Trace dynamics, octonions, and unification

- From quantum foundations to the $E_8 \times \omega E_8$ octonionic theory of unification -

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Motivation behind this unification program

- In quantum theory, classical spacetime is an extrinsic concept, possible only when the universe is dominated by macroscopic, classical bodies.

- This makes quantum theory depend on its own limit, and a more general formulation will remove classical time from quantum theory [*at all energy scales*]

- We develop such a generalized formalism in the following steps :

1. Raise classical dynamical degrees to the status of matrices/operators [Trace Dynamics]

2. Replace every spacetime point by the 16D split-bioctonion.

3. Describe evolution by the so-called Connes time - unique to a non-commutative geometry and absent in Riemannian geometry.

4. Use the Chamseddine-Connes spectral action principle to prescribe a trace dynamics Lagrangian for the theory.

5. Prescribe a gauge symmetry group for the Lagrangian, which we choose to be $E_8 \times \omega E_8$.

This leads to a falsifiable pre-spacetime, pre-quantum theory of unification of the fundamental forces.

- Spontaneous symmetry breaking at the electroweak scale leads to the emergence of the standard model of particle physics, and of general relativity.

- Quantum theory and general relativity are emergent phenomena, with classical spacetime emerging only when the universe becomes dominated by classical bodies.

- The formalism, based on the exceptional Jordan algebra of the octonions, permits us to derive mass ratios of quarks and charged leptons from first principles.

- *We now briefly explain the aforesaid steps* -

Some references

- *Trace dynamics, octonions, and unification, TPS,* arXiv:2501.18139 (2025)

- *An $E_8 \times E_8$ unification of the standard model and pre-gravitation, on an exceptional Lie-algebra valued space,* Priyank Kaushik, Vatsalya Vaibhav, TPS, arXiv:2206.06911 (2022)

- *Fermion mass ratios from the exceptional Jordan algebra, TPS,* arXiv:2508.10131 (2025)

- *Spacetime and internal symmetry space from split bioctonions and the two extra SU(3)'s of $E_8 \times E_8$, TPS, https://www.preprints.org/manuscript/202510.0437/v1 (2025)*

- *Gravi-weak unification in 6D spacetime SO(3,3),* Torsten-Asselmeyer Maluga, Niels Gresnigt, Jose M. Isidro, Antonino Marciano, Claudio F. Paganini, TPS, P Samuel Wesley [*in preparation]*

- *Causal fermion systems, trace dynamics and the spectral action principle,* Felix Finster, Shane Farnsworth, Claudio F. Paganini, TPS (in preparation)

1. Raise classical dynamical degrees to the status of matrices/operators [Trace Dynamics]

- From Newtonian dynamics to trace dynamics -

$$S = \sum_i \int d\tau \, \frac{1}{2} m_i \left(\frac{dq_i}{d\tau} \right)^2$$

From real numbered dynamical variables to matrices: $q \rightarrow \mathbf{q}$

$$S = \sum_i \int d\tau \, Tr \left[\frac{1}{2} \frac{L_p^2}{L^2} \left(\frac{d\mathbf{q}_i}{d\tau} \right)^2 \right]$$

Novel conserved charge: $\tilde{C} = \sum_i [\mathbf{q}_{Bi}, \mathbf{p}_{Bi}] - \sum_j \left\{ \mathbf{q}_{Fj}, \mathbf{p}_{Fj} \right\}$: Pre-quantum theory

(Adler-Millard charge)

1. Trace dynamics —> Quantum (field) theory as an emergent phenomenon

- Assume trace dynamics to hold at some time resolution τ_P
- It is a Lorentz-invariant matrix-valued Lagrangian / Hamiltonian dynamics.
- Ask: what is the emergent dynamics at time resolution $\tau \gg \tau_P$? (Assuming H is self-adjoint)
- Statistical thermodynamics is used to show that the novel charge is equipartitioned.

$$[q_B, p_B] = i\hbar \quad ; \quad \{q_F, p_F\} = i\hbar$$

- Heisenberg equations of motion, and QFT, emerge at thermodynamic equilibrium.

