Canonical Quantum Gravity

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- Classical Hamiltonian formulation of GR
- Canonical quantisation principles
- Loop quantum gravity (LQG) realisation

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Canonical approach to classical & quantum GR has long tradition

- Classical: Initial value formulation, numerical integration of Einstein equations. BH merger simulations ... [Arnowitt, Deser, Misner...]
- Quantum: QG as a constrained QFT [Bergmann, DeWitt, Dirac, Komar, Wheeler,...]
- Key assumption: Globally hyperbolic spacetimes (M, g)
- $\bigcirc \Rightarrow M \cong \mathbb{R} \times \sigma$ [Geroch, Sanchez & Bilal]
- *M* can be foliated by leaves $t \mapsto \Sigma_t \cong \sigma$ ("3+1" split: space+time)
- Lapse, shift parametrisation of foliation

$$ds^2 = -[I \ dt]^2 + q_{ab}[dx^a + s^a \ dt] [dx^b + s^b \ dt]$$

- Legendre transform for constrained systems (Dirac algorithm): $(g, \dot{g}) \mapsto (g, p)$ (3-metric on σ , conj. momentum), Poisson brackets $\{.,.\}$. Similar for matter.
- plus: spatial diffeomorphism and Hamiltonian constraints V_a ; a = 1, 2, 3, S (temporal-spatial, temporal-temporal components of Einstein egns)

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- Let $Q := \sqrt{\det(q)}$, smeared constraints: $V[u] = \int_{\sigma} d^3z \ u^a \ V_a$; $S[f] = \int_{\sigma} d^3z \ f \ S$
- Classical HDA h [Hojman, Kuchar, Teitelboim]

$$\{V[u], V[v]\} = -V[[u, v]], \{V[u], S[f]\} = -S[u[f]],$$
$$\{S[f], S[g]\} = -V[\mathbf{q}^{-1} (M dN - N dM)]$$

Observations:

encodes local spacetime diffeomorphism covariance
 finot a Lie algebra due to g⁻¹; "open algebra, algebroid"

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- Totally constrained Hamiltonian C(T) := V[s] + S[I]
- V, S generator of gauge transf. rather than physical motion
- Observables, physical motion? ("problem of time"
- Relational (Dirac) observables

 Gauge fixing
- Split geometry, matter canonical pairs into 2 sets (q, p) = ((Q, P), (X, Y)) called "true" (observable) and "gauge" (not observable)
- Solve constr. C = 0 for Y = -h(X; Q, P), impose gauge cond. G := X k = 0
- Solve stability condition $\frac{d}{dt}G = 0 \Leftrightarrow \{C(T), X\} = \partial_t k \text{ for } = T$
- lacktriangledown Reduced (true, physical,...) Hamiltonian for $\mathcal{O}=\mathcal{O}(Q,P)$ [Hanson, Regge, Teitelboim]

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- Background independence: no metric singled out by Einstein eqns.
- Continuum theory: no ad hoc discrete input
- Standard QFT: construct QFT of gravitational field in 3+1
- Algebraic QFT (AQFT) methods: Algebras \mathfrak{A} , exp. val. states ω
- Standard AQFT: Fix classical background g_0 , construct $\mathfrak{A}g_0(O)$ If O, O' causally disjoint wrt g_0 then $[\mathfrak{A}g_0(O), \mathfrak{A}g_0(O')] = 0$
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- AQFT: g_0 distinguished, used to construct $\mathfrak{A}_{g_0}(O)$
- How to construct \(\O(O) \) in CQG? Like in usual QFT in CST:
 Solve Heisenberg picture EOM wrt physical H from "time zero" fields

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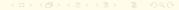
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- Study states ω on $\mathfrak A$ (e.v. functionals): $(\rho, \mathcal H, \Omega)$ GNS data
- Select ω s.t. $\rho(C(T))$, $\rho(\{C(T), C(T')\})$ resp. $\rho(H)$ densely defined operators on $\mathcal{D} := \rho(\mathfrak{A})\Omega$.
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 - \bullet QR: \mathcal{D} inv. for $\rho(\mathcal{C}(T))$ and anomaly freeness of \mathfrak{h}
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- Additional SU(2) Gauss constraint Z(r)
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- Spinorial matter requires densitised triad E_j^a ; $\delta^{jk}E_j^aE_k^b=Q\ q^{ab}$, conj. SU(2) connection momentum \mathcal{A}_a^j (canonical transf.) [Ashtekar,Barbero]
- Additional SU(2) Gauss constraint Z(r)
- Suggests Weyl elements based on holonomies, fluxes $\mathcal{P}(e^{\int_{\mathcal{P}} A})$, $e^{\int_{\mathcal{S}} *E}$ (c.f. YM)
- Every single term in Z, H (vacuum GR, cosm. const., matter) couples to E_i^a
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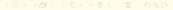
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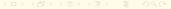
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- ρ(Z(r)) densely def., explicit normalisable soln: SU(2) inv. intertwiner subspace (CG coeff.)
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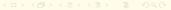
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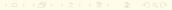
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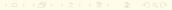
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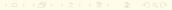
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