castro-Ruiz, Brukner ‘17; de la Hamette, Galley ‘20; de la Hamette, Kabel, Castro-Ruiz, Brukner ’22,…]

equiv. to strong ext. frame indep. for ideal QRFs

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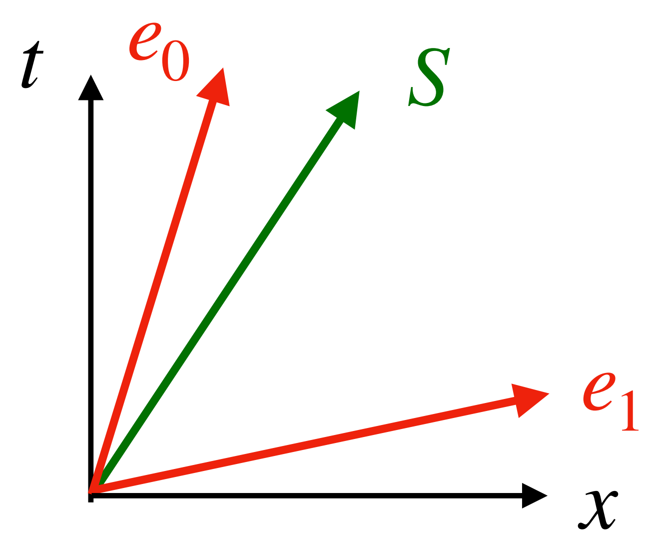
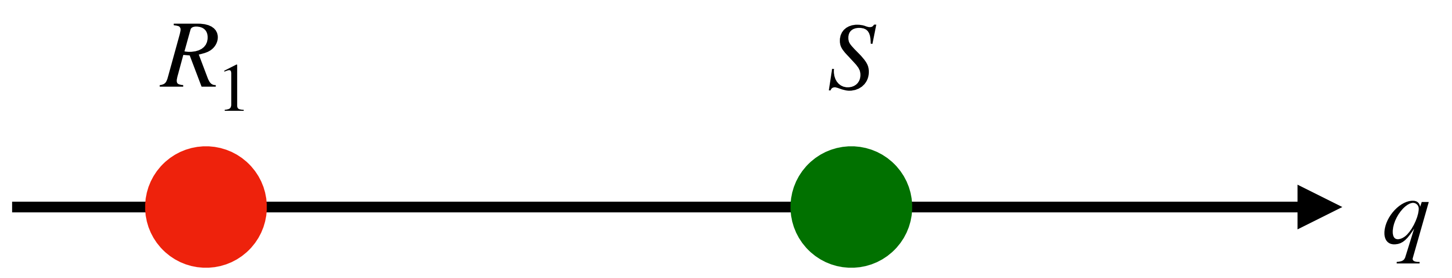
internal frames in SR vs. QT





internally distinguishable configurations (extern frame indep.)	$\{S_\mu S^\mu, \;\; S_\mu e^\mu_a, \;\; e^\mu_a e^b_\mu\}$ **relational/dressed observables**	**weak or strong indep. (dep. on approach)** $[\rho_{\text{inv}}, U^g_{RS}] = 0$ **or** $U^g_{RS}\lvert\psi) = \lvert\psi)$ **relational observables** $\int_G dg U^g_{RS}\left(\lvert g'\rangle\langle g'\rvert \otimes f_S\right)(U^g_{RS})^\dagger$ (f_S when QRF in orientation g')

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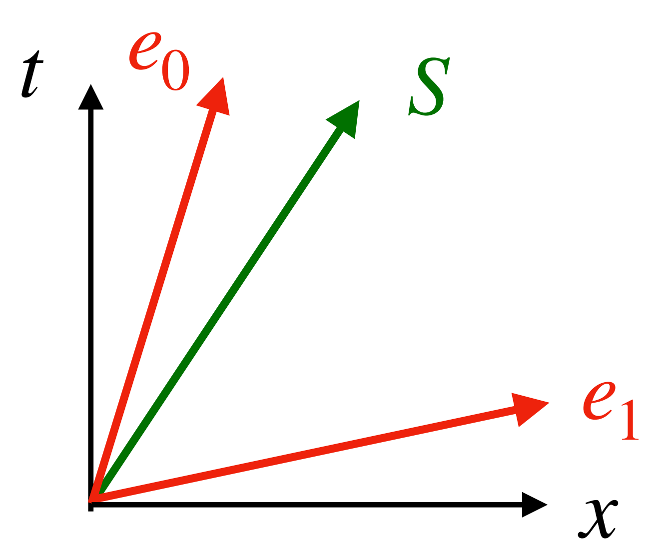

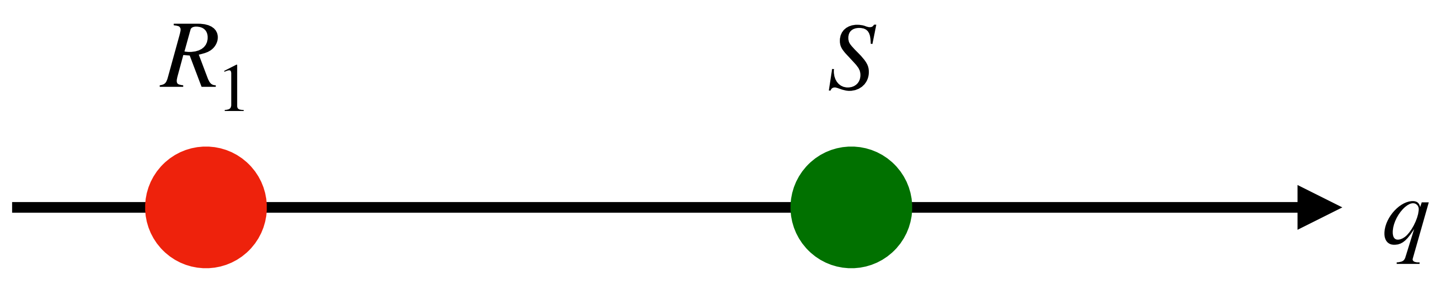

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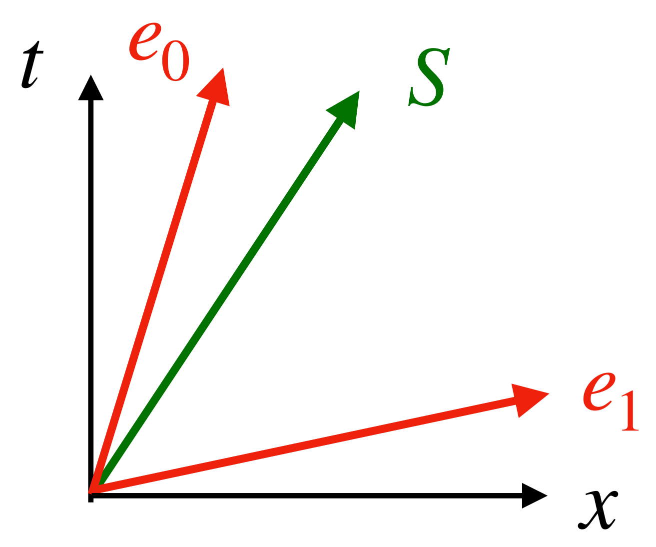
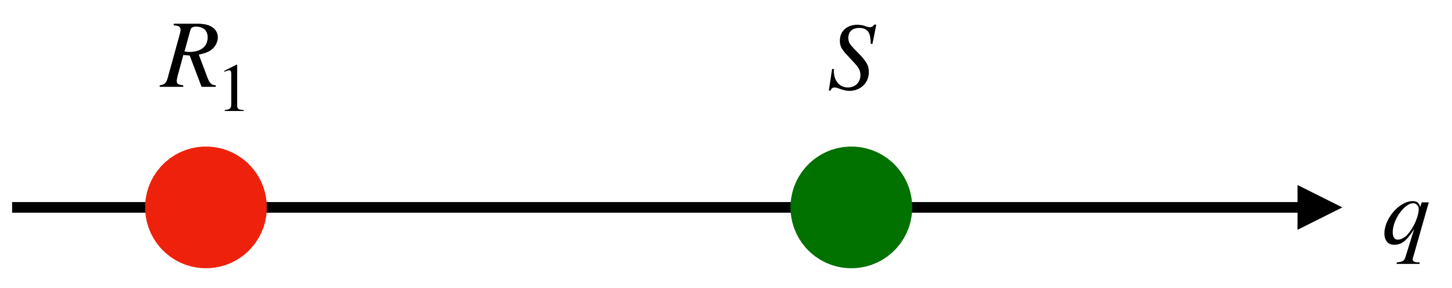
“jumping into internal frame”	**a) relational observables**	

internal frames in SR vs. QT





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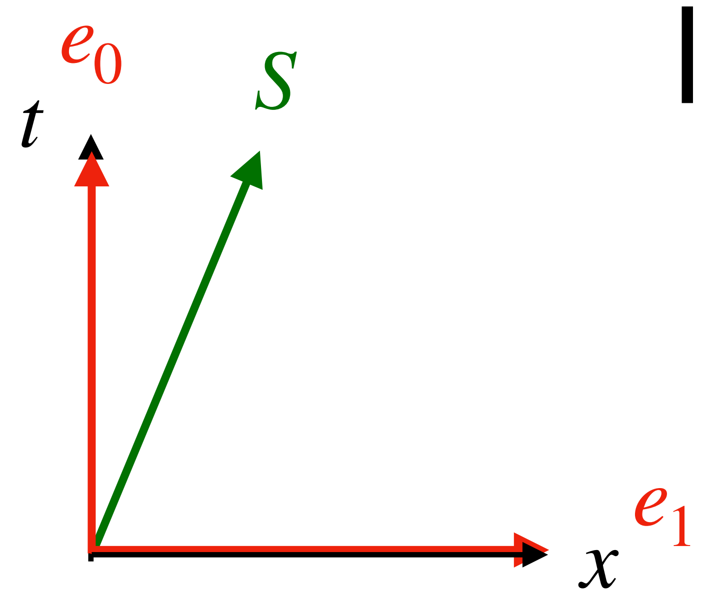

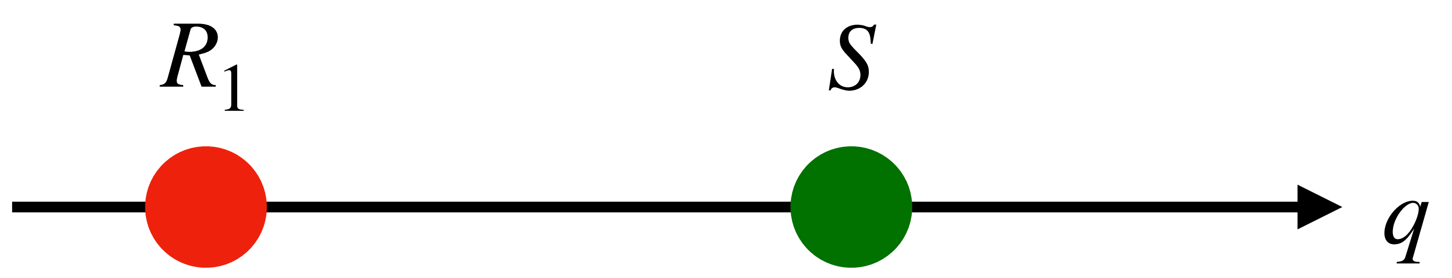

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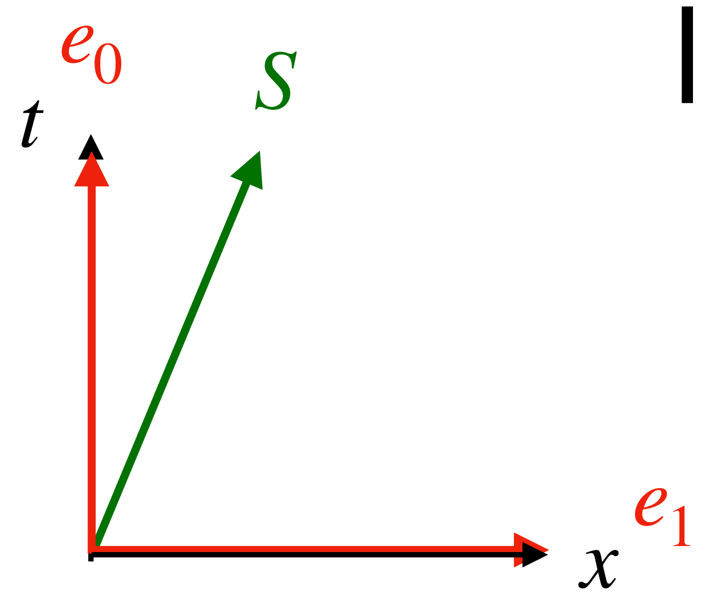
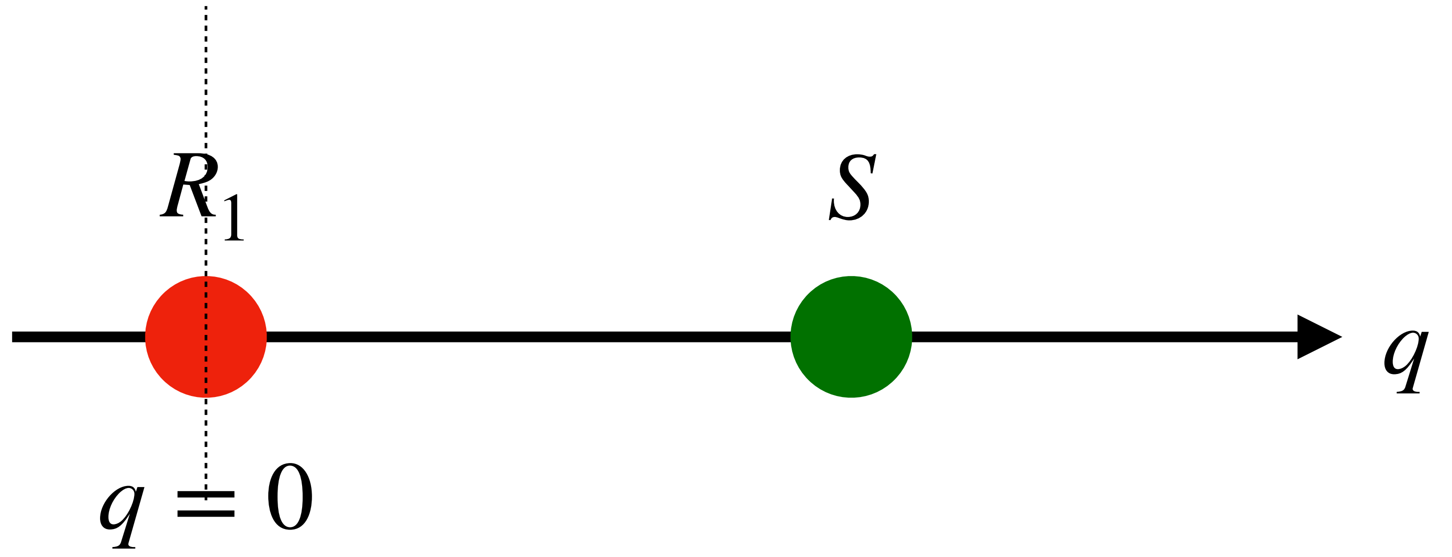


internal frames in SR vs. QT



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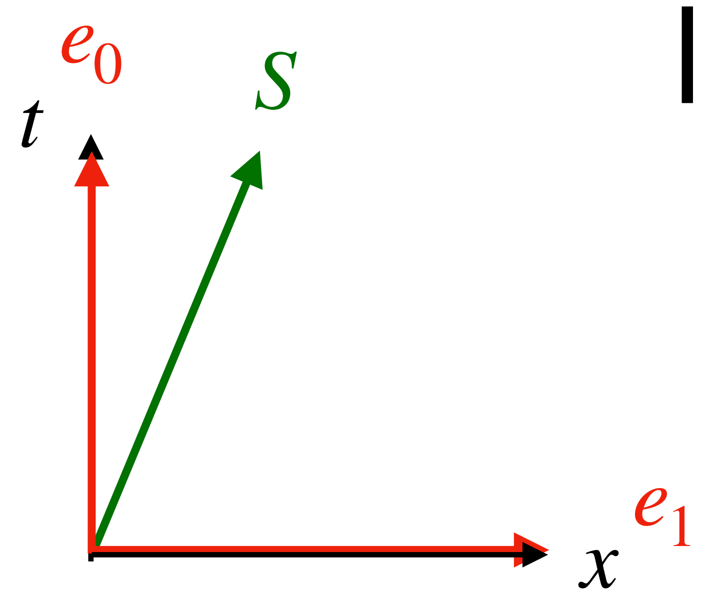