2. Replace every spacetime point by the 16D split-bioctonion

- Octonions as coordinate systems: a non-commutative manifold -

- Internal space + spacetime ——> Octonionic Space

- Algebra automorphisms and unification: no change in energy scale.

- Normed division algebras: reals, complex numbers, quaternions, octonions. $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$

- Quaternions: $q = a_0 + a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

Dirac operator is `gradient operator'

$$\nabla = \hat{i}\frac{d}{dx} + \hat{j}\frac{d}{dy} + \hat{k}\frac{d}{dz}$$

- Octonions: $O = a_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3 + a_4\mathbf{e}_4 + a_5\mathbf{e}_5 + a_6\mathbf{e}_6 + a_7\mathbf{e}_7$

- Lorentz groups: $SL(2,\mathbb{C}) \sim SO(1,3)$; $SL(2,\mathbb{H}) \sim SO(1,5)$; $SL(2,\mathbb{O}) \sim SO(1,9)$

- Bosons and fermions live in octonionic space: ——-> Split bioctonionic space: chiral fermions

$$Q_B = Q_0 + Q_1\mathbf{e}_1 + Q_2\mathbf{e}_2 + Q_3\mathbf{e}_3 + Q_4\mathbf{e}_4 + Q_5\mathbf{e}_5 + Q_6\mathbf{e}_6 + Q_7\mathbf{e}_7$$

3. Describe evolution by the so-called Connes time - unique to a non-commutative geometry and absent in Riemannian geometry.

- Choosing the Lagrangian: Spectral action principle -

- The eigenvalues of the Dirac operator D on a curved space-time manifold:

$$Tr[L_P^2 D^2] \sim L_P^{-2} \int d^4x \, \sqrt{g} \, R + \mathcal{O}(L_P^0) = \sum_i \lambda_i^2$$

- An `atom' of space-time: $\lambda_i \to \hat{\lambda}_i \equiv \dot{q}_{Bi}$ $\qquad \lambda_i^2 \to \int d\tau \, Tr[\dot{q}_{Bi}^2]$ Connes Time τ

- An atom of space-time-matter (an `aikyon') :

$$\frac{S}{\hbar} = \int \frac{d\tau}{\tau_{Pl}} \, Tr\left\{ \frac{L_P^2}{L^2} \left[\dot{q}_B^\dagger + \frac{L_P^2}{L^2} \beta_1 \dot{q}_F^\dagger \right] \times \left[\dot{q}_B + \frac{L_P^2}{L^2} \beta_2 \dot{q}_F \right] \right\}$$

- Including Yang-Mills fields

$$\frac{S}{\hbar} = \int \frac{d\tau}{\tau_{Pl}} \, Tr\left\{ \frac{L_P^2}{L^2} \left[\left(\dot{q}_B^\dagger + i\frac{\alpha}{L} q_B^\dagger \right) + \frac{L_P^2}{L^2} \beta_1 \left(\dot{q}_F^\dagger + i\frac{\alpha}{L} q_F^\dagger \right) \right] \times \left[\left(\dot{q}_B + i\frac{\alpha}{L} q_B^\dagger \right) + \frac{L_P^2}{L^2} \beta_2 \left(\dot{q}_F + i\frac{\alpha}{L} q_F \right) \right] \right\}$$

4. A trace dynamics Lagrangian for the unified theory

$$\frac{S}{\hbar} = \frac{1}{2}\int \frac{d\tau}{\tau_{Pl}}\, Tr\left[\frac{L_p^2}{L^2}\dot{Q}_1^\dagger\, \dot{Q}_2\right]$$: An atom of space-time-matter

Bosons and fermions: $\dot{Q}_1^\dagger = \dot{Q}_B^\dagger + \frac{L_p^2}{L^2}\beta_1 \dot{Q}_F^\dagger; \qquad \dot{Q}_2 = \dot{Q}_B + \frac{L_p^2}{L^2}\beta_2 \dot{Q}_F$

Dotted & undotted variables: $\dot{Q}_B = \frac{1}{L}(i\alpha q_B + L\dot{q}_B); \qquad \dot{Q}_F = \frac{1}{L}(i\alpha q_F + L\dot{q}_F);$

