Causality Violations in QFT? A Ping-Pong Ball Test

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This talk is based on the following articles:

1. A. Much, RV, *Superluminal local operations in quantum field theory: A ping-pong ball test*, Universe **9**, 447 (2023)
2. C.J. Fewster, RV, *Quantum fields and local measurements*, Commun. Math. Phys. **378**, 851-889 (2020)
3. H. Bostelmann, C.J. Fewster, M. Ruepp, *Impossible measurements require impossible apparatus*, Phys. Rev. **D 103**, 025017, 14 (2021)
4. C.J. Fewster, RV, *Measurement in quantum field theory*, in: Encyclopedia in Mathematical Physics, 2025 edition (Elsevier), arXiv:2304.13356

- AQFT framework
- Sorkin's "impossible measurements (operations)" in QFT
  $\longleftrightarrow$ local "superluminal operations" in QFT
- Local superluminal operations in classical relativistic field theory
  ping-pong ball test
- Absence of local superluminal operations in the FV approach

When someone presents a paradox as being rooted in quantum physics, replace the term quantum mechanical particle by ping-pong ball everywhere. If the paradox persists, it is unrelated to quantum physics.

Due to Reinhard Werner (oral version)

In algebraic quantum field theory (or algebraic *classical* field theory), there is a local structure for the observables:

$$\mathcal{A} = \text{ *-algebra of (or: generated by) observables,}$$

formed by *-subalgebras

$$\mathcal{A}(O) = \text{ algebra of observables that can be}$$

measured in the spacetime region O

with the properties:

- $O_1 \subset O_2 \quad \Longrightarrow \quad \mathcal{A}(O_1) \subset \mathcal{A}(O_2)$
- O_2 and O_1 causally disjoint $\quad \Longrightarrow \quad [A_1, A_2] = 0$ for all $A_j \in \mathcal{A}(O_j)$
- For every symmetry (isometry) $L : M \to M$ of the spacetime, there is an automorphism $\alpha_L : \mathcal{A} \to \mathcal{A}$ so that

$$\alpha_L(\mathcal{A}(O)) = \mathcal{A}(L(O)) \quad \text{and} \quad \alpha_{L_1} \circ \alpha_{L_2} = \alpha_{L_1 L_2}$$

The algebra $\mathcal{A}$ may be non-commutative (quantum case) or commutative (classical case)

Typical situation in QFT:

- $\mathcal{A}(O)$ are weakly closed *-subalgebras of $B(\mathcal{H})$ ("von Neumann algebras")
- Set of (physical) states $\omega \in \mathcal{S}$ given by density matrices ϱ on $\mathcal{H}$:

$$\omega(A) = \langle A \rangle_{\varrho} = \mathrm{Tr}(\varrho A)$$

- $$\alpha_L(A) = U_L A U_L^* \quad \text{with continuous unitary group repr } L \mapsto U_L$$
- There is a unit vector $\psi_0 \in \mathcal{H}$ with $U_L \psi_0 = \psi_0$
- static and geodesic time-translations have positive generators: I.e. if $U_t = e^{itH}$ implements time-shifts of an inertial time-coordinate, then $H \geq 0$.

This is the setting we will adopt in the following, mainly for M = Minkowski spacetime.

- A **channel** is a CP map $T : \mathcal{A} \to \mathcal{A}$, typically of the form

$$T(A) = \sum_j W_j A W_j^* \qquad (A \in \mathcal{A}) \quad \text{with} \quad W_j \in \mathcal{A}\,, \quad \sum_j W_j W_j^* \leq \mathbf{1}$$

- **nonselective channel**: $T(\mathbf{1}) = \mathbf{1} \qquad (\sum_j W_j W_j^* = \mathbf{1})$

- A channel T is **localized** in $O \subset M$ if

$$T(\mathcal{A}(O)) \subset \mathcal{A}(O) \quad \text{and} \quad T(A') = A'\,, \quad A' \in \mathcal{A}(O')\,, \quad O \text{ and } O' \text{ causally disjoint}$$