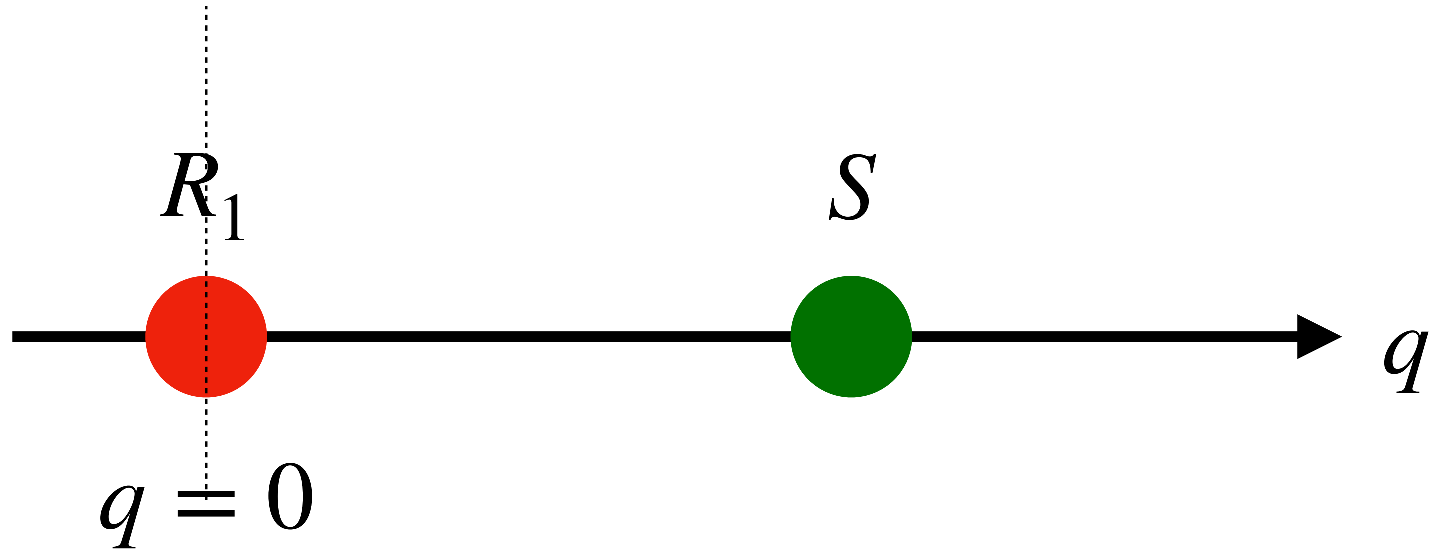

	SR	QT
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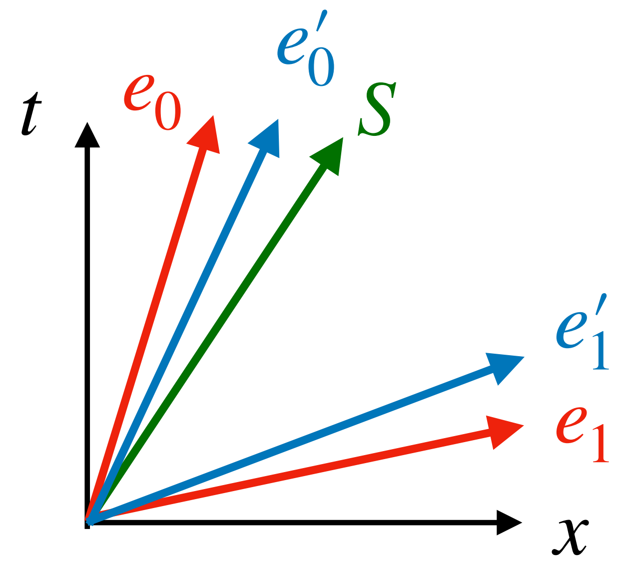
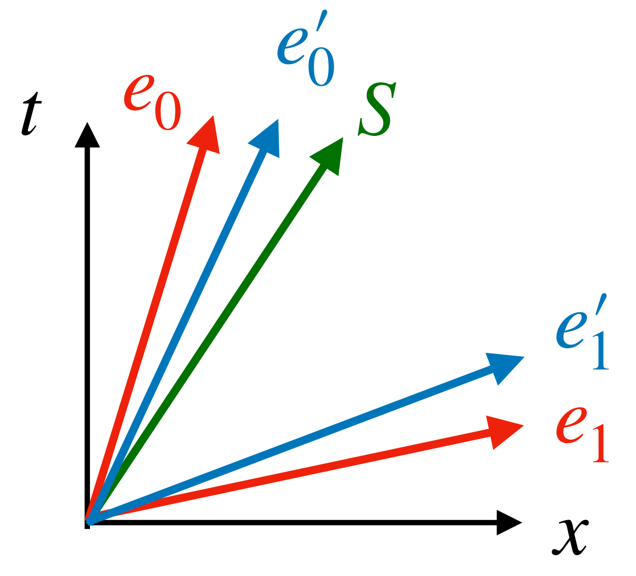
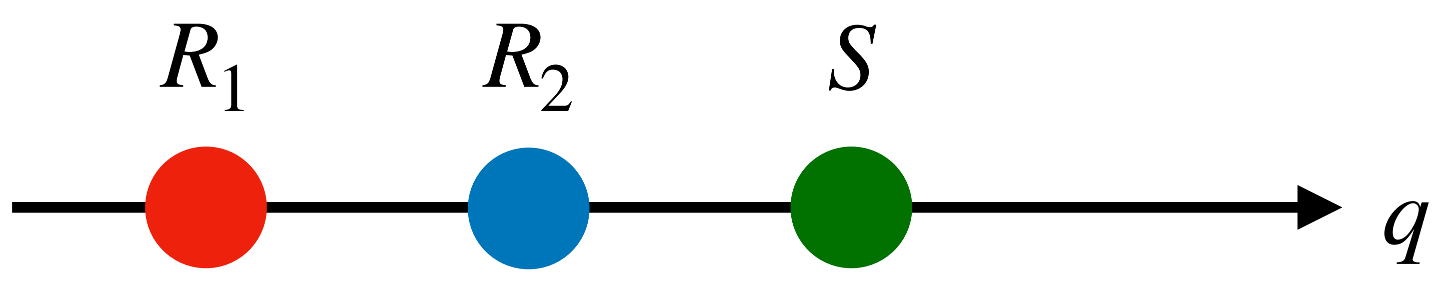
Internal frame changes in SR and the PN approach



$$
\begin{tikzcd}
 & S_a = S_\mu \, e^\mu_a \arrow[dl, "e^\mu_a = \delta^\mu_a"'] \arrow[dr, "e^{\mu'}_{a'} = \delta^{\mu'}_{a'}"] & \\
S_a \equiv S_\mu \arrow[rr, "\Lambda_{\mu'}{}^{\mu}"'] & & S_{a'} \equiv \Lambda_{\mu'}{}^{\mu} S_\mu
\end{tikzcd}
$$

Internal frame changes in SR and the PN approach





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\end{tikzcd}$$

$$\begin{tikzcd}
& \int dg\, U^g_{R_1R_2S} \left(|g'\rangle\langle g'|_1 \otimes I_2 \otimes f_S \right) \left(U^g_{R_1R_2S} \right)^\dagger \arrow[dl, "\hat{\mathscr{R}}_1^e"'] \arrow[dr, "\hat{\mathscr{R}}_2^e"] & \\
\tilde{f}_S \arrow[rr, "\hat{\mathscr{R}}_2^e \circ \left(\hat{\mathscr{R}}_1^e \right)^{-1}"'] & & \Pi_2 \int dg\, U^g_{R_1S} \left(|g'\rangle\langle g'|_1 \otimes f_S \right) \left(U^g_{R_1S} \right)^\dagger
\end{tikzcd}$$

Summary: perspective-neutral QRF framework

externally distinguishable states

$$\mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$$

removing external frame information

$$\Pi_{\text{phys}} = \int dg\, U^g_{12S}$$

perspective-neutral state space
(encodes and links all possible QRFs)

$$\begin{tikzcd}
 & \mathscr{H}_{\text{pn}} \arrow[dl, "\mathscr{R}^e_1 = \langle e|_{R_1} \oplus I_{2S}"'] \arrow[dr, "\mathscr{R}^e_2 = \langle e|_2 \oplus I_{1S}"] & \\
\mathscr{H}_{|R_1} \arrow[rr, "\mathscr{R}^e_2 \circ \left(\mathscr{R}^e_1\right)^{-1}"'] & & \mathscr{H}_{|R_2}
\end{tikzcd}$$

QRF covariance here

internal perspective rel. to R_1

internal perspective rel. to R_2

QRF changes as
“quantum coordinate transformations”

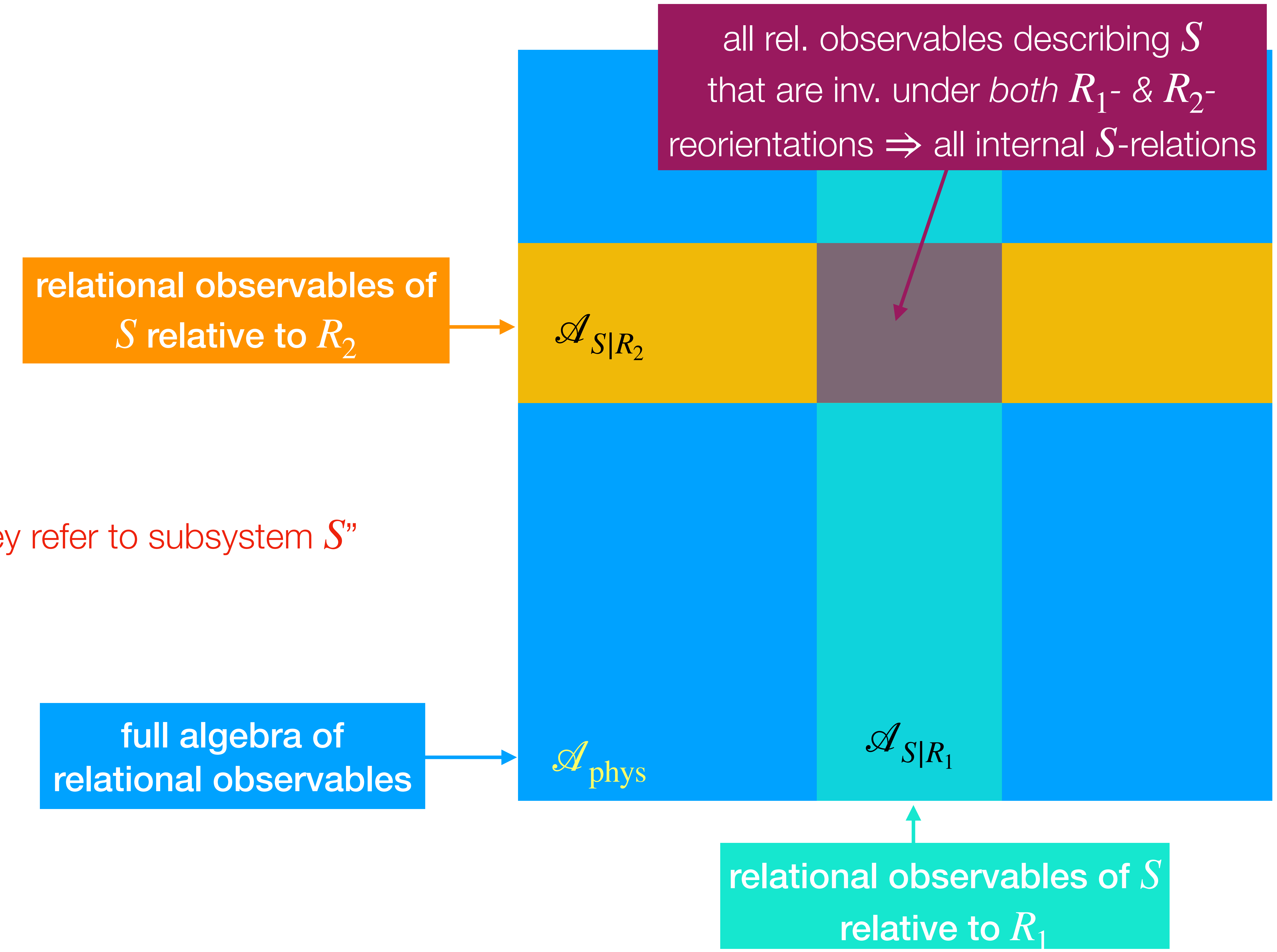
Subsystem relativity

[Ahmad, Galley, PH, Lock, Smith ’21; Araújo-Regado, PH, Sartini ’25;
Castro-Ruiz, Oreshkov ’21; PH, Kotecha, Mele ’23; De Vuyst, Eccles, PH, Kirklin ’24; ….]

Relativity of subsystems

$$\mathscr{A}_{S|R_1} \cap \mathscr{A}_{S|R_2} = \mathscr{A}_S^G = \{\mathrm{internal\ rel.obs.of\ S}\}$$

"frames R_1 and R_2 mean different gauge inv. DoFs when they refer to subsystem S"



Relativity of subsystems

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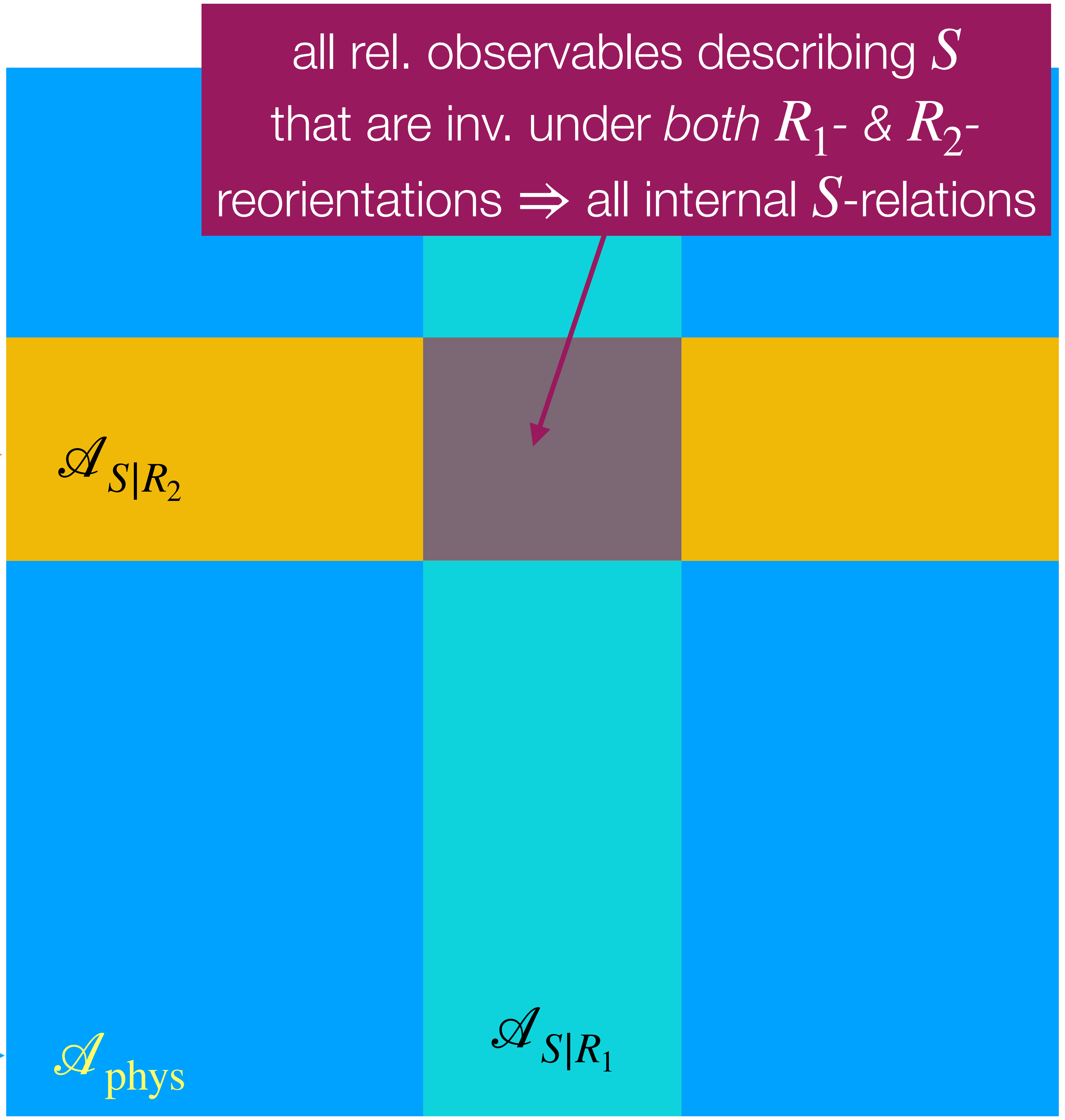
relational observables of S relative to R_2

full algebra of relational observables



relational observables of S relative to R_1

"frames R_1 and R_2 mean different gauge inv. DoFs when they refer to subsystem S"

$\Rightarrow$ **correlations, entropies, thermodynamic properties, … become QRF-dependent**

[systematic investigation in PH, Kotecha, Mele '23]

QRF perspectives are TPSs on the PN-space

Kotecha, Mele, PH *'23;*
De Vuyst, Eccles, PH, Kirklin *'24;*
Araújo-Regado, PH, Sartini '25

TPS relative to external frame

$$\mathscr{H}_{\mathrm{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$$

removing external frame information $\Big\downarrow \Pi_{\mathrm{phys}}$

perspective/TPS-neutral state space
(does not inherit kin. TPS)

$$\mathscr{H}_{\mathrm{pn}}$$

ideal (sharp) QRFs ($\langle g \mid g' \rangle = \delta(g, g')$)

QRF perspective is a gauge-inv. subsystem partition

$$\begin{tikzcd} & \mathscr{H}_{\mathrm{pn}} \arrow[dl, "\mathscr{R}_1^e"'] \arrow[dr, "\mathscr{R}_2^e"] & \\ \mathscr{H}_{|R_1} = \mathscr{H}_{R_2} \otimes \mathscr{H}_S \arrow[rr, "\mathscr{R}_2^e \circ \left(\mathscr{R}_1^e\right)^{-1}"'] & & \mathscr{H}_{|R_2} = \mathscr{H}_{R_1} \otimes \mathscr{H}_S \end{tikzcd}$$

internal TPS rel. to R_1

internal TPS rel. to R_2

QRF transformations are **changes of TPS**

QRF perspectives are TPSs on the PN-space

Kotecha, Mele, PH '23;
De Vuyst, Eccles, PH, Kirklin '24;
Araújo-Regado, PH, Sartini '25