Define: $q_1 = q_B + \beta_1 \frac{L_P^2}{L^2} q_F\ ; \qquad q_2 = q_B + \beta_2 \frac{L_P^2}{L^2} q_F$

Equations of Motion: $\ddot{q}_1 = -\frac{\alpha^2}{L^2} q_1\ ; \qquad \ddot{q}_2 = -\frac{\alpha^2}{L^2} q_2$

Assume for the Lagrangian: $E_8 \times \omega E_8$ symmetry

5. Prescribe a gauge symmetry group for the Lagrangian, which we choose to be $E_8 \times \omega E_8$

- Spontaneous symmetry breaking at the electroweak scale leads to the emergence of the standard model of particle physics, and of general relativity.

- The assumed decomposition of the unified gauge group:

$$E_{8L} \rightarrow SU(3)_I^{geom} \times E_{6L} \xrightarrow{E_{6L}} SU(3)_c \times SU(3)_{F,L} \times SU(3)_L \xrightarrow{SU(3)_L} SU(2)_L \times U(1)_Y$$

$$E_{8R} \rightarrow SU(3)_{II}^{geom} \times E_{6R} \xrightarrow{E_{6R}} SU(3)_{c'} \times SU(3)_{F,R} \times SU(3)_R \xrightarrow{SU(3)_R} SU(2)_R \times U(1)_{Ydem}$$

- Here, $SU(3)_I^{geom} \times \omega SU(3)_{II}^{geom}$ give rise to spacetime and internal symmetry space.
- $E_L^6 \times E_R^6$ gives rise to the SM, GR, a new force $U(1)_{Ydem}$, a 2nd Higgs, and sterile ν
- Atoms of spacetime-matter ——-> spacetime + internal space + matter content

(https://www.preprints.org/manuscript/202510.0437/v1)

Theoretically

Derived:

$$\delta = \sqrt{\frac{3}{8}}$$

SQUARE-ROOT MASS RATIO	THEORETICAL DERIVATION	THEORY VALUE (DECIMALS)	EXXPERIMENTAL @M_Z
$\sqrt{m_u/m_e}$	2	2.00	1.59 ± 0.14 (RG: 1.50 - 2.10)
$\sqrt{m_d/m_e}$	3	3.00	2.34 ± 0.08
$\sqrt{m_s/m_d}$	$\frac{1+\delta}{1-\delta}$	4.159592	4.46 ± 0.25
$\sqrt{m_b/m_s}$	$\frac{1+\delta}{1-\delta}\cdot(1+\delta)$	6.706811	7.37 ± 0.34
$\sqrt{m_\tau/m_\mu}$	$\frac{1+\delta}{1-\delta}$	4.159592	4.11930 ± 0.00006
$\sqrt{m_\mu/m_e}$	$\frac{1+\delta}{1-\delta}\cdot\frac{\delta+1/3}{\delta-1/3}$	14.097486	14.543 ± 0.000
$\sqrt{m_c/m_u}$	$\frac{2/3+\delta}{2/3-\delta}$	23.557551	22.43 ± 2.45
$\sqrt{m_t/m_c}$	$\frac{2/3}{2/3-\delta}$	12.278775	16.65 ± 1.14 (RG: 15.96-17.19)

We derive mass ratios of quarks and charged leptons: arXiv:2508.10131 [hep-ph]

Left-Right symmetry breaking: emergence of classical spacetime and gravitation

Why 4D?

$Cl(7) \rightarrow Cl(3)$

- Critical entanglement amongst fermions leads to the quantum - classical transition, and emergence of 4D classical curved space-time. $SU(2)_L \times SU(2)_R$ breaks.

