Constructive gravity

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Classical models of matter and gravity

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Postulate the action for matter fields	Postulate the action for the underlying geometry

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History of the standard model:
Keep postulating actions
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History of general relativity
Einstein told us so.

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Postulate the action for matter fields	Postulate the action for the underlying geometry
History of the standard model: Keep postulating actions until phenomenology explained	History of general relativity Einstein told us so. Alternatives not mainstream

Constructive gravity

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Central result

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Postulate the
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provided
physically weak
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$$S_{\mathrm{matter}}[\Phi, G] \qquad + \qquad S_{\mathrm{gravity}}[G]$$

Postulate the
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Obtain action
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Central result

common canonical evolution

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Warm-up:

Maxwell implies Einstein

$$\boxed{\mathcal{L}_{\text{gravity}}[G]}$$

$$\boxed{\mathcal{L}_{\text{matter}}[\Phi; G)}$$

$$\boxed{\mathcal{L}_{\text{gravity}}[G]}$$

Maxwell theory

$$\boxed{\mathcal{L}_{\text{matter}}[\Phi; G)} = \sqrt{-\det G}\, G^{ac} G^{bd}\, F_{ab} F_{cd}$$

Einstein-Hilbert

$$\boxed{\mathcal{L}_{\text{gravity}}[G]} = \kappa\sqrt{-\det G}\,(R[G] - 2\Lambda)$$

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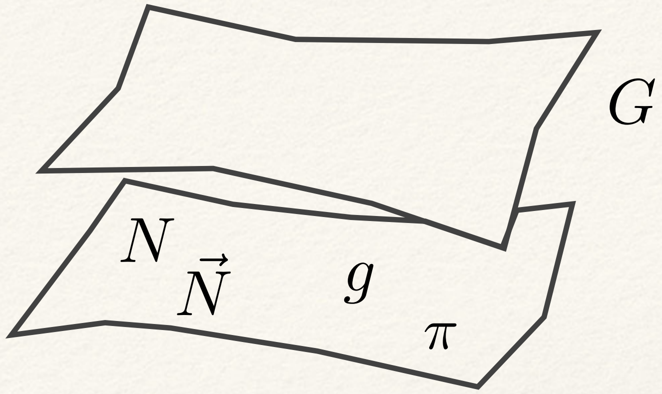

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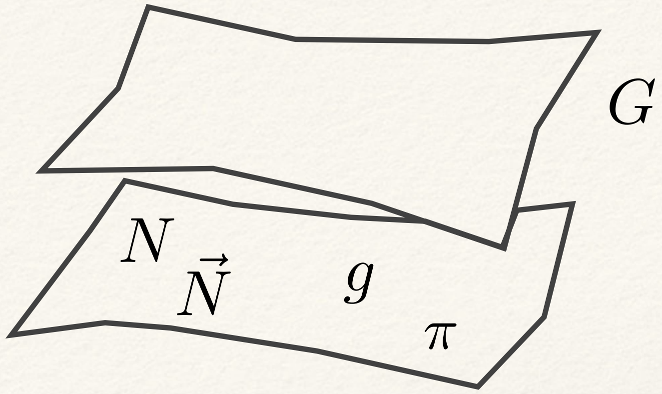
initial value formulation



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$\Downarrow\Uparrow$ ADM '60

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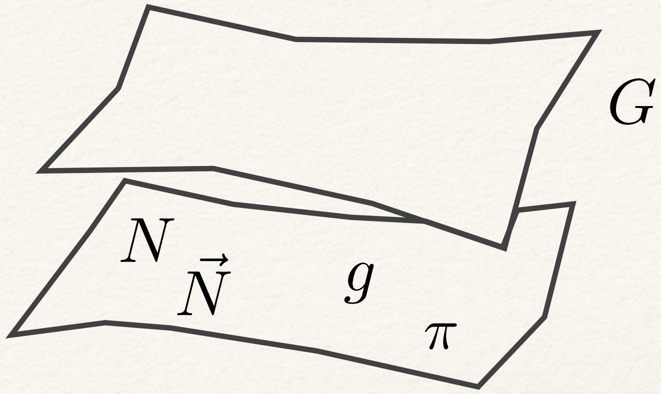
$$C_{[g,\pi]}(N) \;\dot{+}\; D_{[g,\pi]}(\vec{N})$$