TPS relative to external frame

removing external frame information

perspective/TPS-neutral state space
(does not inherit kin. TPS)

$$\begin{tikzcd}
 & \mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1}\otimes\mathscr{H}_{R_2}\otimes\mathscr{H}_S \arrow[d, "\Pi_{\text{phys}}"] & \\
 & \mathscr{H}_{\text{pn}} \arrow[dl, "\mathscr{R}_1^e"'] \arrow[dr, "\mathscr{R}_2^e"] & \\
\mathscr{H}_{|R_1} = \mathscr{H}_{R_2}\otimes\mathscr{H}_S \arrow[rr, "\mathscr{R}_2^e\circ\left(\mathscr{R}_1^e\right)^{-1}"'] & & \mathscr{H}_{|R_2} = \mathscr{H}_{R_1}\otimes\mathscr{H}_S
\end{tikzcd}$$

ideal (sharp) QRFs ($\langle g\,|\,g'\rangle = \delta(g,g')$)

QRF perspective is a gauge-inv. subsystem partition

generated by relational observable algebras

$$\mathscr{A}_{\text{pn}} \simeq \mathscr{A}_{S|R_1}\otimes\mathscr{A}_{R_2|R_1} \simeq \mathscr{A}_{S|R_2}\otimes\mathscr{A}_{R_1|R_2}$$

internal TPS rel. to R_1

internal TPS rel. to R_2

QRF transformations are **changes of TPS**

Example: particles on a line

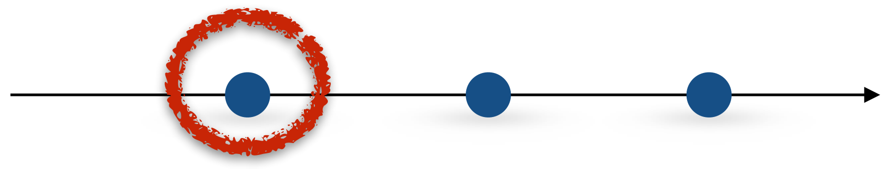
Vanrietvelde, PH, Giacomini, Castro-Ruiz ‘19

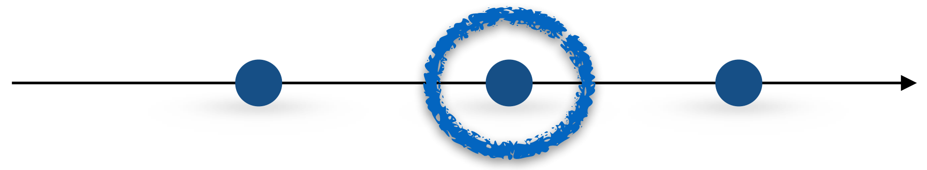
perspective-neutral space

$$
\begin{tikzcd}
 & \mathscr{H}_{\mathrm{pn}} \arrow[dl, "\mathcal{R}_1^e = \langle q_1 = 0| \oplus 1_{2S}"'] \arrow[dr, "\mathcal{R}_2^e = \langle q_2 = 0| \oplus 1_{1S}"] & \\
\mathscr{H}_{|R_1} = L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S \arrow[rr] & & \mathscr{H}_{|R_2} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_S
\end{tikzcd}
$$

internal TPS rel. to R_1

internal TPS rel. to R_2





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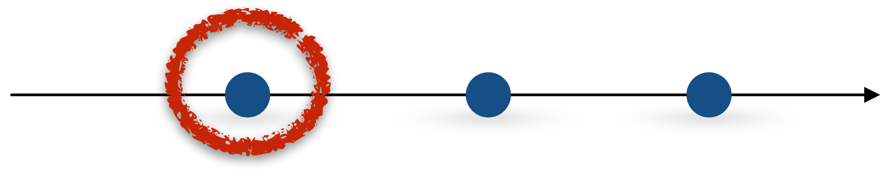
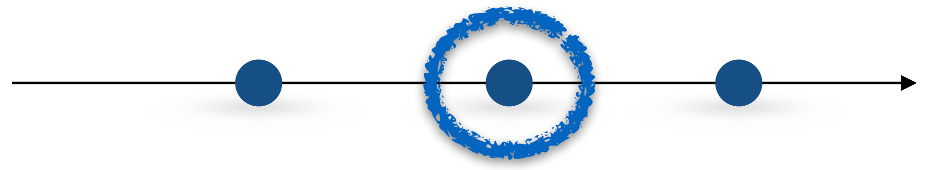
$$\begin{tikzcd} & \mathcal{H}_{\rm pn} \arrow[dl, "\mathcal{R}_1^e = \langle q_1 = 0| \oplus I_{2S}"'] \arrow[dr, "\mathcal{R}_2^e = \langle q_2 = 0| \oplus I_{1S}"] & \\ \mathcal{H}_{|R_1} = L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S \arrow[rr] & & \mathcal{H}_{|R_2} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_S \end{tikzcd}$$

internal TPS rel. to R_1

internal TPS rel. to R_2

$$\mathcal{R}_2^e \circ \left(\mathcal{R}_1^e\right)^{-1} = \mathbb{F}_{12} \int dq\, |-q\rangle\langle q|_{R_2} \otimes U_S(q)$$





Example: particles on a line

Vanrietvelde, PH, Giacomini, Castro-Ruiz ‘19

perspective-neutral space

superposition and entanglement of subsystem S QRF relative

$$
\begin{array}{ccc}
 & \mathscr{H}_{\text{pn}} & \\
\swarrow^{\mathscr{R}_1^e = \langle q_1 = 0 | \otimes I_{2S}} & & \searrow^{\mathscr{R}_2^e = \langle q_2 = 0 | \otimes I_{1S}} \\
\mathscr{H}_{|R_1} = L^2(\mathbb{R})_{R_2} \otimes L^2(\mathbb{R})_S & \longrightarrow & \mathscr{H}_{|R_2} = L^2(\mathbb{R})_{R_1} \otimes L^2(\mathbb{R})_S
\end{array}
$$

internal TPS rel. to R_1

internal TPS rel. to R_2

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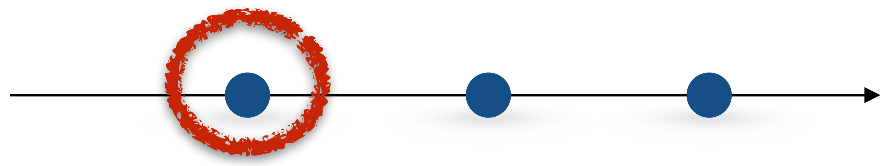
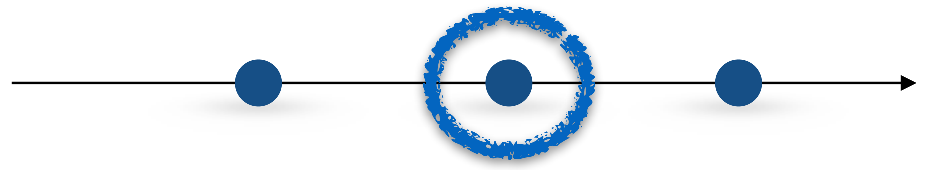


$(|q_1\rangle_{R_2} + |q_2\rangle_{R_2}) \otimes |x\rangle_S$

superposition + product



$| -q_1\rangle_{R_1} \otimes |x - q_1\rangle_S + | -q_2\rangle_{R_1} \otimes |x - q_2\rangle_S$

entanglement

QRF perspectives are TPSs on the PN-space

Kotecha, Mele, PH *'23;*
De Vuyst, Eccles, PH, Kirklin *'24;*
Araújo-Regado, PH, Sartini '25

fuzzy QRFs

TPS relative to external frame

$$\mathscr{H}_{\mathrm{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$$

removing external frame information

$$\Big\downarrow \, \Pi_{\mathrm{phys}}$$

perspective/TPS-neutral state space
(does not inherit kin. TPS)

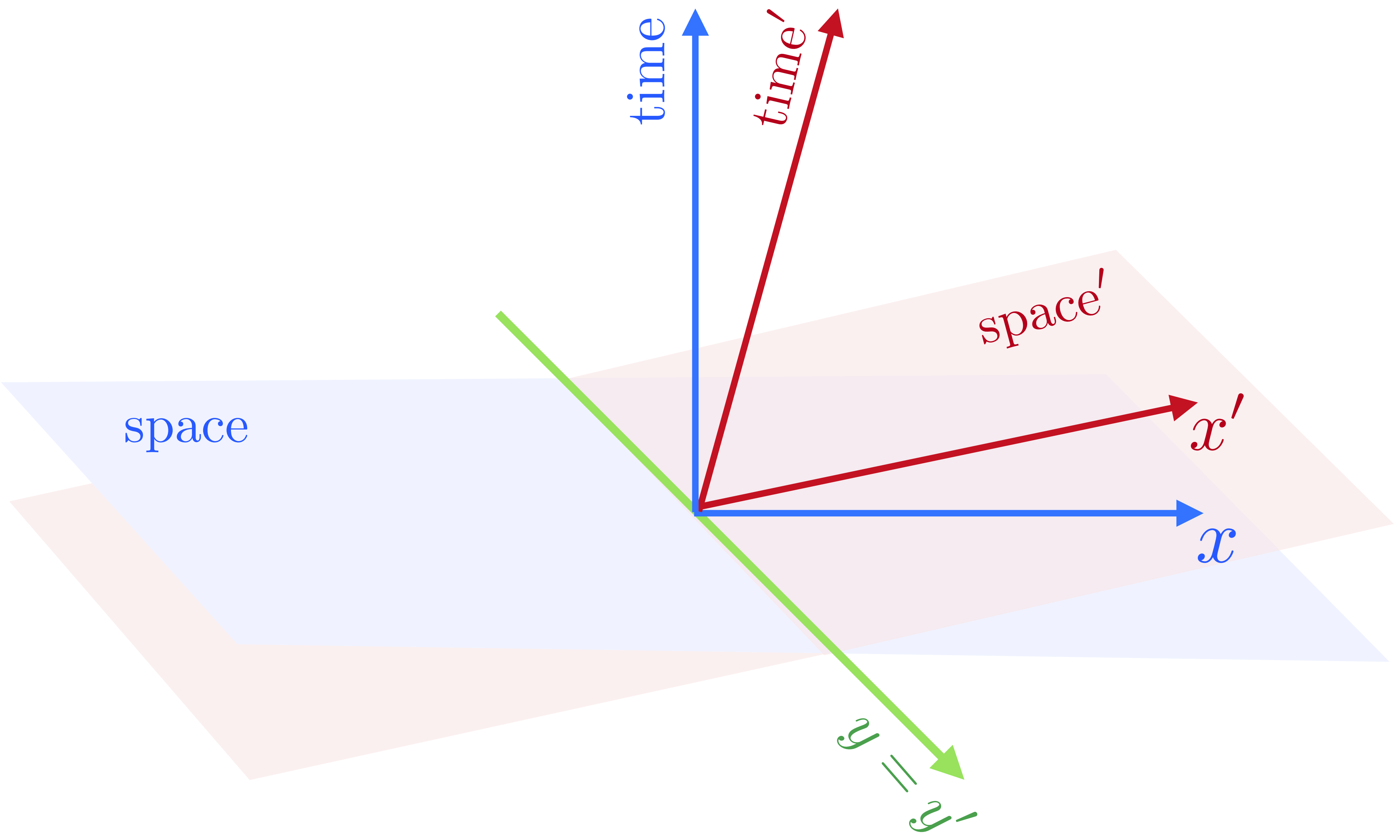
$$\begin{tikzcd}
 & \mathscr{H}_{\mathrm{pn}} \arrow[dl, "\mathscr{R}_1^e"'] \arrow[dr, "\mathscr{R}_2^e"] & \\
\mathscr{H}_{|R_1} = \bigoplus_i \mathscr{H}^i_{R_2} \otimes \mathscr{H}^i_S \arrow[rr, "\mathscr{R}_2^e \circ \left(\mathscr{R}_1^e\right)^{-1}"'] & & \mathscr{H}_{|R_2} = \bigoplus_j \mathscr{H}^j_{R_1} \otimes \mathscr{H}^j_S
\end{tikzcd}$$

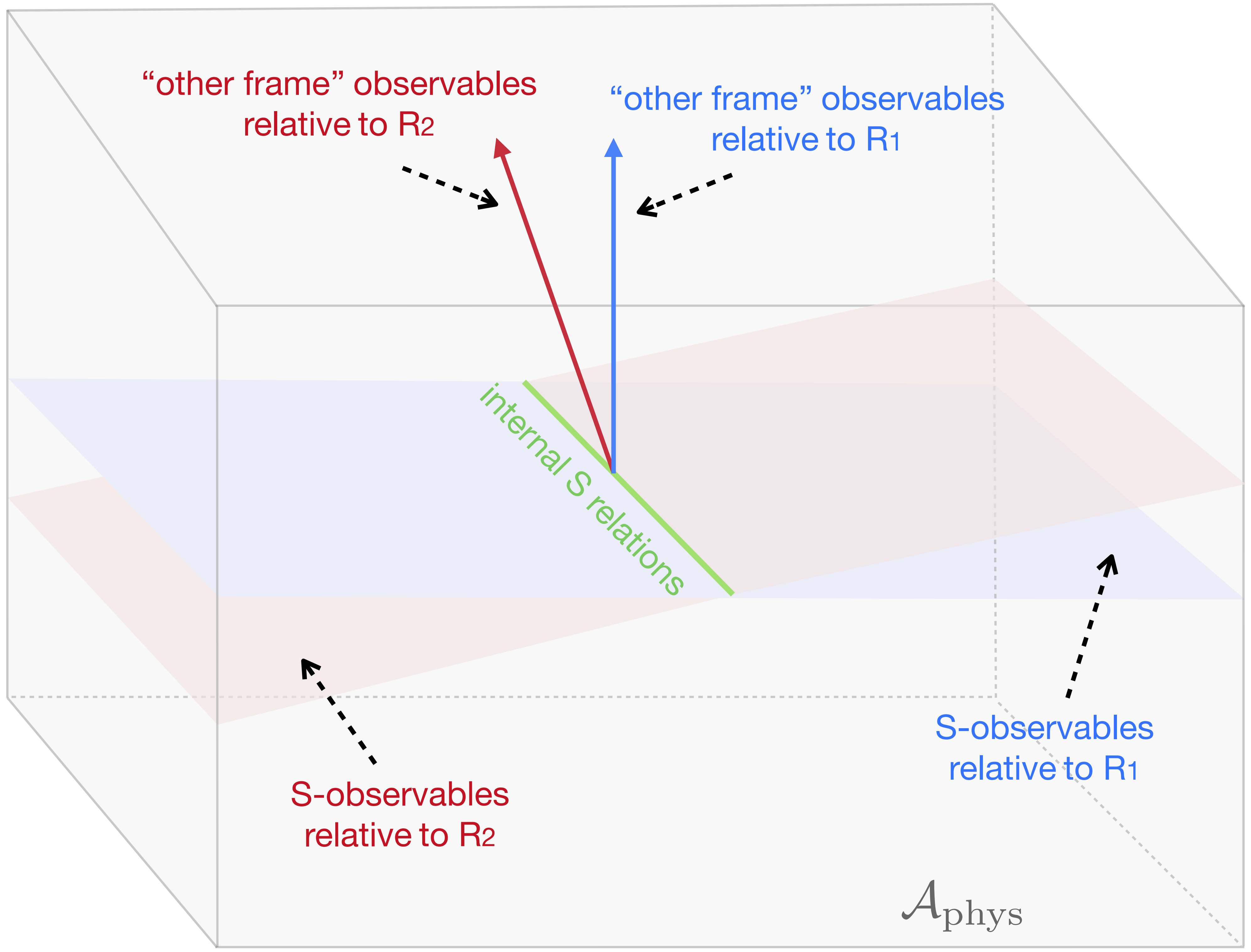
internal TPS rel. to R_1

internal TPS rel. to R_2

QRF transformations are **changes of TPS**

Relativity of simultaneity vs. subsystem relativity





⇒ physical consequences should be seen in similar light:
Just like the relativity of simultaneity is the root of all characteristic special relativistic phenomena,
so is subsystem relativity root of all novel QRF relative effects

Gauge and physical QRFs in lattice gauge theory

[Araújo-Regado, PH, Sartini 2506.23459]

