Canonical Quantum Gravity

Thomas Thiemann[1]

[1] Inst. f. Quantengravitation (IQG), FAU Erlangen – Nürnberg

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FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG
NATURWISSENSCHAFTLICHE FAKULTÄT

ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

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Initial value formulation

- Canonical approach to classical & quantum GR has long tradition
- Classical: Initial value formulation, numerical integration of Einstein equations, BH merger simulations ... [Arnowitt,Deser,Misner,..]
- Quantum: QG as a constrained QFT [Bergmann, DeWitt, Dirac, Komar, Wheeler,..]
- Key assumption: Globally hyperbolic spacetimes (M, g)
- $\Rightarrow M \cong \mathbb{R} \times \sigma$ [Geroch, Sanchez & Bilal]
- M can be foliated by leaves $t \mapsto \Sigma_t \cong \sigma$ ("3+1" split: space+time)
- Lapse, shift parametrisation of foliation

$$ds^2 = -[l\ dt]^2 + q_{ab}[dx^a + s^a\ dt]\ [dx^b + s^b\ dt]$$

- Legendre transform for constrained systems (Dirac algorithm): $(g, \dot{g}) \mapsto (q, p)$ (3-metric on σ, conj. momentum), Poisson brackets $\{.,.\}$. Similar for matter.
- plus: spatial diffeomorphism and Hamiltonian constraints $V_a;\ a = 1, 2, 3, \quad S$ (temporal-spatial, temporal-temporal components of Einstein eqns)

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Classical Hypersurface Deformation Algebroid (HDA)

- Let $Q := \sqrt{\det(q)}$, smeared constraints:
  $V[u] = \int_\sigma \; d^3z \; u^a \; V_a; \quad S[f] = \int_\sigma \; d^3z \; f \; S$
- Classical HDA $\mathfrak{h}$ [Hojman, Kuchar, Teitelboim]

$$\{V[u], V[v]\} = -V[[u,v]], \;\; \{V[u], S[f]\} = -S[u[f]],$$

$$\{S[f], \; S[g]\} = -V[q^{-1} \; (M \; dN - N \; dM)]$$

- Observations:
  - encodes local spacetime diffeomorphism covariance
  - $\mathfrak{h}$ not a Lie algebra due to q^{-1}: "open algebra, algebroid"
- Anomaly-free representation of $\mathfrak{h}$ major challenge in quantum reduction (QR) approach to CQG

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Classical reduction approach

- Totally constrained Hamiltonian $C(T) := V[s] + S[l]$
- V, S generator of gauge transf. rather than physical motion
- Observables, physical motion? ("problem of time")
- Relational (Dirac) observables $\Leftrightarrow$ Gauge fixing
- Split geometry, matter canonical pairs into 2 sets $(q, p) = ((Q, P), (X, Y))$ called "true" (observable) and "gauge" (not observable)
- Solve constr. $C = 0$ for $Y = -h(X; Q, P)$, impose gauge cond. $G := X - k = 0$
- Solve stability condition $\frac{d}{dt} G = 0 \quad \Leftrightarrow \quad \{C(T), X\} = \partial_t k$ for $= T_*$
- Reduced (true, physical,...) Hamiltonian for $\mathcal{O} = \mathcal{O}(Q, P)$ [Hanson, Regge, Teitelboim]

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Generalities

- Non-perturbative: PT applied to QG: non-renormalisable [Goroff, Sagnotti 85]
- Background independence: no metric singled out by Einstein eqns.
- Continuum theory: no ad hoc discrete input
- Standard QFT: construct QFT of gravitational field in 3+1
- Algebraic QFT (AQFT) methods: Algebras $\mathfrak{A}$, exp. val. states ω
- Standard AQFT: Fix classical background g_0, construct $\mathfrak{A}_{g_0}(O)$
  If O, O' causally disjoint wrt g_0 then $[\mathfrak{A}_{g_0}(O), \mathfrak{A}_{g_0}(O')] = 0$
- BI violation, in QG $g = g_0$ not fixed, even operator, fluctuating causality
- Algebraic QG (AQG):
  Suppose 1. $\omega(g^2) - \omega(g)^2$ "small" (semiclassical), 2. O, O' causally disjoint wrt $\omega(g)$ then show $\omega([\mathfrak{A}(O), \mathfrak{A}(O')])$ small
- Difference:
  AQFT: g_0 distinguished, used to construct $\mathfrak{A}_{g_0}(O)$
  AQG: g_0 $\nexists$, $\mathfrak{A}(O)$ BI, quantum locality rel. to state only
- How to construct $\mathfrak{A}(O)$ in CQG? Like in usual QFT in CST:
  Solve Heisenberg picture EOM wrt physical H from "time zero" fields

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Algebra and representation

- Spinorial matter requires densitised triad E^a_j; $\delta^{jk} E^a_j E^b_k = Q\, q^{ab}$, conj. SU(2) connection momentum A^j_a (canonical transf.) [Ashtekar,Barbero]
- Additional SU(2) Gauss constraint $Z(r)$
- Suggests Weyl elements based on holonomies, fluxes $\mathcal{P}(e^{\int_p A})$, $e^{\int_S *E}$ (c.f. YM)
- Every single term in Z, H (vacuum GR, cosm. const., matter) couples to E^a_j
- 1st state motivation: Z, H densely defined if vacuum s.t. $E^a_j\, \Omega = 0$
- Proposition: This fixes an irregular state of $\mathfrak{A}$ of [Narnhofer-Thirring] type
- 2nd state motivation: for suitable matter, H inv. under automorphisms α_g, β_φ
- Proposition: The GNS rep. of the LQG state defined by

$$\omega(W[f,F]) = \delta_{f,0}$$

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