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Kuchar '74

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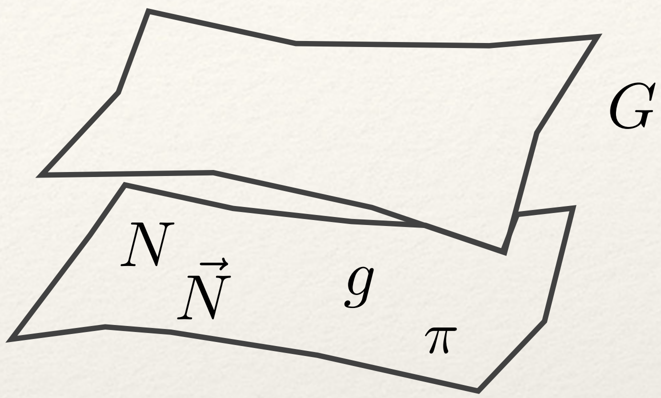

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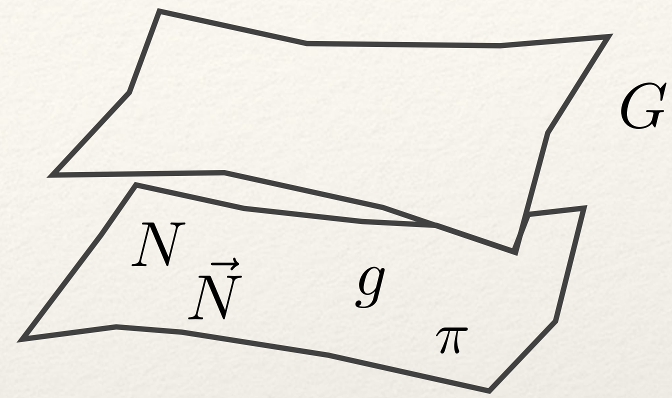

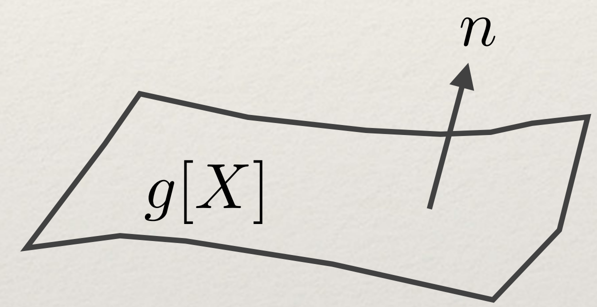

$$\mathcal{C}(N) = \int N\, n^a \frac{\delta}{\delta X^a}$$

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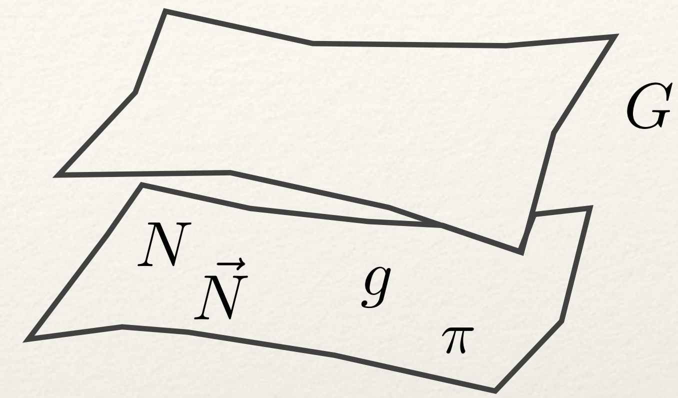