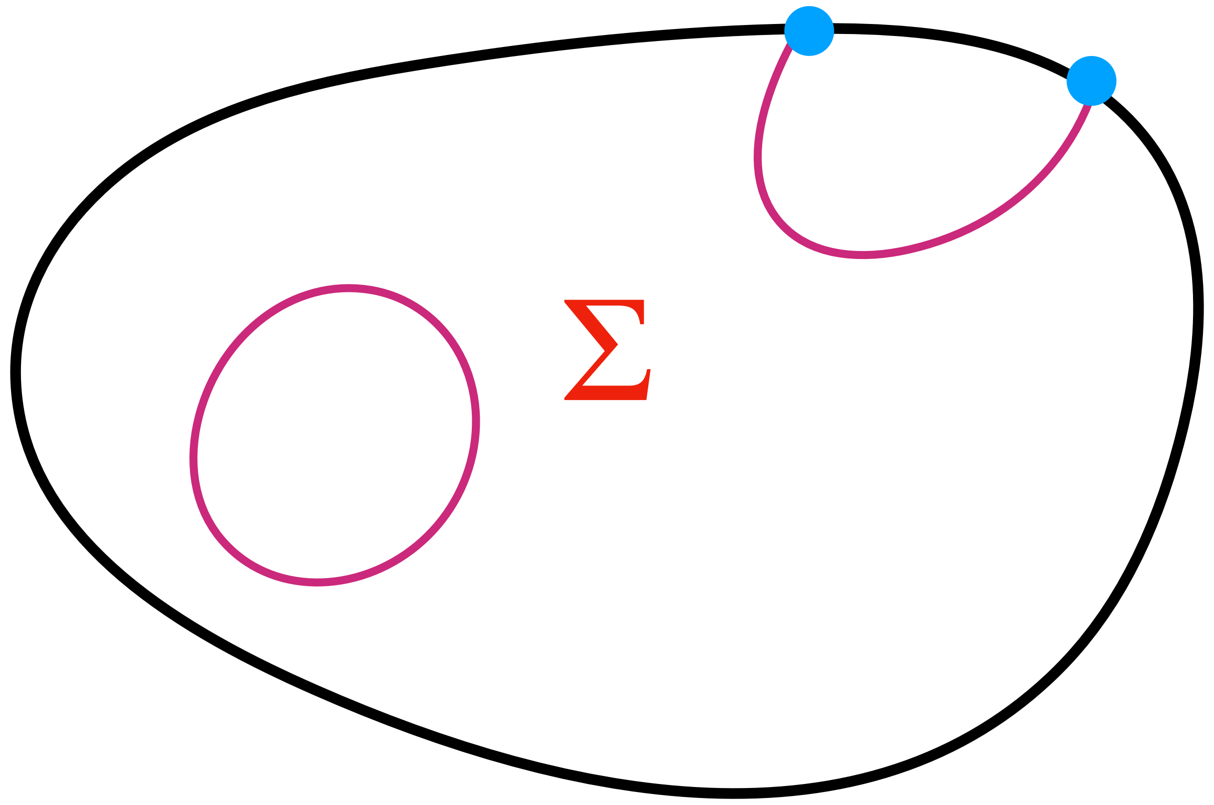
Edge modes and corner symmetries

small gauge transf. generated by Gauß law (redundancy)

$$C[\alpha] = \int_\Sigma \mathrm{Tr}(\alpha \mathrm{d_A} \star \mathrm{F})$$

electric corner symmetries (physical)

$$Q[\rho] = \int_{\partial\Sigma} \mathrm{Tr}(\rho \star \mathrm{F}_\Phi)$$



edge mode Φ

[Donnelly, Freidel ’16; Geiller, Jai-akson ’19; Ball, Law, Wong ’24;…]

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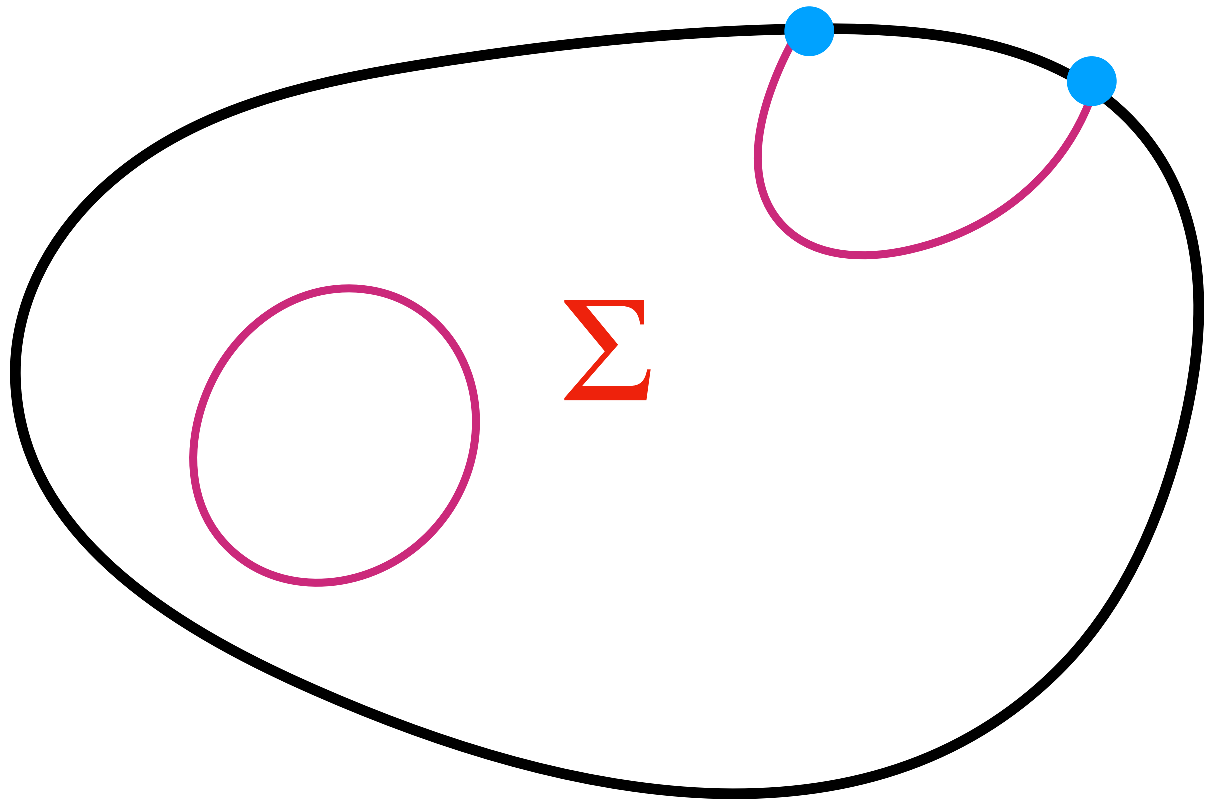

edge mode Φ

[Donnelly, Freidel ’16; Geiller, Jai-akson ’19; Ball, Law, Wong ’24;…]

⇒ how do you realize them?

Edge modes as QRFs

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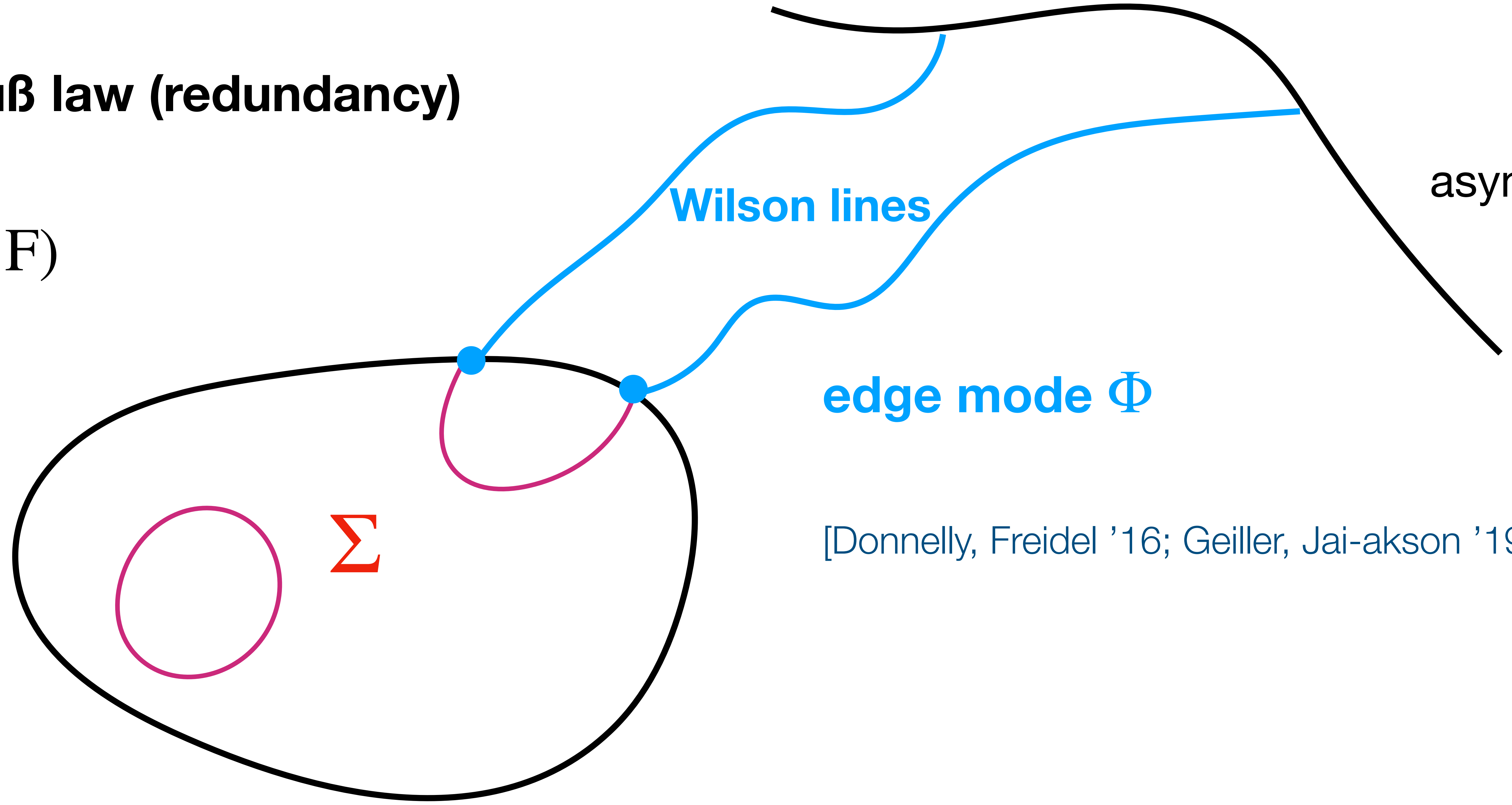

[Donnelly, Freidel ’16; Geiller, Jai-akson ’19; Ball, Law, Wong ’24;…]

$\Rightarrow$ **how do you realize them?** **as QRFs**

extrinsic edge mode QRFs: **e.g. via extrinsic Wilson lines** [Carrozza, PH ’21; Araújo-Regado, PH, Sartini, Tomova ’24]

Edge modes as QRFs

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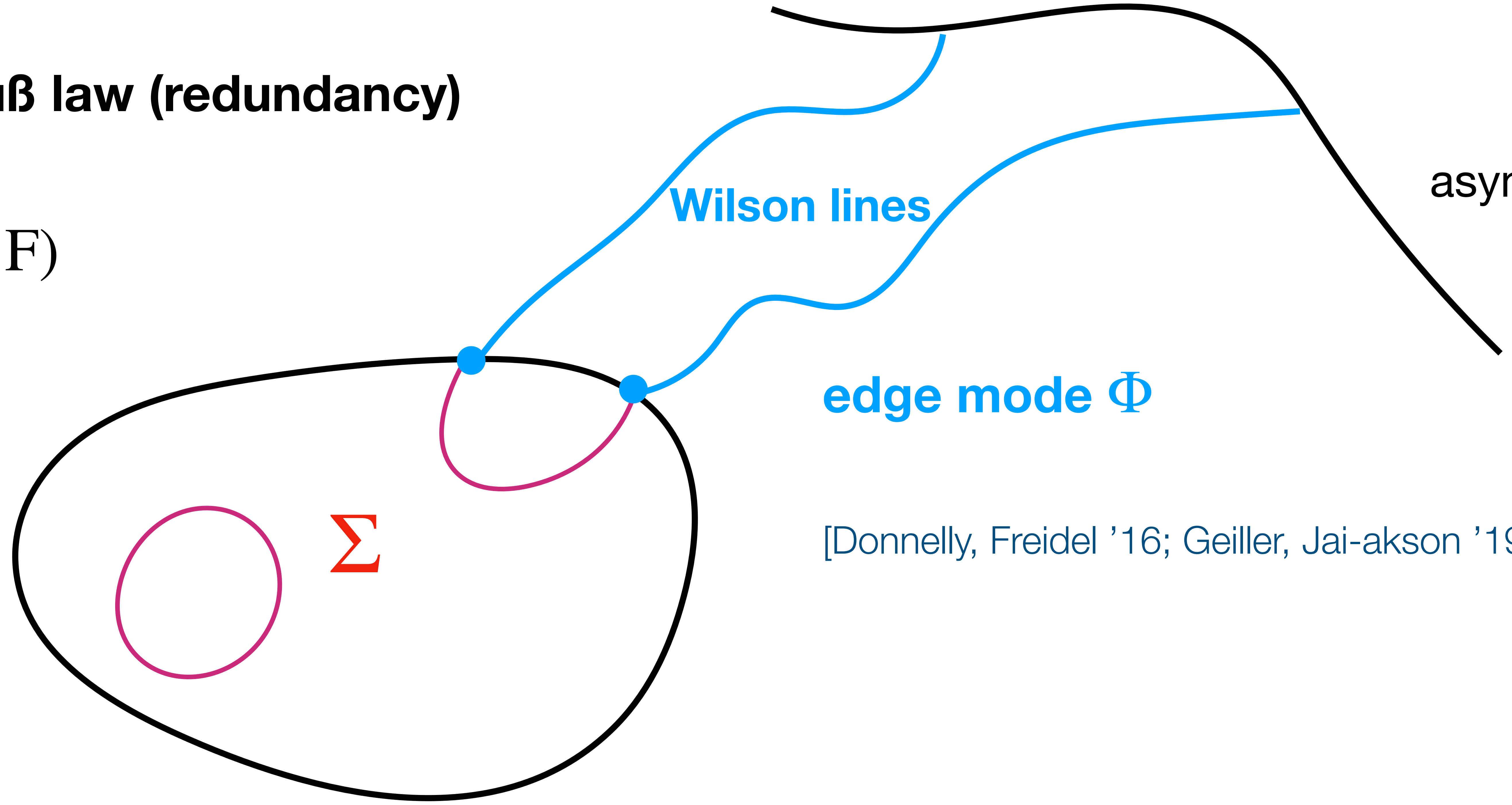

[Donnelly, Freidel ’16; Geiller, Jai-akson ’19; Ball, Law, Wong ’24;…]

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intrinsic edge mode QRFs: **e.g. via Wilson lines anchored on the corner $\partial\Sigma$ or a Hodge decomp.**

[Araújo-Regado, PH, Sartini, Tomova ’24]

Edge modes as QRFs

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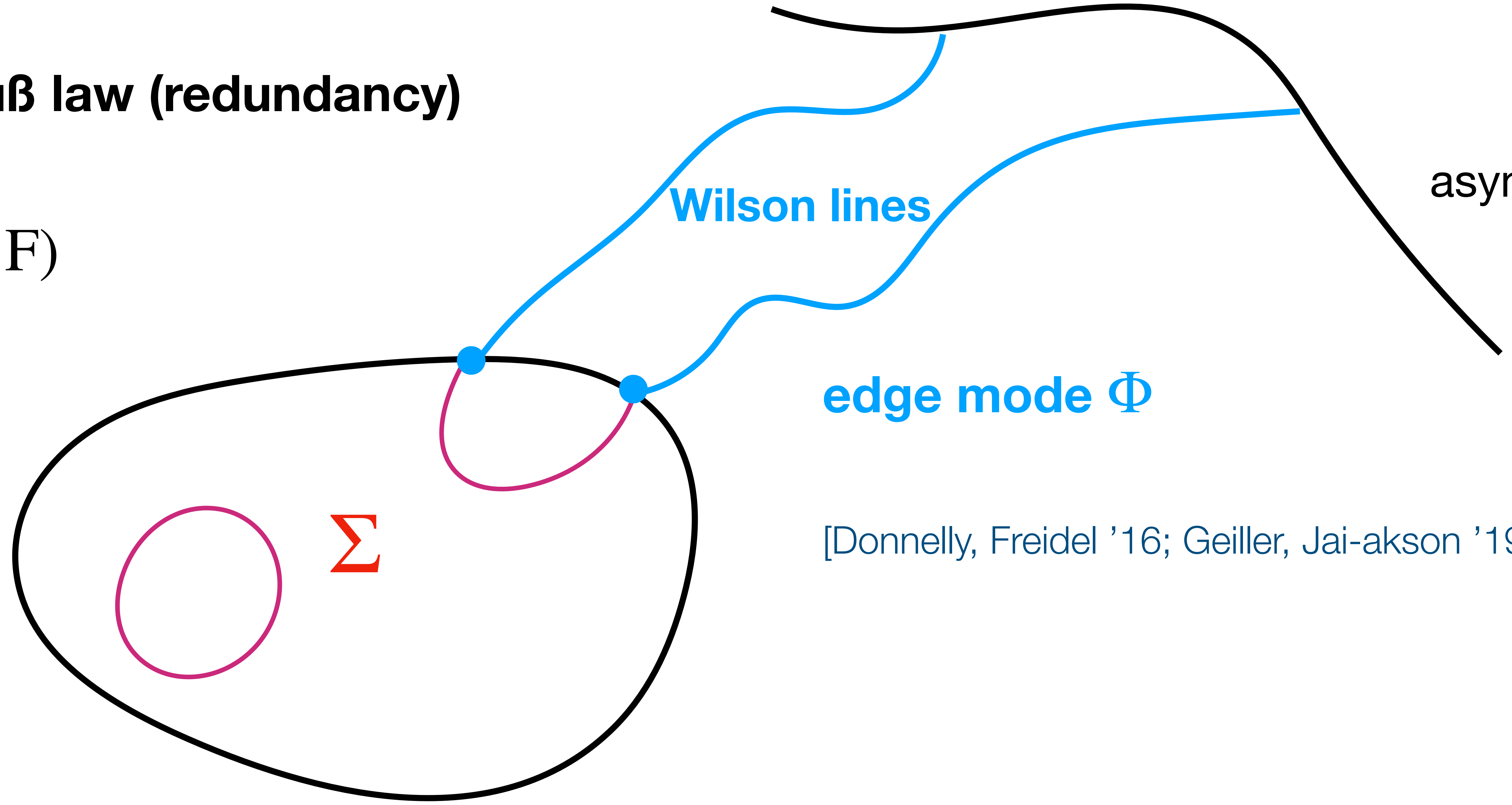
⇒ are reorientations of extrinsic gauge QRFs!