- An **operation** is a convex map $\tau : \mathcal{S} \to \mathcal{S}$ that arises as the dual of a channel,

$$\tau(\omega)(A) = \omega(T(A)) \qquad (A \in \mathcal{A}\,, \quad \omega \in \mathcal{S})$$

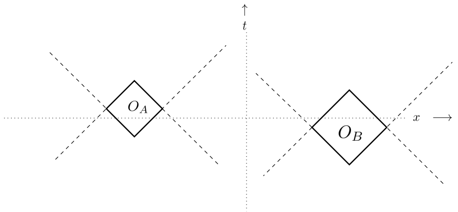
- τ is nonselective/localized in O if so is the pre-dual channel T

The simplest example of a non-selective channel localized in O is

$$T_U(A) = UAU^* \quad (A \in \mathcal{A}) \quad \text{where} \;\; U \in \mathcal{A}(O) \;\; \text{is unitary}$$

If O_a and O_b are causally disjoint then any unitary operation τ_U induced by T_U with $U \in \mathcal{A}(O_a)$ has no effect on $\mathcal{A}(O_b)$:

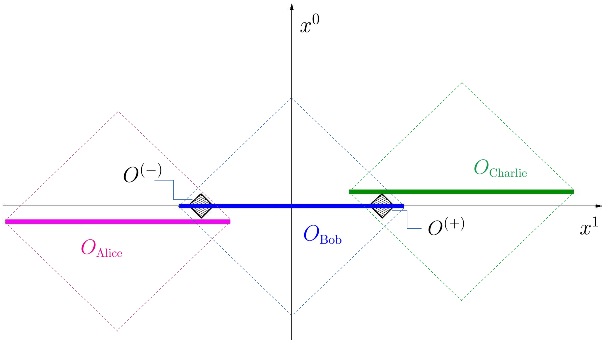
$$\tau_U(\omega)(B) = \omega(T_U(B)) = \omega(UBU^*) = \omega(UU^*B) = \omega(B) \qquad (B \in \mathcal{A}(O_b))$$



Can all such local unitary operations be physically performed?

"Impossible measurements/operations scenario" [Sorkin (1993)]:

Consider 3 spacetime regions, named after experimenters carrying out measurements/operations therein:



Since $O_{\rm Alice}$ and $O_{\rm Charlie}$ are causally disjoint, Charlie cannot know by measuring in $O_{\rm Charlie}$ if Alice has carried out a unitary operation $\tau_{U_{\rm Alice}}$ with $U_{\rm Alice} \in \mathcal{A}(O_{\rm Alice})$:

$$\tau_{U_{\rm Alice}}(\omega)(C) = \omega(U_{\rm Alice} C U^*_{\rm Alice}) = \omega(C) \quad \text{for all } C \in \mathcal{A}(O_{\rm Charlie})\,, \quad \omega \in \mathcal{S}$$

But if **first Alice carries out a unitary operation, and then Bob**:

$$\tau_{U_{\rm Bob}} \circ \tau_{U_{\rm Alice}}(\omega)(C) = \omega(U_{\rm Alice} U_{\rm Bob} C U^*_{\rm Bob} U^*_{\rm Alice}) \quad \text{for all } C \in \mathcal{A}(O_{\rm Charlie})$$

In general, $U_{\rm Bob} \in \mathcal{A}(O_{\rm Bob})$ won't commute with all $C \in \mathcal{A}(O_{\rm Charlie})$ nor with all $U_{\rm Alice} \in \mathcal{A}(O_{\rm Alice})$ since

$O_{\rm Alice}$ causally overlaps with $O_{\rm Bob}$ and $O_{\rm Bob}$ causally overlaps with $O_{\rm Charlie}$

Hence, one can choose $U_{\rm Alice}$, $U_{\rm Bob}$, C and ω such that

$$\tau_{U_{\rm Bob}} \circ \tau_{U_{\rm Alice}}(\omega)(C) \neq \tau_{U_{\rm Bob}}(\omega)(C)$$

This means, Charlie can determine by measuring the observable C in $O_{\rm Charlie}$ if Alice has carried out an operation $\tau_{U_{\rm Alice}}$ in $O_{\rm Alice}$, if Bob carries out a suitable operation $\tau_{U_{\rm Bob}}$ in $O_{\rm Bob}$.