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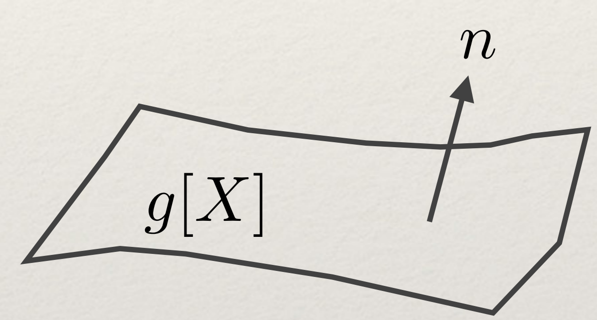

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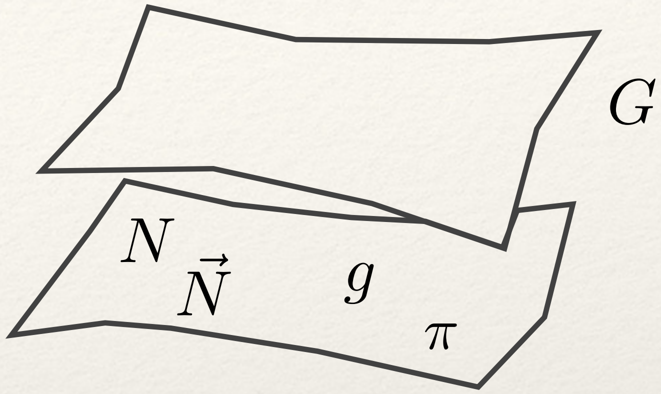

HKT '76

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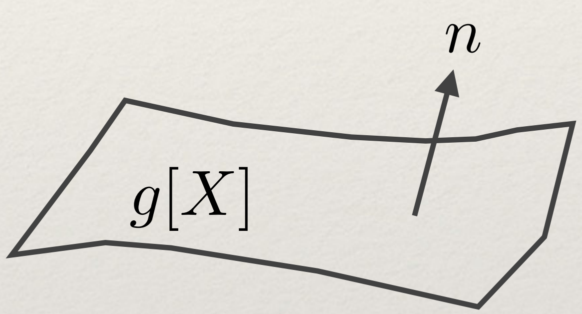

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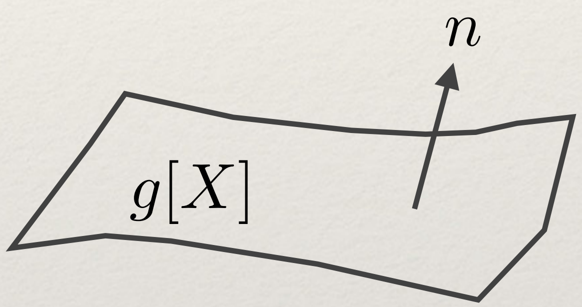

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HKT '76

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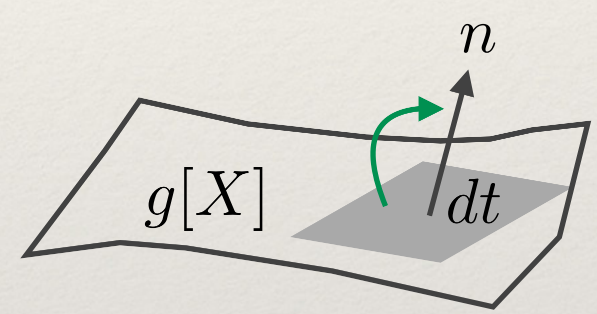

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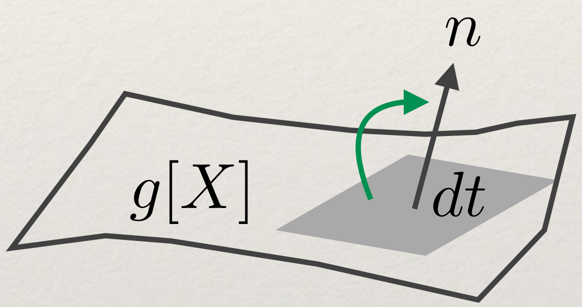