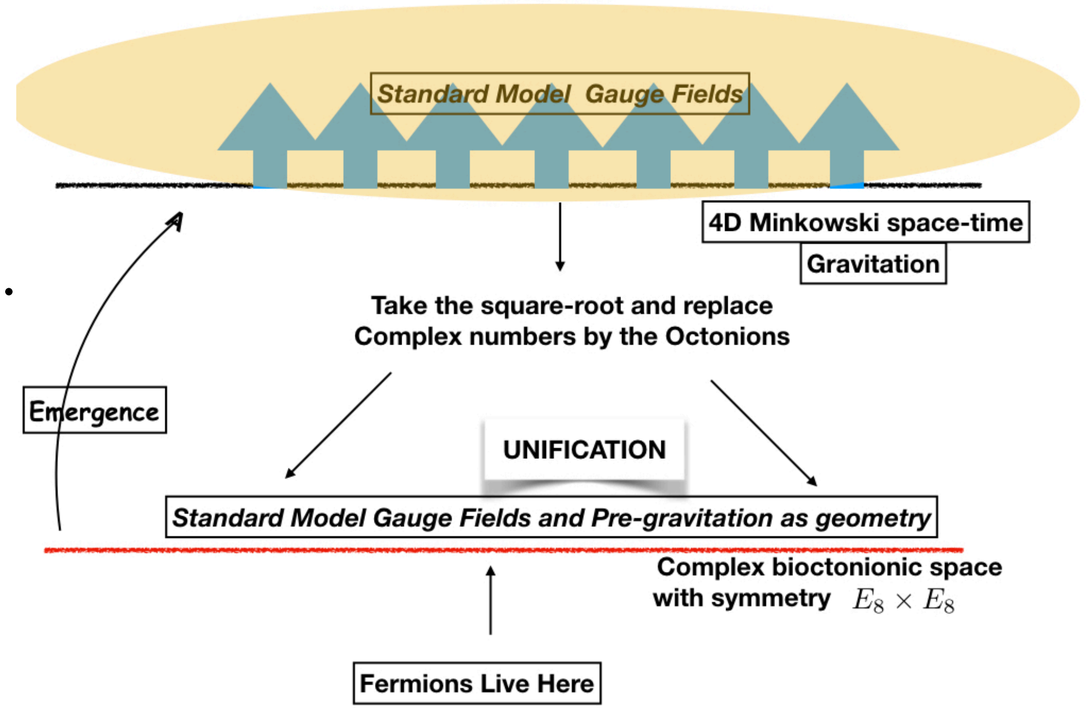
$$\int d\tau\, Tr\left[D_1^2 + D_2^2 + D_3^2 + \ldots\right] \longrightarrow \int d\tau \left[\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \ldots\right] = \int d\tau\, Tr[D^2] \propto \int d\tau \int d^4x \sqrt{-g}\, [R + \mathscr{L}_{matter}]$$

- Only classical systems live in 4D spacetime [compactification without compactification].

(Gravi-Weak is 6D)

- Quantum systems obey L-R symmetry (group $E_8 \otimes E_8$).

- The mechanism in the early universe is the same as is in the laboratory today: quantum entanglement.



[*This slide is work in progress!*]

Predictions, and Outlook

1. Mass ratio prediction: $\sqrt{\frac{m_\tau}{m_\mu}} = \sqrt{\frac{m_s}{m_d}}$
2. The neutrino is Majorana.
3. There is a second, BSM Higgs.
4. There is a fifth force: dark electromagnetism (dark photon) : sourced by $\pm\sqrt{m}$
5. There is a deterministic, local theory underlying quantum mechanics (Bell OK).
6. We predict spontaneous collapse [collapse models].
7. Karolyhazy length uncertainty relation: $(\Delta l)^3 = L_p^2\, l$
8. Still to be understood: origin of neutrino masses, nature of dark matter and dark energy, universe before EW symmetry breaking; why only fermions localise?

Summary

- There must exist, at all energy scales, a reformulation of quantum field theory which does not depend on classical time.
- We have constructed one such formulation, using Adler's Trace Dynamics.
- Furthermore, spacetime points are replaced by the 16D split bioctonions.
- Connes time is used to describe evolution.
- The trace dynamics Lagrangian is motivated by the spectral action principle.
- $E_8 \times E_8$ unification gauge symmetry is proposed - symmetry breaking.
- Spacetime, gravitation, and quantum theory, are emergent phenomena.
- We have a falsifiable candidate theory of unification [the octonionic / aikyon theory].