[Donnelly, Freidel '16; Geiller, Jai-akson '19; Ball, Law, Wong '24;…]

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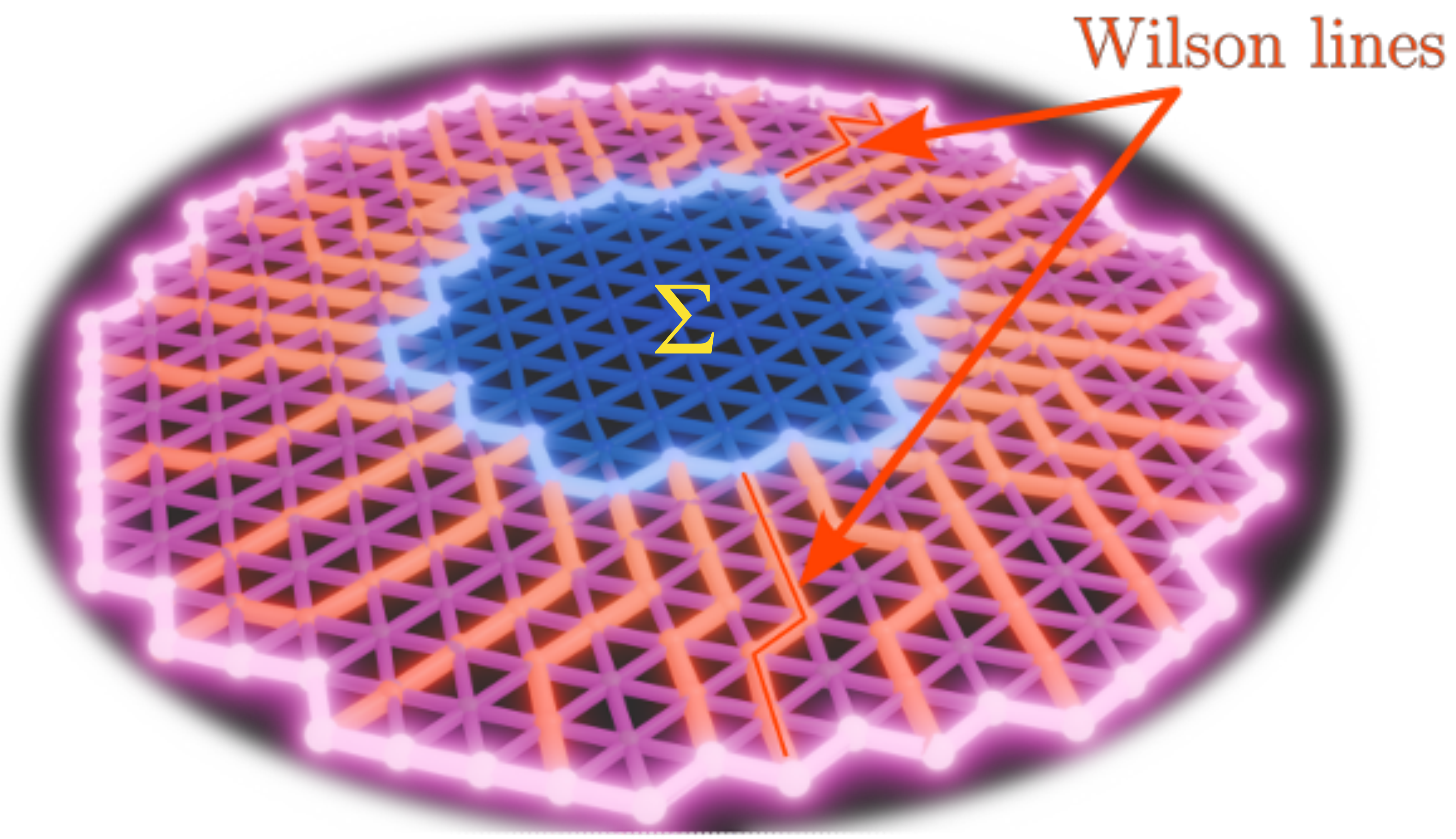
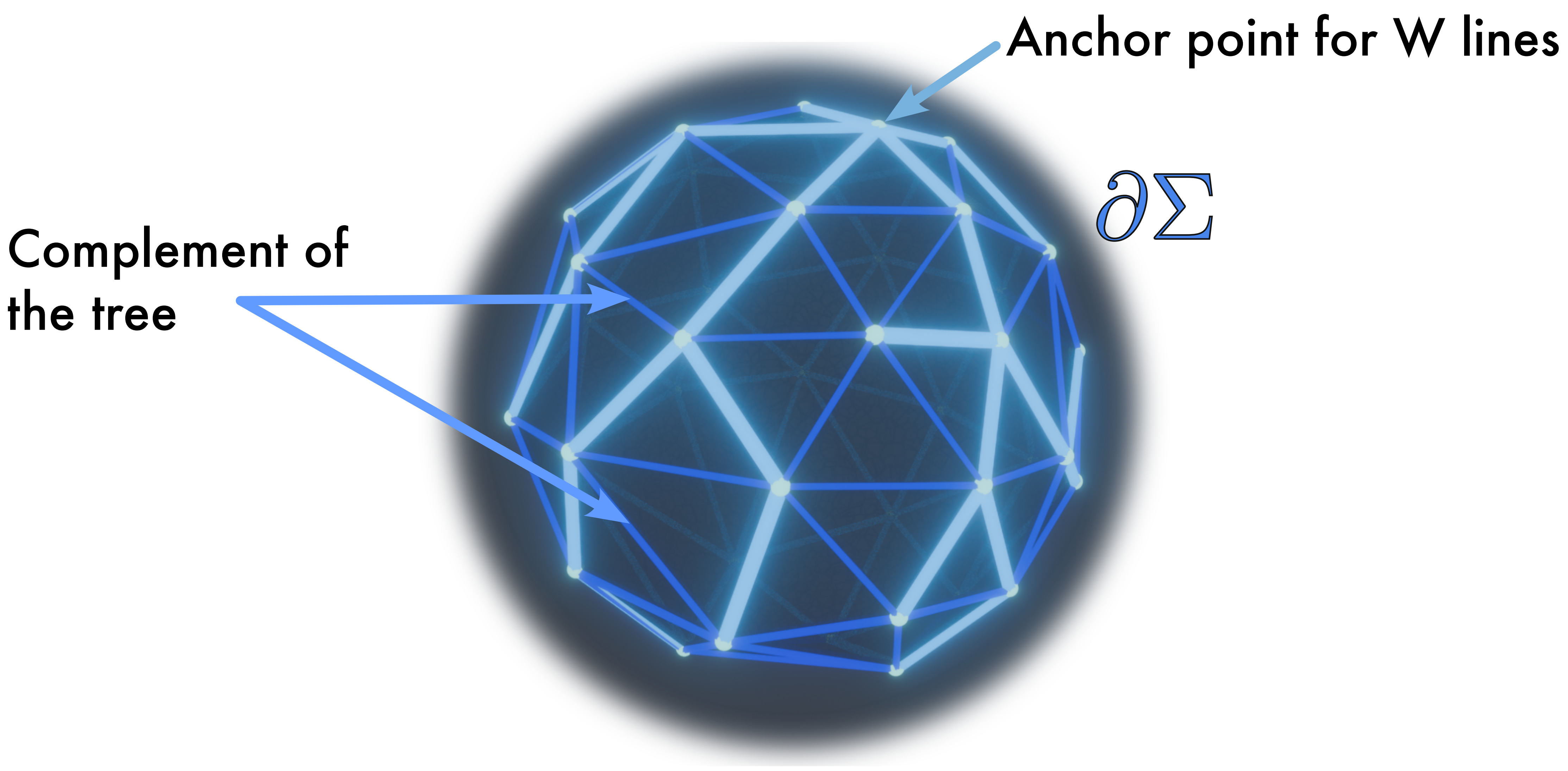
[Araújo-Regado, PH, Sartini, Tomova '24]

Extrinsic and intrinsic gauge QRFs on the lattice

Extrinsic: congruence of Wilson lines in the complement



Spanning tree anchored on node N of the corner



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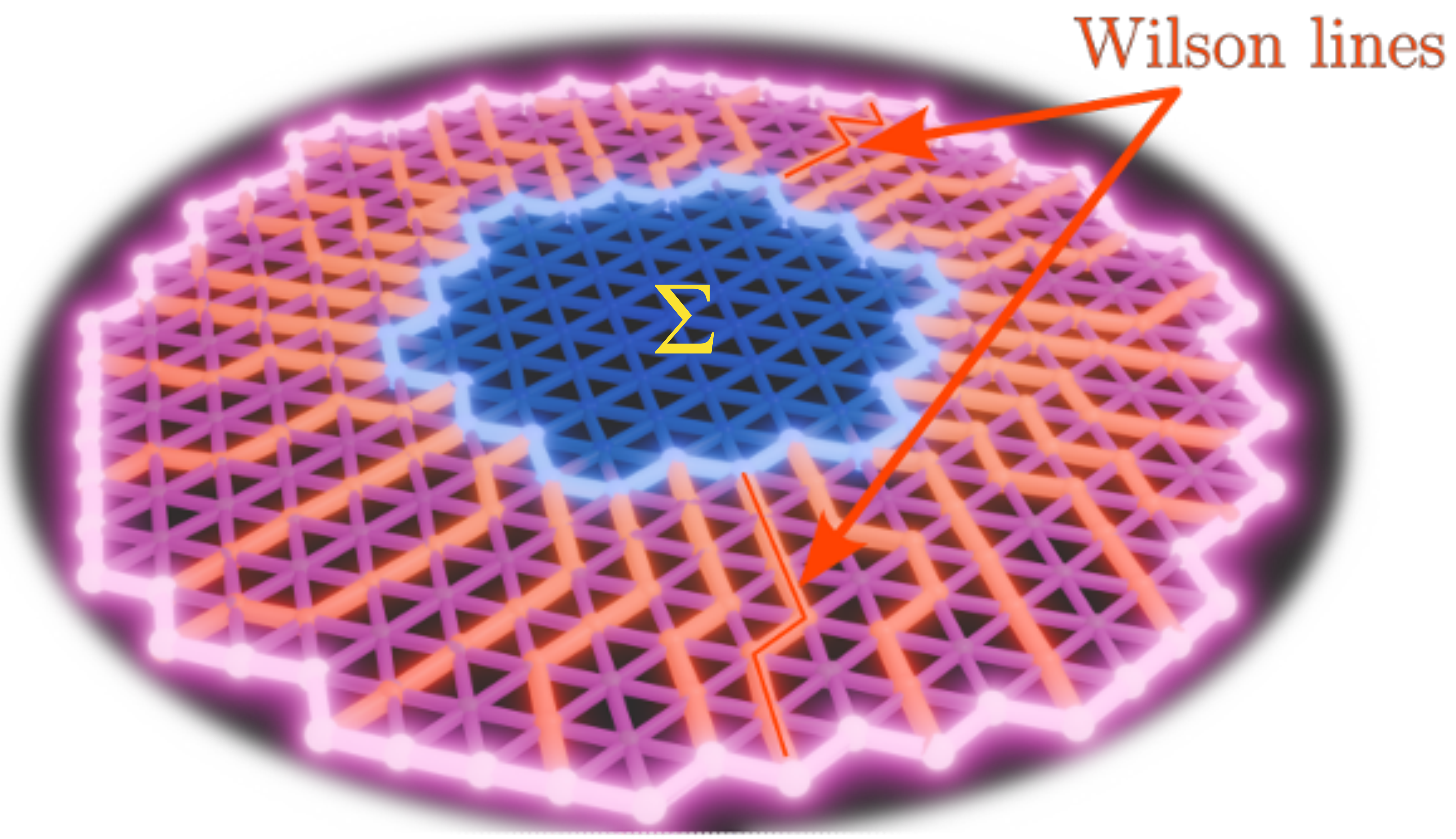
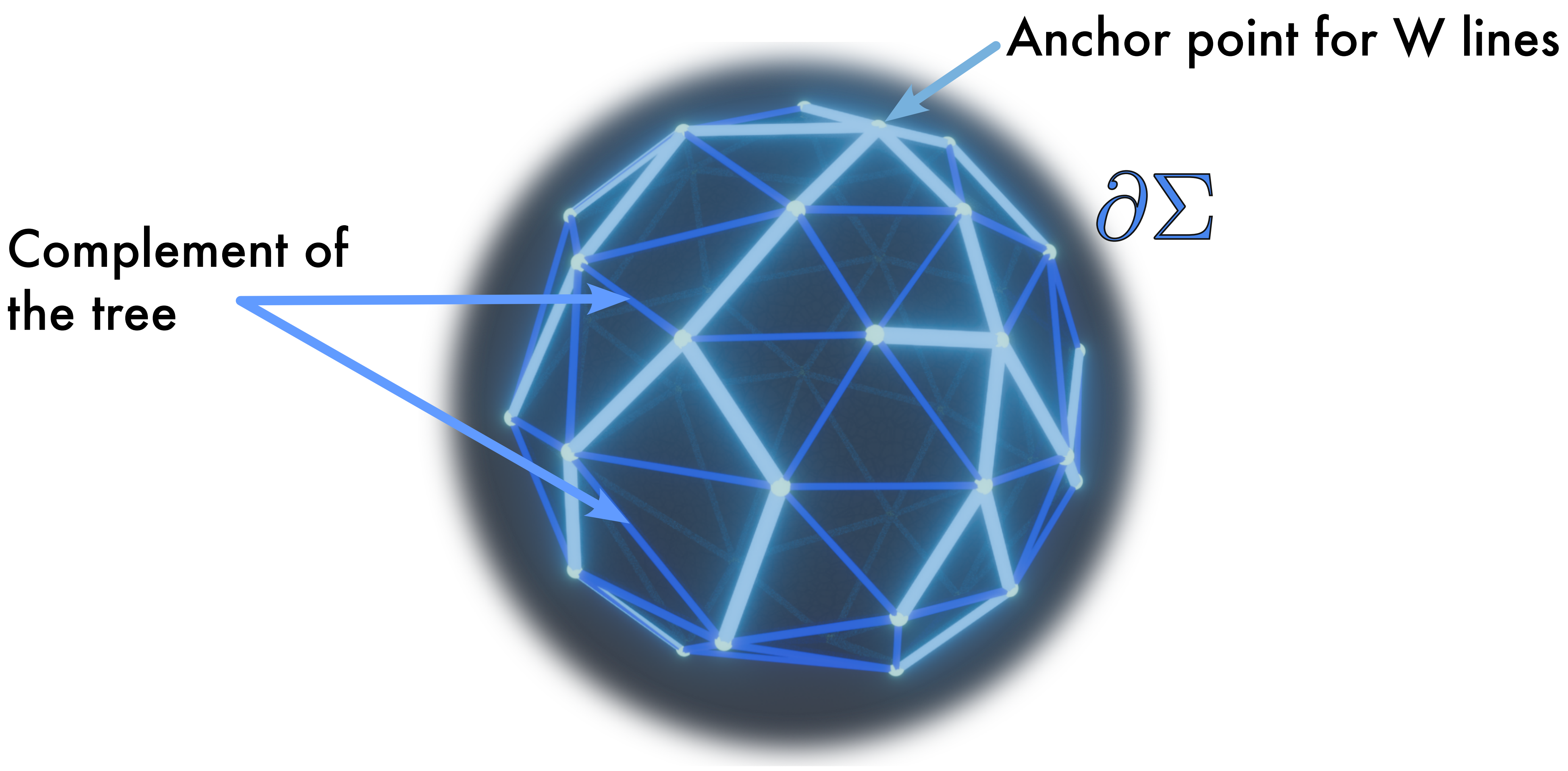