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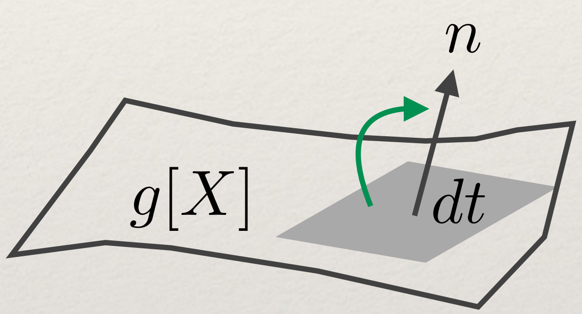

GSSW '12

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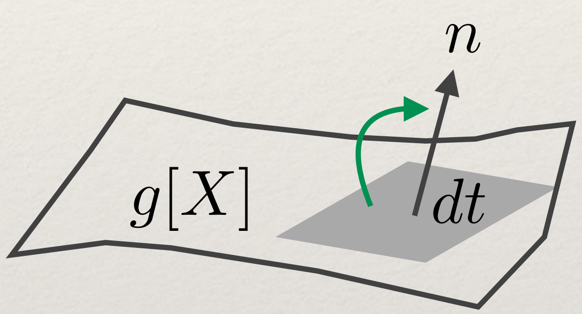

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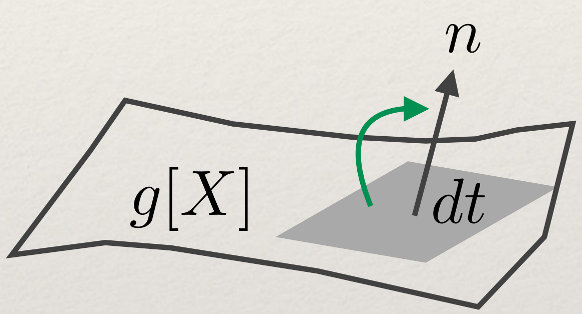

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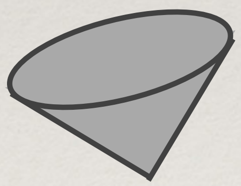

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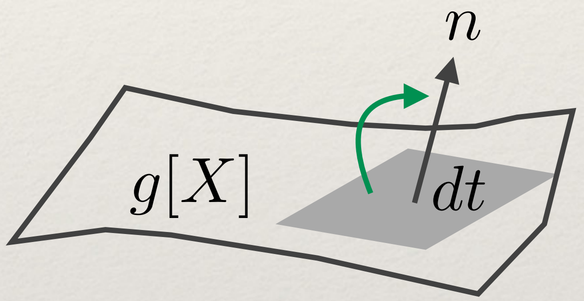

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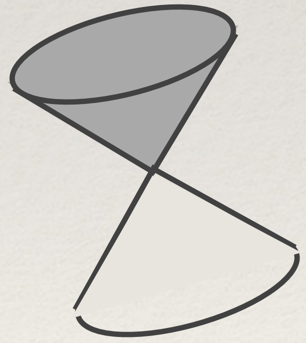

$T_x^* M$

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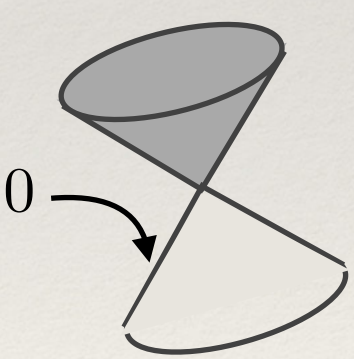
GSSW '12

Legendre map formulation

$$n := \frac{G^{-1}(dt, \cdot)}{G^{-1}(dt, dt)}$$

$P = 0$



T_x^*M

Maxwell theory

$$\mathcal{L}_{\text{matter}}[\Phi; G] = \sqrt{-\det G}\; G^{ac}G^{bd}\,F_{ab}F_{cd}$$

Einstein-Hilbert

$$\mathcal{L}_{\text{gravity}}[G] = \kappa\sqrt{-\det G}\,(R[G] - 2\Lambda)$$

ADM '60

initial value formulation

$$C_{[g,\pi]}(N) \quad \dot{+} \quad D_{[g,\pi]}(\vec{N})$$

Kuchar '74

constraint algebra formulation

$$\begin{aligned}
\{D(\vec{N}), D(\vec{M})\} &= D([\vec{N}, \vec{M}]) \\
\{D(\vec{N}), C(M)\} &= C(\vec{N}M) \\
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\end{aligned}$$