This would mean a superluminal transfer of information since $O_{\rm Alice}$ and $O_{\rm Charlie}$ are causally disjoint.

Examples are given in: R. Sorkin (1993); L. Bosten, I. Jubb, G. Kells, PRD 104 (2021); I. Jubb, PRD 105 (2022).

The issue: $\tau_{U_{\rm Bob}}$ amounts to a superluminal communication channel between $O_{\rm Alice}$ and $O_{\rm Charlie}$ which is unphysical.

But such superluminal communication channels arise also in classical field theory, e.g. by local, kinematical symmetries.

Theorem (A. Much & RV (2023))

Let $\mathcal{A}(O)$ be the local observable algebras of the classical or the quantized Klein-Gordon field on Minkowski spacetime M, with field equation $(\Box + m^2)\varphi = 0$.

Then there are states ω and operations τ_{Alice} and τ_{Bob} together with observables $C \in \mathcal{A}(O_{\mathrm{Charlie}})$ so that

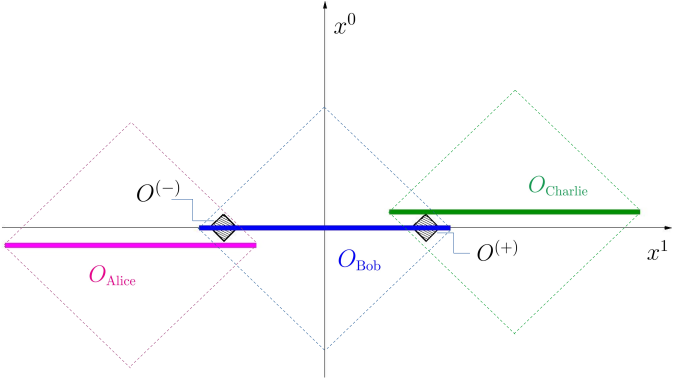
$$\tau_{\mathrm{Bob}} \circ \tau_{\mathrm{Alice}}(\omega)(C) \neq \tau_{\mathrm{Bob}}(\omega)(C)$$

τ_{Alice} and τ_{Bob} are localized in O_{Alice} and O_{Bob}.

Specifically, τ_{Bob} can be chosen so that it corresponds to an instantaneous rotation around the x^3-axis by 180 degrees, flipping $O^{(-)} \leftrightarrow O^{(+)}$ (local kinematical symmetry).

For the quantized Klein-Gordon field, there is a unitary $U_{\mathrm{Bob}} \in \mathcal{A}(O_{\mathrm{Bob}})$ so that

$$\tau_{\mathrm{Bob}}(\omega)(\,.\,) = \omega(U_{\mathrm{Bob}}\,.\,U_{\mathrm{Bob}}^*)$$



τ_{Bob} has the effect of flipping $O^{(-)}$ instantaneously to $O^{(+)}$ and vice versa.

Remarks

- The approach of describing classical field theory in terms of a local algebra framework has been developed by Brunetti, Duetsch, Fredenhagen and Rejzner (and co-authors). See:

  K. Rejzner: *Perturbative Algebraic Quantum Field Theory*, Springer, 2016

  M. Duetsch: *From Classical Field Theory to Perturbative Quantum Field Theory*, Birkhäuser, 2019

- In the *classical* case, $\tau_{\rm Bob}$ and $\tau_{\rm Alice}$ are not implemented by unitaries in the local algebras since the local algebras are commutative — they are formed by (certain) functions on the phase space.