complete ideal QRF: $\mathscr{H}_\Phi = L^2(G^{V_{\partial\Sigma}})$

Spanning tree anchored on node N of the corner

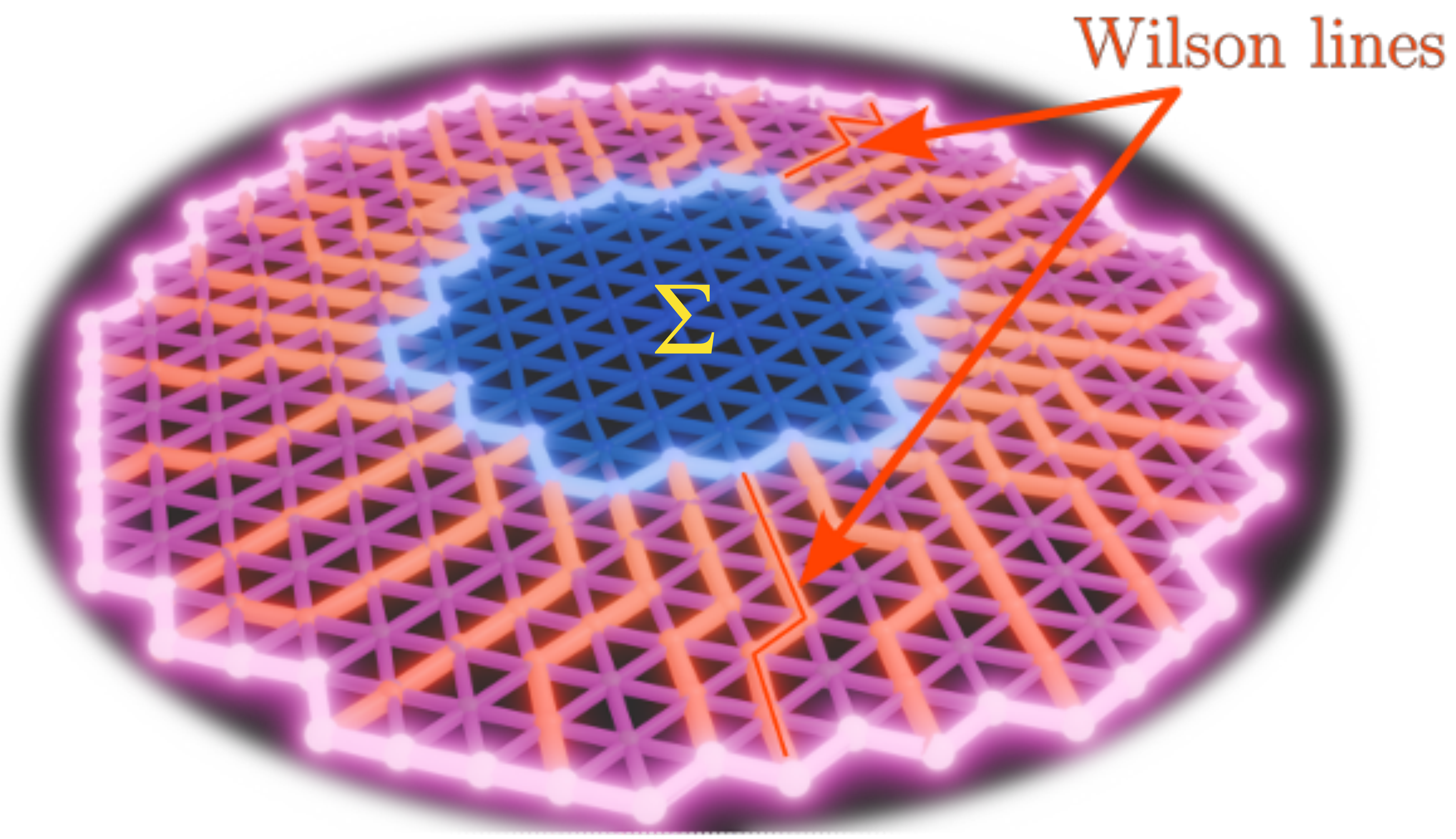


incomplete ideal QRF: $\mathscr{H}_{\tilde{\Phi}} = L^2(G^{V_{\partial\Sigma}-1})$

$\Rightarrow$ can't deparametrize/dress at anchor point N

Extrinsic and intrinsic gauge QRFs on the lattice

Extrinsic: congruence of Wilson lines in the complement



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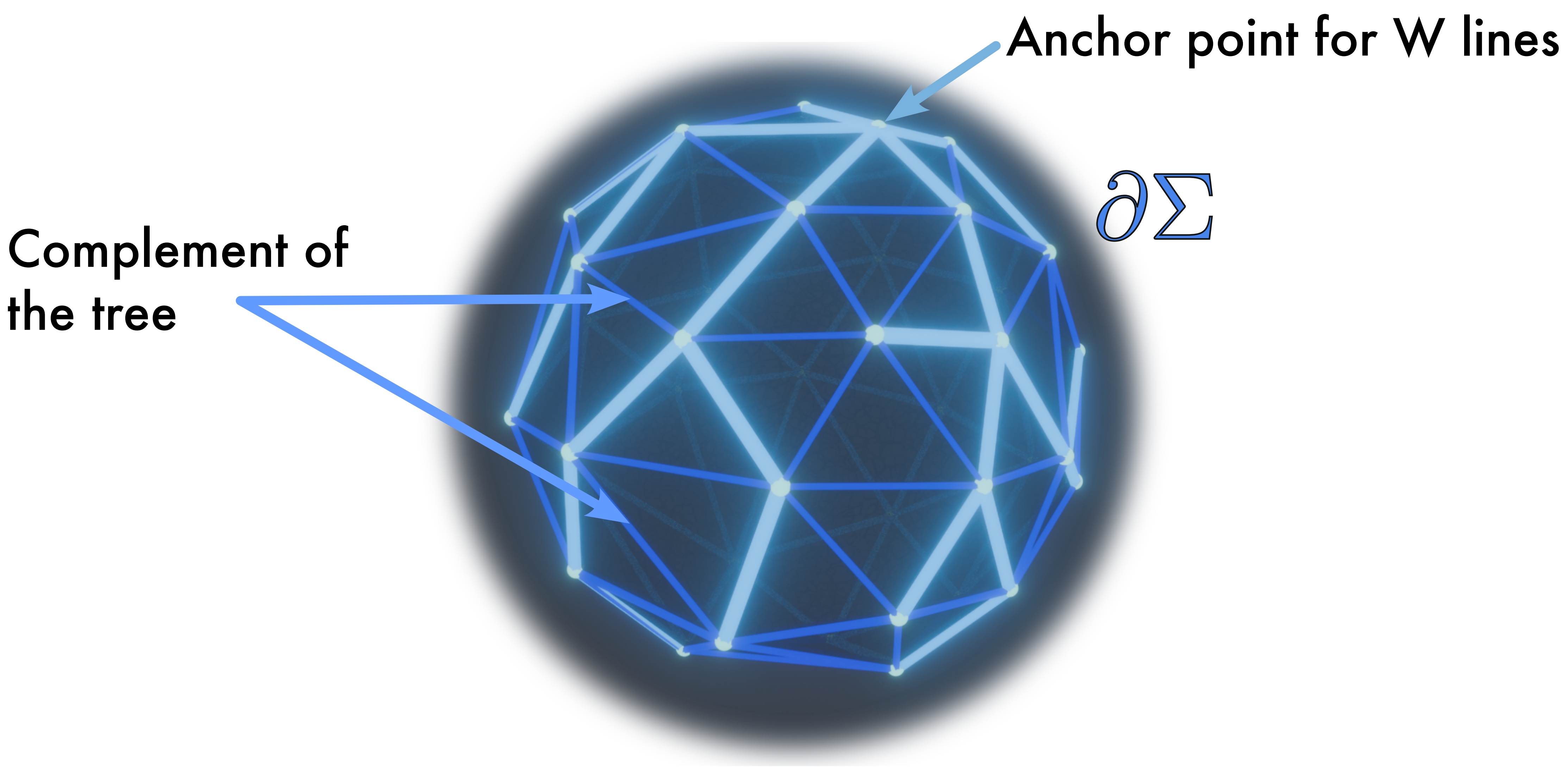


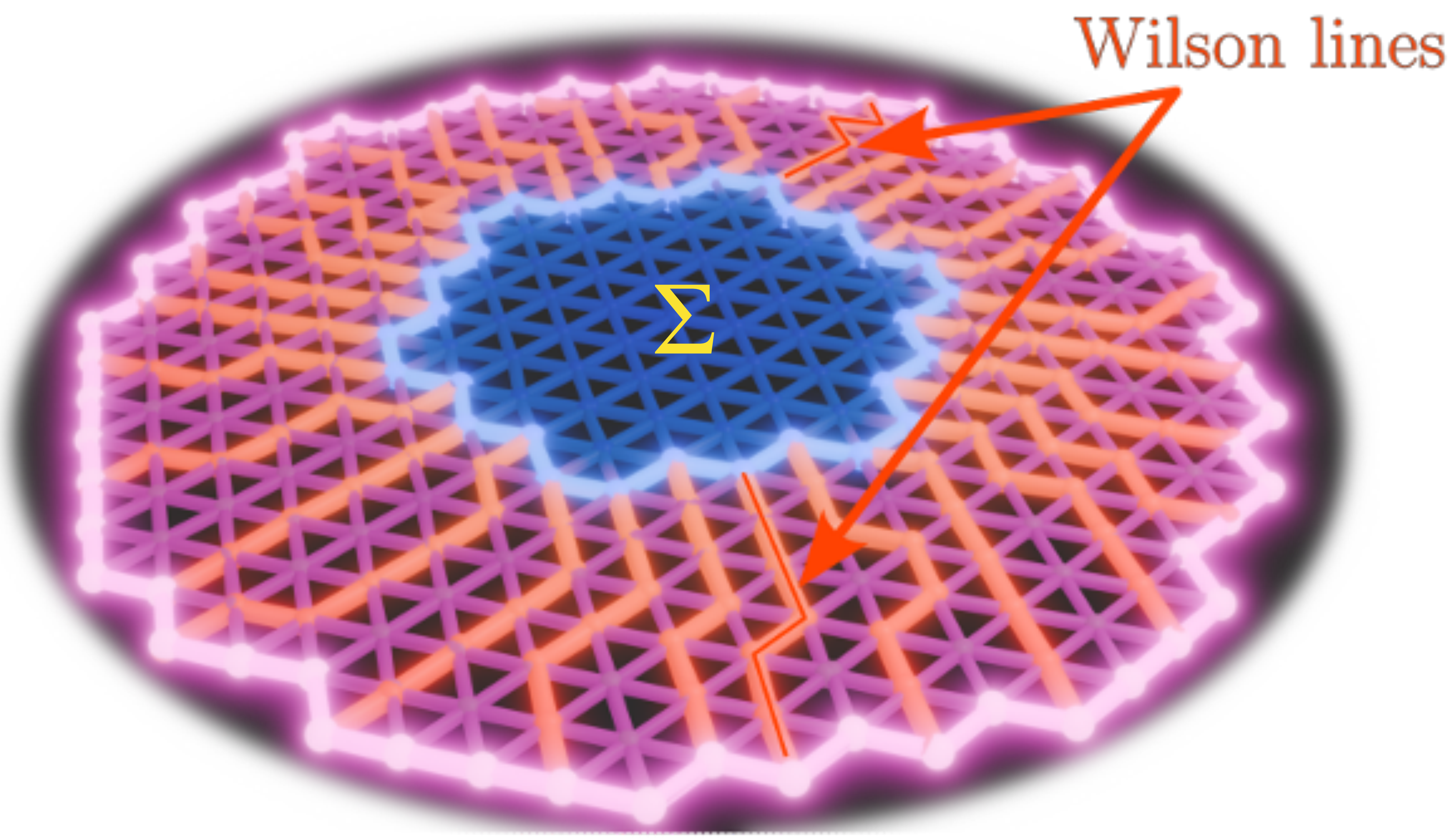
complete ideal QRF: $\mathscr{H}_{\Phi} = L^2(G^{V_{\partial\Sigma}})$

incomplete ideal QRF: $\mathscr{H}_{\tilde{\Phi}} = L^2(G^{V_{\partial\Sigma}-1})$

$\Rightarrow$ **both can be used to build gauge-inv. description (algebras) for subregion, overall impose strong G-invariance**

How do you build physical QRFs for the corner group?

Extrinsic: congruence of Wilson lines in the complement



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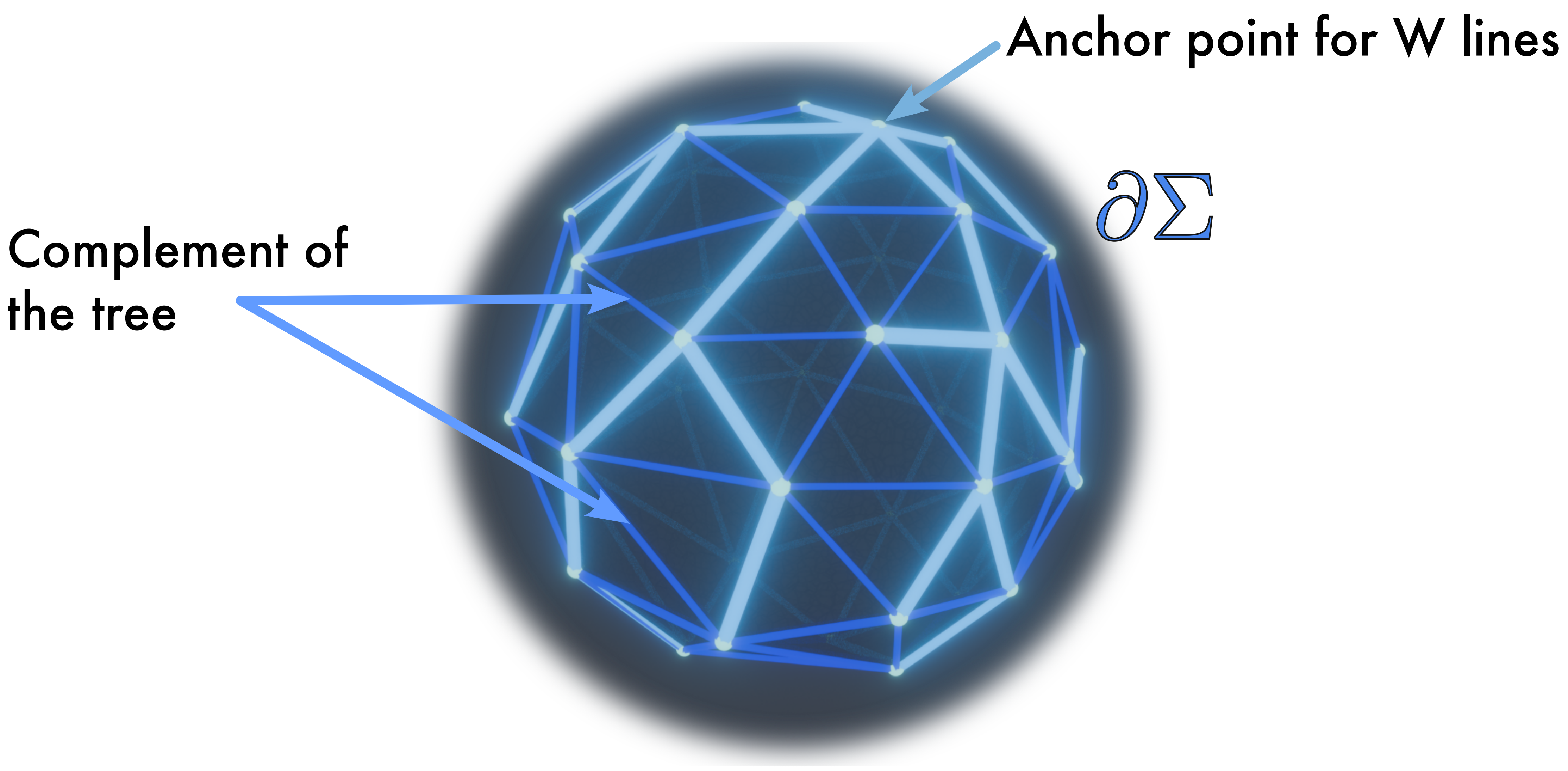
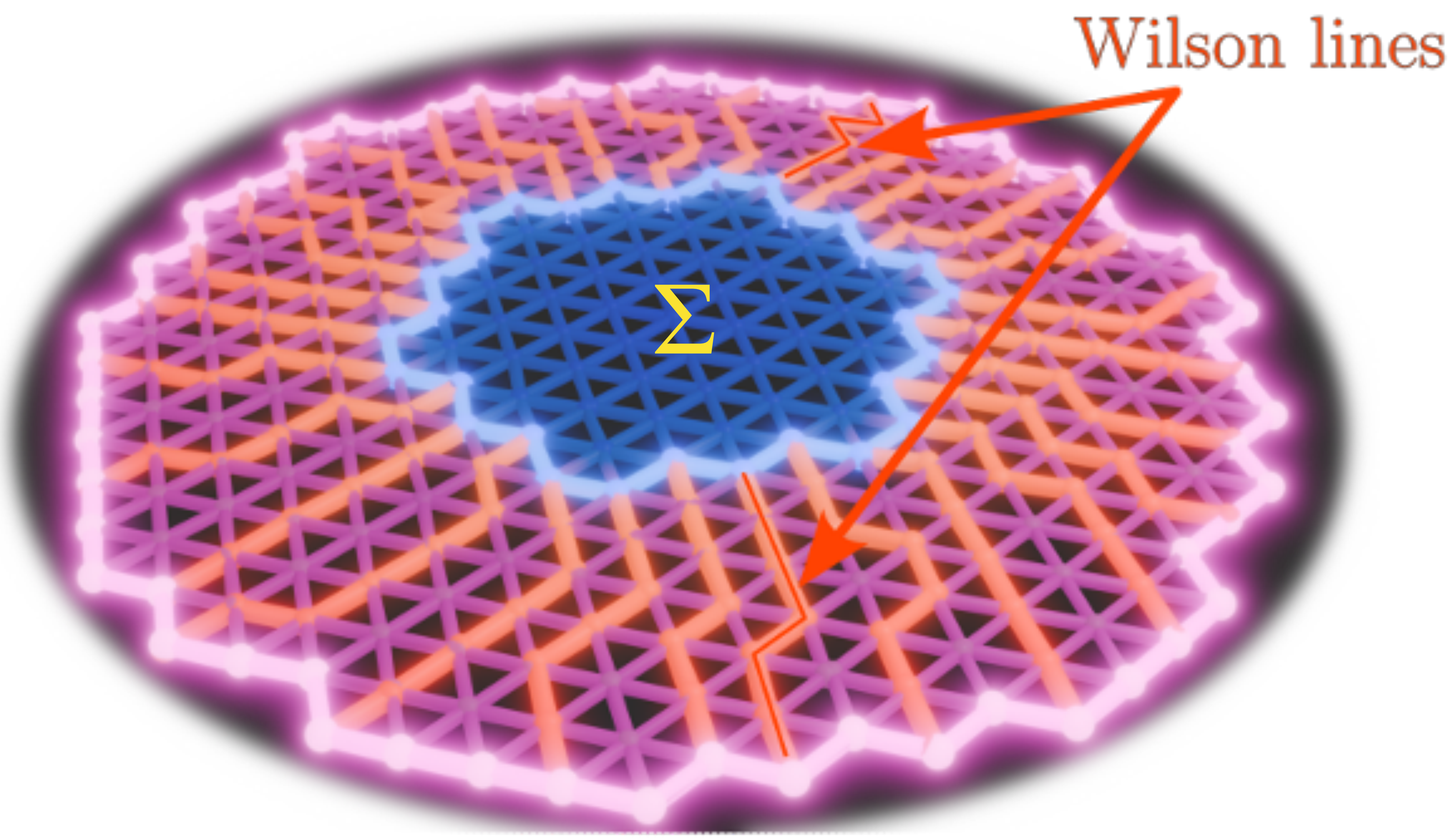


How do you build physical QRFs for the $\boxed{\text{corner group}}$?