HKT '76

deformation algebra formulation

$$\begin{aligned}
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RRS '10

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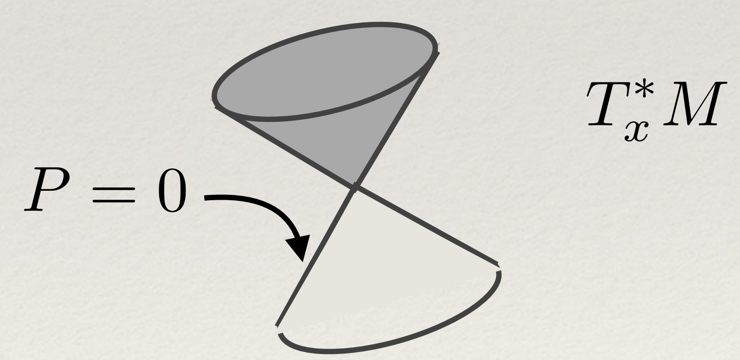

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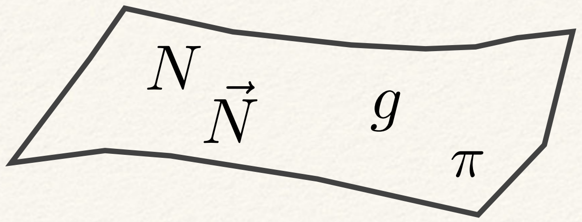

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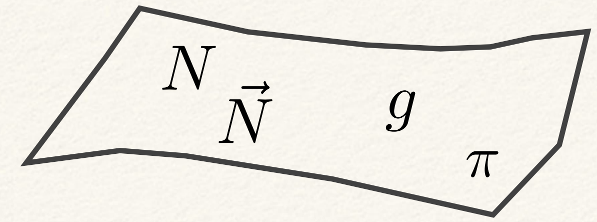

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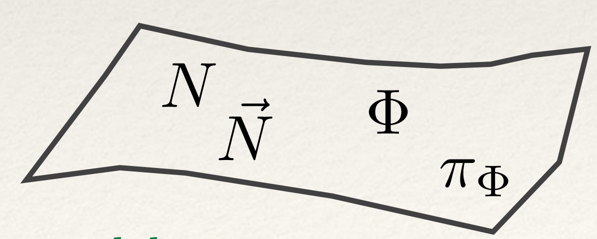

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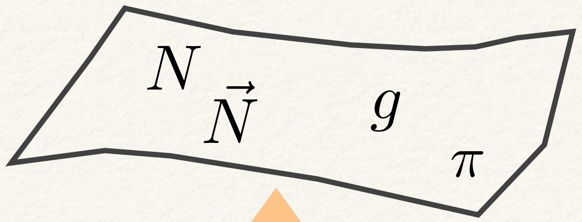
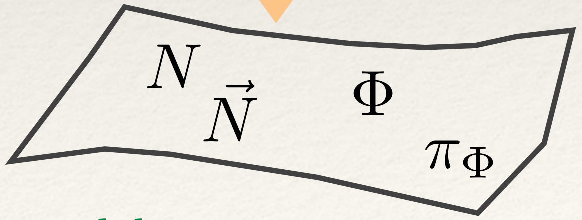


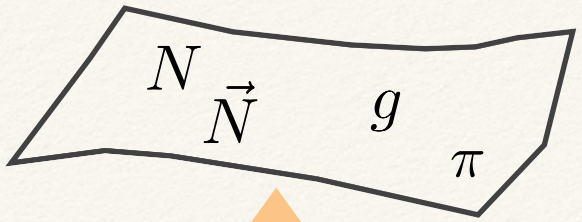
must be shared
for common evolution



Einstein-Hilbert — $\mathcal{L}_{\text{gravity}}[G] = \kappa\sqrt{-\det G}\,(R[G] - 2\Lambda)$