  The generator of $\tau_{\rm Bob}$ can be obtained with the help of a *Peierls bracket*, generalising the Poission bracket of Hamiltonian mechanics.

- QFT $\mathcal{A}$ with local algebras $\mathcal{A}(O)$ **system**
- QFT $\mathcal{P}$ with local algebras $\mathcal{P}(O)$ **probe**
- can be combined into a new QFT $\mathcal{Q} = \mathcal{P} \otimes \mathcal{A}$, local algebras $\mathcal{Q}(O) = \mathcal{P}(O) \otimes \mathcal{A}(O)$ (uncoupled)

Idea: Measure observables of the system by coupling the system dynamically to the probe in a specified **interaction region** O_I, and measuring observables of the probe.

In case you are wondering why it might be a good idea to measure a quantum field by coupling it to another quantum field — this is common practice in elementary physics. The detectors that are available are basically sensitive to electrically charged particles (like electrons) or photons. But how do you measure neutrinos? They must first be brought into interaction with other kinds of particles for which detectors are available.

The effect of the coupling is described by a **scattering morphism** $\Theta : \mathcal{Q} \to \mathcal{Q}$. Under general conditions, this is given by the action of the S-matrix:

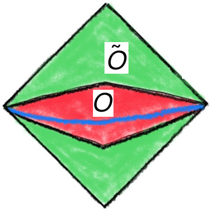
$$\Theta(P \otimes A) = S(P \otimes A)S^{-1}$$

Assumption: The interaction is only active in a bounded interaction region O_I (i.e. over finite time and over a compact region in space).

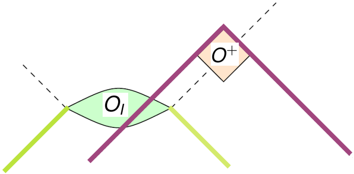
Scattering morphism can be shown to exist, and localization properties derived, if all the QFTs obey the **time-slice property**:

If $\tilde{O}$ is a globally hyperbolic sub-spacetime of the ambient spacetime M, and if O is an open neighbourhood in $\tilde{O}$ of a Cauchy-surface for $\tilde{O}$, then

$$\mathcal{A}(O) = \mathcal{A}(\tilde{O}) \qquad \text{(same for } \mathcal{P} \text{ and } \mathcal{Q})$$



time slice property



Localization properties of scattering morphism Θ relative to O_I:

- If O' and O_I causally disjoint $\Longrightarrow$

$$\Theta(P \otimes A) = P \otimes A \qquad (P \otimes A \in \mathcal{Q}(O'))$$

- If O^+ is not in the causal past of O_I $\Longrightarrow$

$$\Theta(\mathcal{Q}(O^+)) \subset \mathcal{Q}(\text{causal past of } O^+)$$

Consider the case

$$\mathcal{P} = \mathcal{P}_{\text{Alice}} \otimes \mathcal{P}_{\text{Bob}}\,, \qquad O_I = O_{I,\text{Alice}} \cup O_{I,\text{Bob}}\,, \quad O_{I,\text{Alice}} \cap J^+(O_{I,\text{Bob}}) = \emptyset$$

Then we have the total scattering morphism Θ as before, and also the scattering morphisms induced by Θ_{Alice} on $\mathcal{P}_{\text{Alice}} \otimes \mathcal{A}$ and Θ_{Bob} on $\mathcal{P}_{\text{Bob}} \otimes \mathcal{A}$:

$$\hat{\Theta}_{\text{Alice:}}\ \mathcal{P}_{\text{Alice}} \otimes \mathcal{P}_{\text{Bob}} \otimes \mathcal{A} \to \mathcal{P}_{\text{Alice}} \otimes \mathcal{P}_{\text{Bob}} \otimes \mathcal{A}\,,$$
$$\hat{\Theta}_{\text{Alice}} = \Theta_{\text{Alice}} \otimes_2 \mathbf{1}$$

$$\hat{\Theta}_{\text{Bob}} : \mathcal{P}_{\text{Alice}} \otimes \mathcal{P}_{\text{Bob}} \otimes \mathcal{A} \to \mathcal{P}_{\text{Alice}} \otimes \mathcal{P}_{\text{Bob}} \otimes \mathcal{A}\,,$$
$$\hat{\Theta}_{\text{Bob}} = \mathbf{1} \otimes_1 \Theta_{\text{Bob}}$$

The label on the tensor product indicates the position where the unit **1** is inserted.