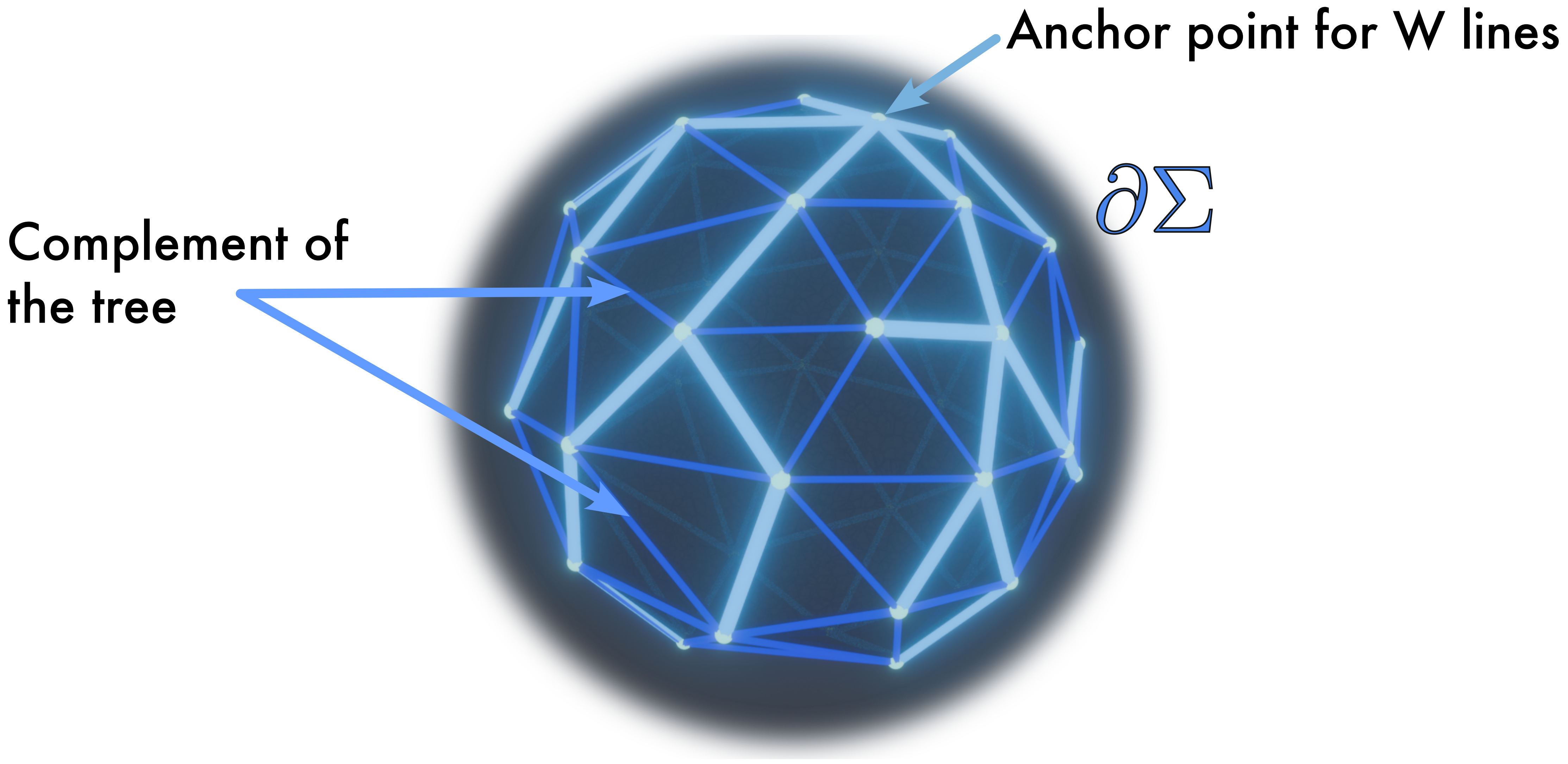
reorientations of extr. QRF Φ

Extrinsic: congruence of Wilson lines in the complement



complete ideal QRF: $\mathscr{H}_{\Phi} = L^2(G^{V_{\partial\Sigma}})$

Spanning tree anchored on node N of the corner



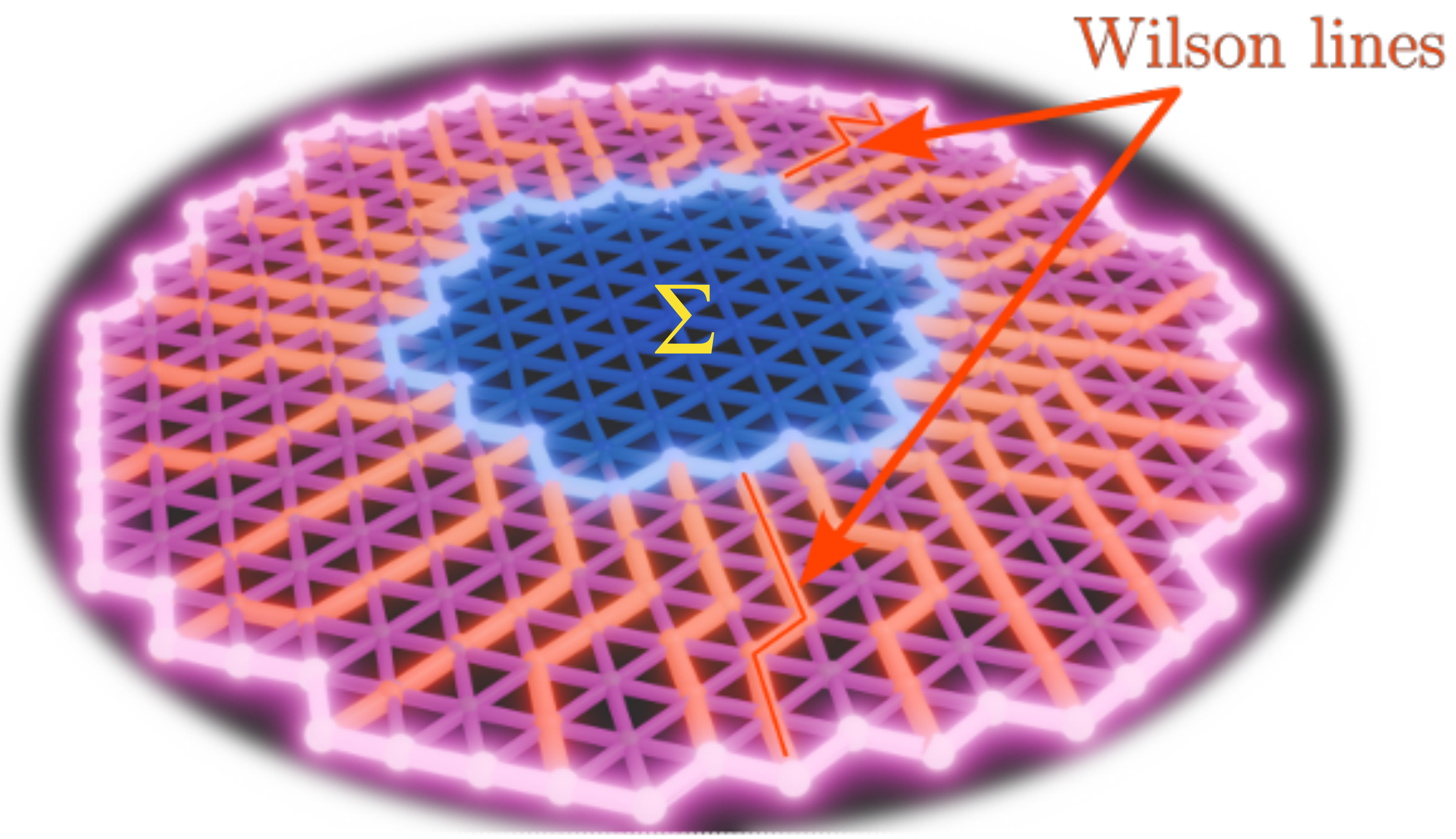
incomplete ideal QRF: $\mathscr{H}_{\tilde{\Phi}} = L^2(G^{V_{\partial\Sigma}-1})$

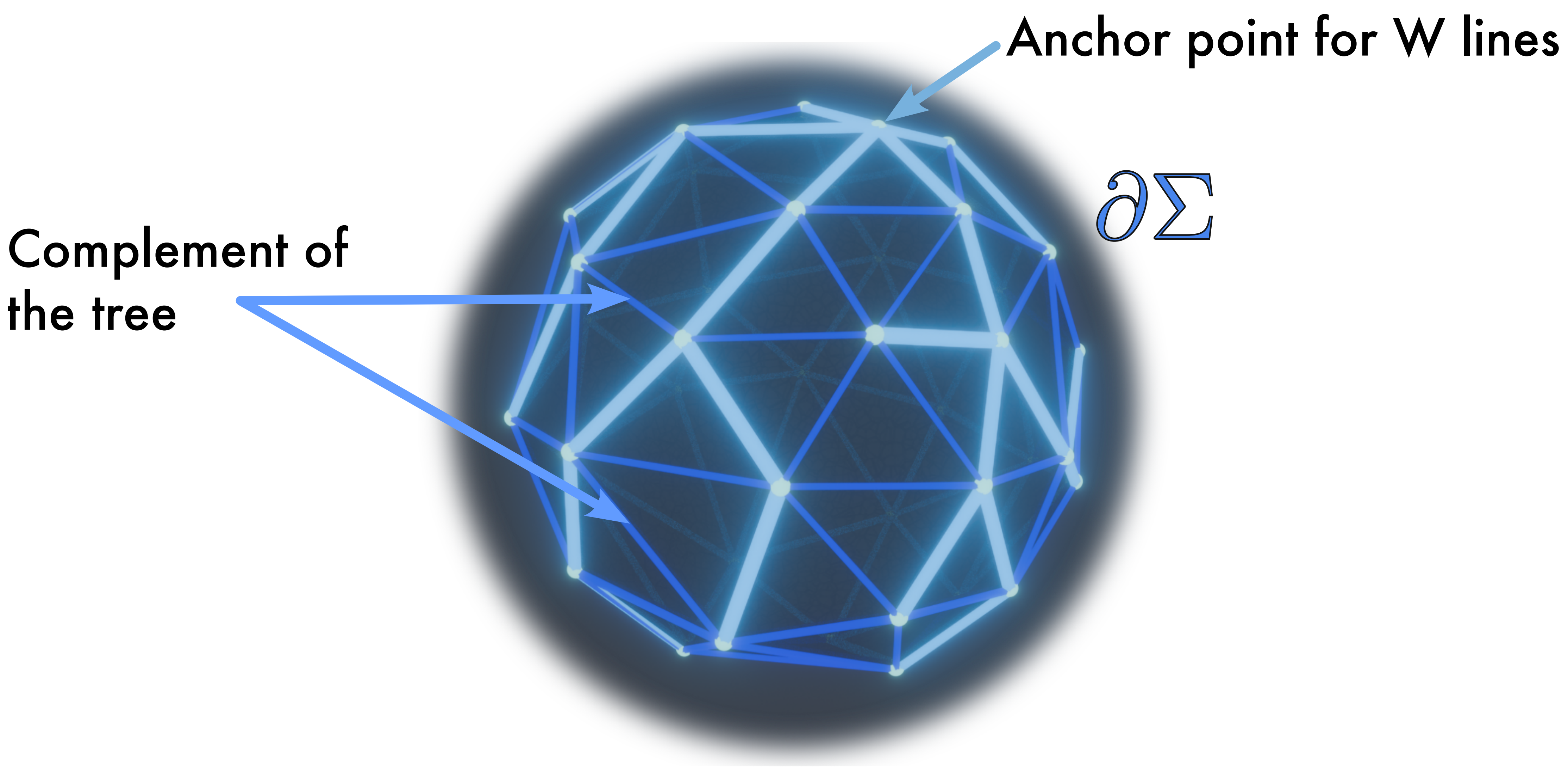
How do you build physical QRFs for the corner group?

reorientations of extr. QRF Φ

Extrinsic: congruence of Wilson lines in the complement

Spanning tree anchored on node N of the corner





complete ideal QRF: $\mathscr{H}_{\Phi} = L^2(G^{V_{\partial\Sigma}})$

incomplete ideal QRF: $\mathscr{H}_{\tilde{\Phi}} = L^2(G^{V_{\partial\Sigma}-1})$

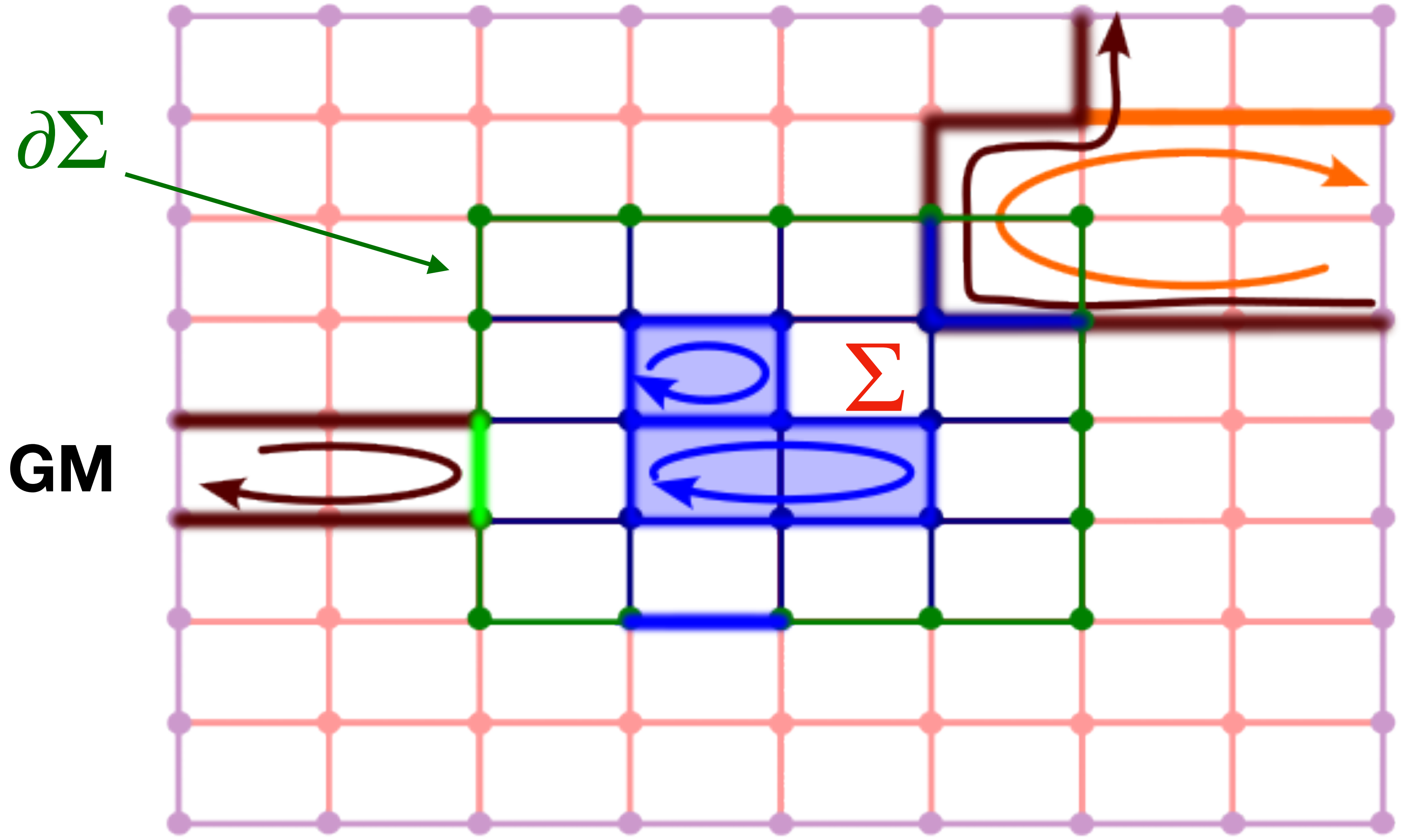
relational observables $O_{|\Phi}(\tilde{\Phi})$ describing intr. relative to extr. gauge QRF constitute a gauge-inv./phys. QRF for the corner group (Goldstone modes)

Averaging over the Goldstone mode

$\mathscr{A}_{\text{ext}} = (\mathscr{A}_\Sigma \otimes \mathscr{A}_\Phi)^G$ **contains the Goldstone mode:**

rel. obs. describing int. rel. to ext. QRF $O^g_{|\Phi}(\tilde{\Phi})$

$\Rightarrow$ **nonlocal gauge-inv. QRF for the electric corner group**



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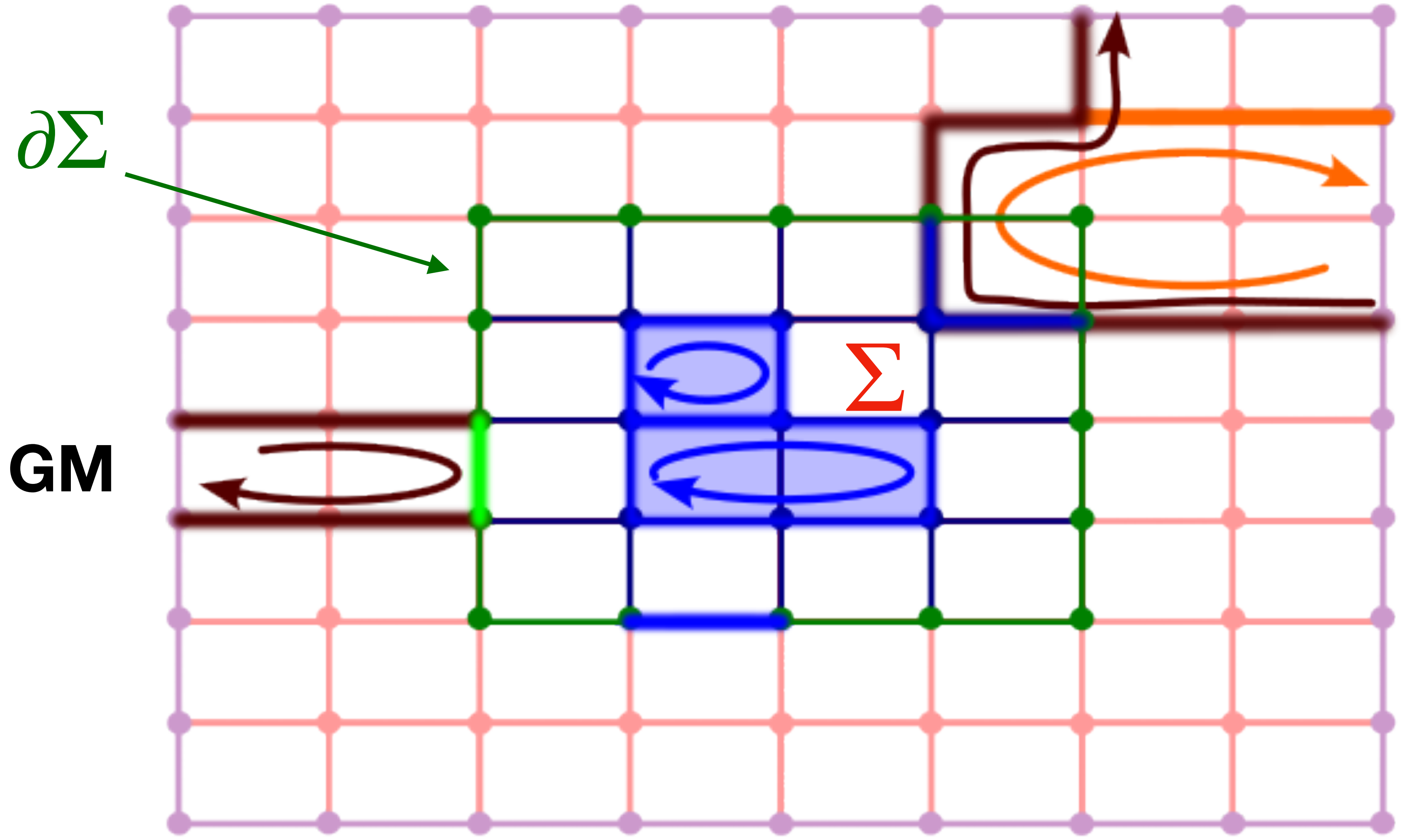
$\Rightarrow$ **nonlocal gauge-inv. QRF for the electric corner group**

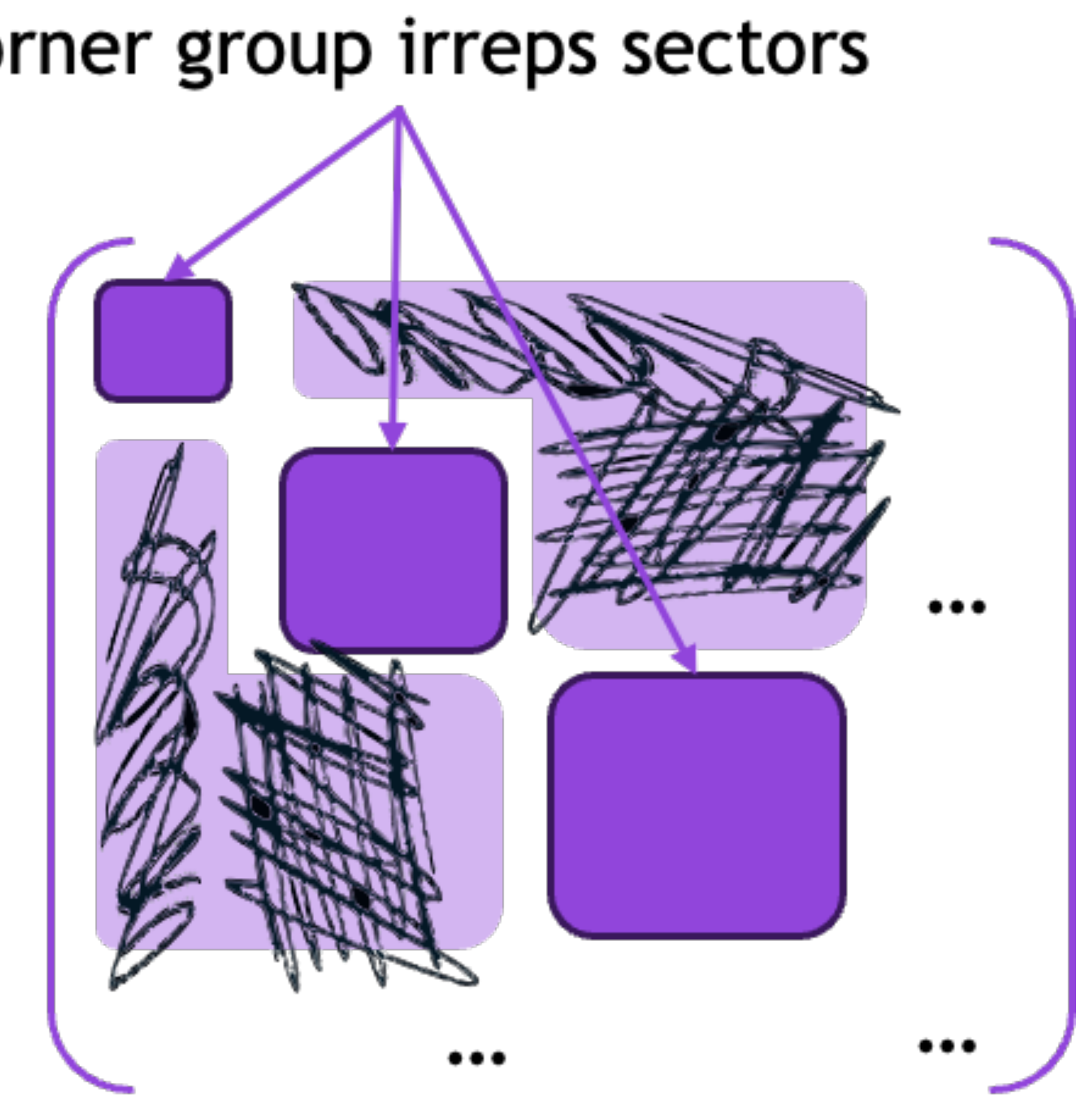
if no access to it $\Rightarrow$ **average over its orientations/corner group**

$$\mathscr{A}_E = \mathbb{G}_{\partial\Sigma}(\mathscr{A}_{\text{ext}}) = \int_{G^{V_{\partial\Sigma}}} dg\, V_\Phi(g)\, \mathscr{A}_{\text{ext}}\, V^\dagger_\Phi(g)$$



electric corner charge superselection:

$\mathscr{A}_E =$



Averaging over the Goldstone mode

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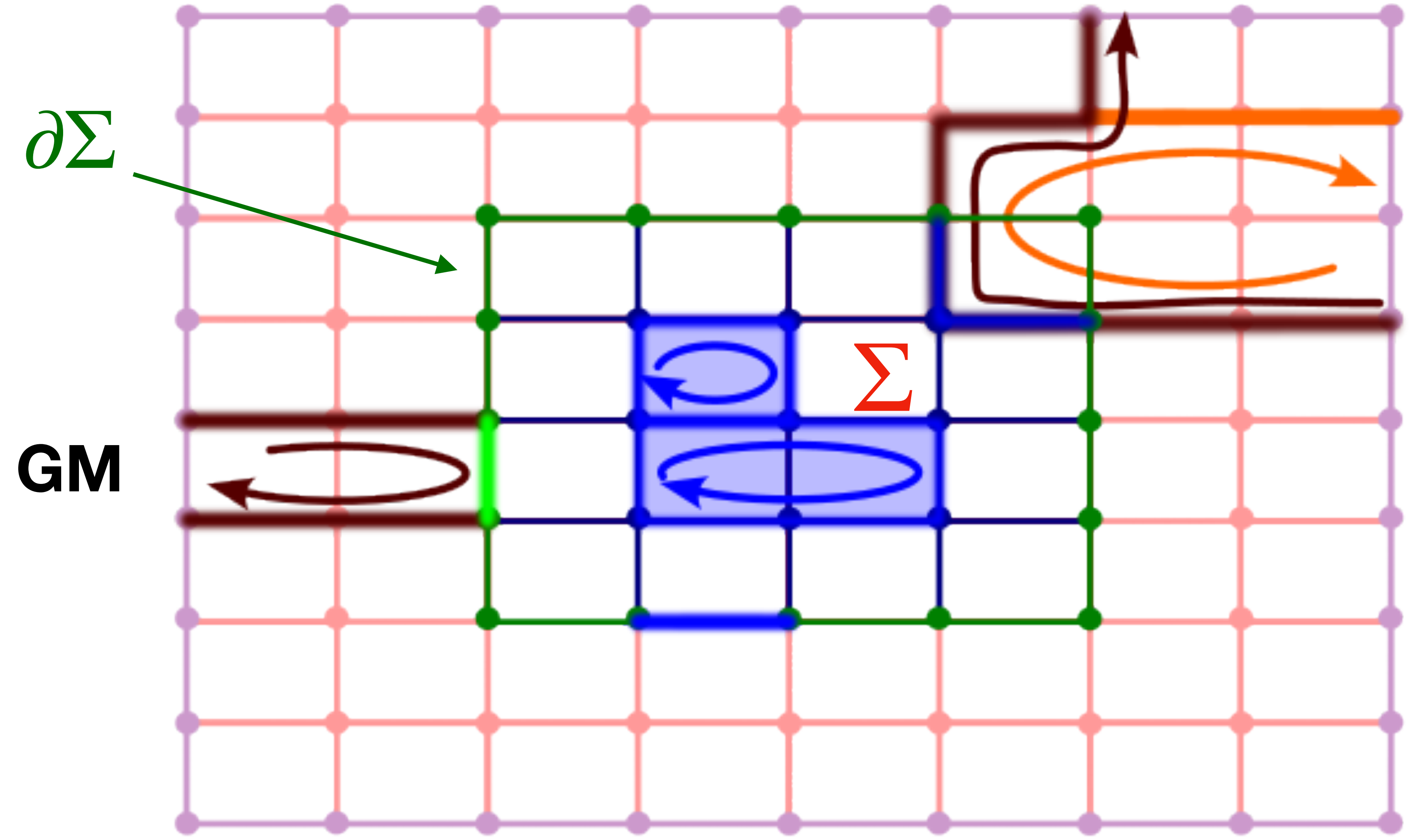
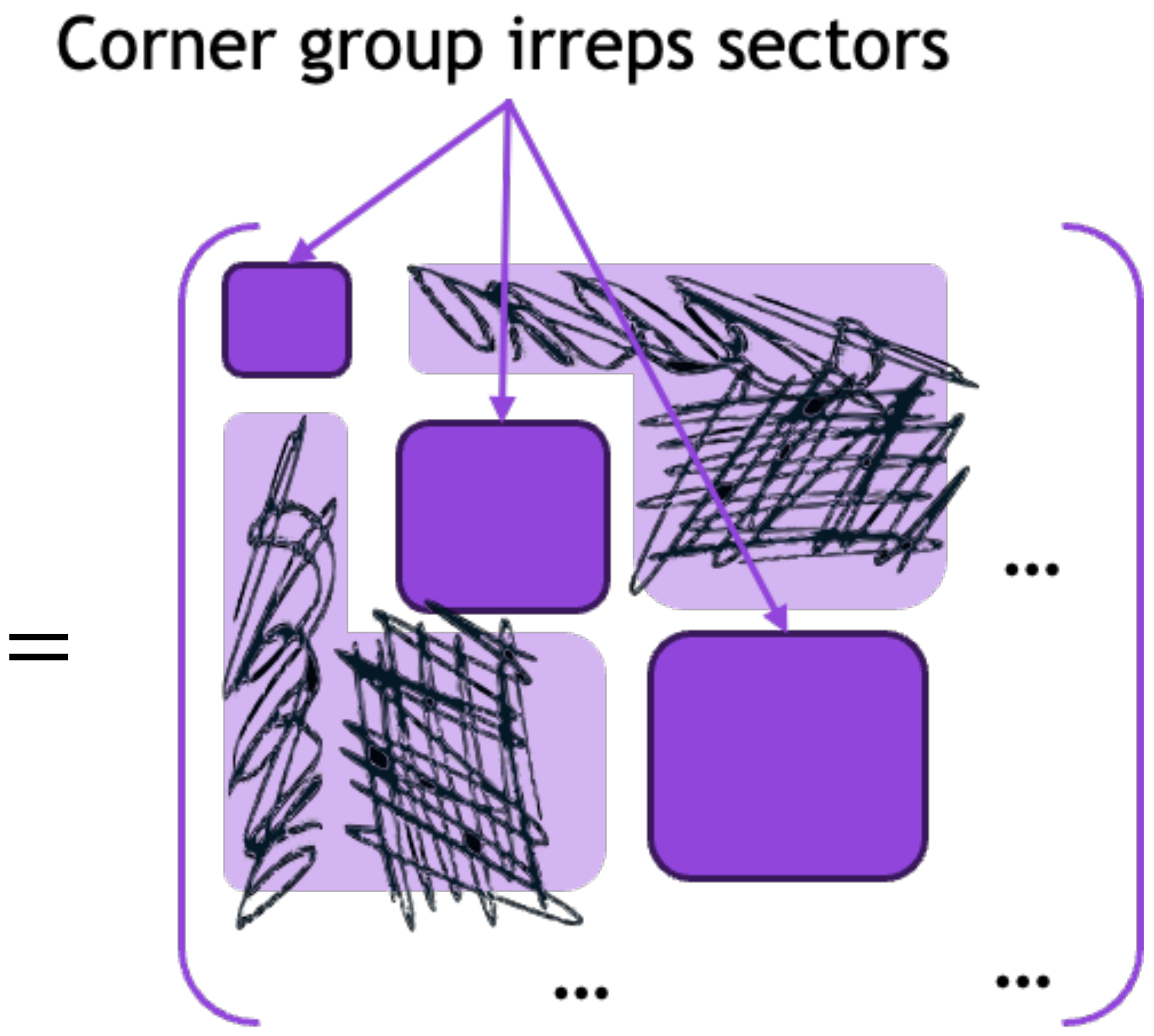


electric corner charge superselection:

$\Rightarrow$ **yields standard electric center algebra of entanglement entropy computations**

$\mathscr{A}_E =$



$$S_\Sigma(\rho) = \sum_c p_c \left(S_{\text{vN}}(\rho_c) + \log d_c\right) - \sum_c p_c \ln p_C$$

[Casini, Huerta, Rosabal ’13; Donnelly ’12; Buividovich, Polikarkov ’08; Ghosh et al ’15; …]

Conclusions

- **Extension of SR covariance structures into quantum realm**
  based on internal QRFs $\Rightarrow$ in terms of group structures really the same as in SR
- **Systematic method for changing QRF perspectives**
  accommodates RFs in relative superposition
- **Gauge-inv. subsystems depend on choice of QRF (subsystem relativity)**
  $\Rightarrow$ correlations, thermal properties, dynamics, …. depend on frame
- **Two distinct notions of external frame independence**
  $\Rightarrow$ depending on whether G is a physical symmetry or redundancy
- **Peaceful coexistence of physical and gauge QRFs in lattice gauge theory**
  $\Rightarrow$ edge vs. Goldstone modes and recovery of standard entanglement entropy computations