Assumption: Causal factorization of the coupling dynamics

$$\Theta = \hat{\Theta}_{\mathrm{Alice}} \circ \hat{\Theta}_{\mathrm{Bob}}$$

or with ω = state on $\mathcal{A}$, σ_{Alice} = state on $\mathcal{P}_{\mathrm{Alice}}$, and σ_{Bob} = state on $\mathcal{P}_{\mathrm{Bob}}$:

$$\tau(\sigma_{\mathrm{Alice}} \otimes \sigma_{\mathrm{Bob}} \otimes \omega)(\mathbf{1} \otimes \mathbf{1} \otimes C) = \hat{\tau}_{\mathrm{Bob}} \circ \hat{\tau}_{\mathrm{Alice}}(\sigma_{\mathrm{Alice}} \otimes \sigma_{\mathrm{Bob}} \otimes \omega)(\mathbf{1} \otimes \mathbf{1} \otimes C)$$

Causal factorization of localized dynamics typically holds in QFT with pointlike interactions. It is a variant of “Bogoliubov’s formula”.

RHS expresses the expectation value of an observable C in $\mathcal{A}$ on the state ω after first Alice couples it dynamically to her probe in state σ_{Alice} and evaluates on her probe observable **1**, followed by Bob coupling his probe in state σ_{Bob} and evaluating on his probe observable **1**. Evaluating on **1** means forming the partial trace, or the corresponding partial state on $\mathcal{A}$.

Theorem (H. Bostelmann, C.J. Fewster, M. Ruepp (2021))

Let the spacetime regions $O_{\rm Alice}$, $O_{\rm Bob}$ and $O_{\rm Charlie}$ be as before.

Suppose that Alice and Bob use probe QFTs $\mathcal{P}_{\rm Alice}$ and $\mathcal{P}_{\rm Bob}$ to couple to the system QFT $\mathcal{A}$.

The interaction regions are $O_{I,{\rm Alice}} = O_{\rm Alice}$ and $O_{I,{\rm Bob}} = O_{\rm Bob}$.

Alice and Bob have prepared the states of their probes in states $\sigma_{\rm Alice}$ and $\sigma_{\rm Bob}$ (prior to coupling)

If $C \in \mathcal{A}(O_{\rm Charlie})$, then

$$\hat{\tau}_{\rm Bob} \circ \hat{\tau}_{\rm Alice}(\sigma_{\rm Alice} \otimes \sigma_{\rm Bob} \otimes \omega)(\mathbf{1} \otimes \mathbf{1} \otimes C) = \tau_{\rm Bob}(\sigma_{\rm Bob} \otimes \omega)(\mathbf{1} \otimes C)$$

This means that Charlie cannot determine by measurement in $O_{\rm Charlie}$ if Alice has done a (probe-measuring induced) measurement in the causally separated spacetime region $O_{\rm Alice}$, even if Bob carries out a (probe-measuring induced) measurement “in between” in the spacetime region $O_{\rm Bob}$.

The *impossible measurements/impossible operations scenario* does not only arise in QFT, but also in classical field theory. (This is a *ping-pong ball test*, and its outcome shows that the impossible measurements scenario of Sorkin is not specific to QFT.)

There are “superluminal” local operations also in classical field theory, e.g. by local kinematical symmetries. Not all local operations in quantum or classical field theory can be “actively” carried out.

Elements in the local algebras should be interpreted as observables, not as (implementers of) local operations.

Local operations induced by the local measurement scheme in the FV framework do not feature the Sorkin scenario, i.e. no superluminal communication effects occur.