ADM '60

initial value formulation — $C_{[g,\pi]}(N) \;\dot{+}\; D_{[g,\pi]}(\vec{N})$



Kuchar '74

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GSSW '12

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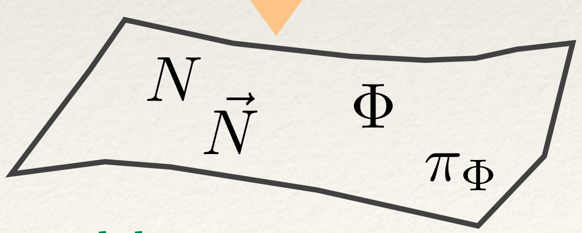
RRS '10

Principal polynomial formulation — $P(k) = G^{-1}(k, k)$



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$$P(k) = G^{-1}(k, k)$$

$$\mathcal{L}_{\text{matter}}[\Phi; G] = \sqrt{-\det G}\; G^{ac}G^{bd}\, F_{ab}F_{cd}$$

Maxwell theory

General case

$$\mathcal{L}_{\text{gravity}}[G]$$

$$\mathcal{L}_{\text{matter}}[\Phi; G)$$

$\mathcal{L}_{\text{gravity}}[G]$

canonically
quantizable

$\mathcal{L}_{\text{matter}}[\Phi; G)$

$$\mathcal{L}_{\text{gravity}}[G]$$

$$P_G(k, k, \ldots, k)$$

$$\uparrow$$

$$\mathcal{L}_{\text{matter}}[\Phi; G)$$

canonically

quantizable

$\mathcal{L}_{\text{gravity}}[G]$

hyperbolic principal polynomial
hyperbolic dual polynomial

canonically
quantizable

$P_G(k, k, \ldots, k)$

$\uparrow$

$\mathcal{L}_{\text{matter}}[\Phi; G)$

$\mathcal{L}_{\text{gravity}}[G]$

Legendre map

$$n := \frac{P_G(dt, dt, \dots, dt, \cdot)}{P_G(dt, \dots, dt)}$$

$\uparrow$ RRS '10

hyperbolic principal polynomial

hyperbolic dual polynomial

$P_G(k, k, \dots, k)$

$\uparrow$

canonically

quantizable

$\mathcal{L}_{\text{matter}}[\Phi; G)$

$$\mathcal{L}_{\text{gravity}}[G]$$

deformation algebra

$$\begin{aligned}
\left[\mathcal{D}(\vec{N}),\mathcal{D}(\vec{M})\right] &= \mathcal{D}([\vec{N},\vec{M}])\\
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\end{aligned}$$

$\uparrow$

Legendre map

$$n := \frac{P_G(dt,dt,\dots,dt,\cdot)}{P_G(dt,\dots,dt)}$$

$\uparrow$ RRS '10

hyperbolic principal polynomial

hyperbolic dual polynomial

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Legendre map

$$n := \frac{P_G(dt, dt, \ldots, dt, \cdot)}{P_G(dt, \ldots, dt)}$$

$$p_G(\cdot, \cdot) := P_G(dt, \ldots, dt, \cdot, \cdot)$$

RRS '10

hyperbolic principal polynomial

hyperbolic dual polynomial

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$$\mathcal{L}_{\text{matter}}[\Phi; G)$$

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Hamiltonian

$$C_{[\varphi^A,\pi_A]}(N) \quad \dot{+} \quad D_{[\varphi^A,\pi_A]}(\vec{N})$$

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RRS '10

hyperbolic principal polynomial
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$$\mathcal{L}_{\text{matter}}[\Phi; G)$$

Lagrangian for the geometry

$$\mathcal{L}_{\text{gravity}}[G]$$

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Construction equations

Lagrangian for the geometry

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Lagrangian for the geometry

$\mathcal{L}_{\text{gravity}}[G]$

„Construction equations“

DSSW '16

hyperbolic principal polynomial

hyperbolic dual polynomial

$P_G(k, k, \dots, k)$

canonically

quantizable

$\mathcal{L}_{\text{matter}}[\Phi; G)$

GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^\mu E^A{}_\mu - \partial_\gamma N^\mu F^A{}_\mu{}^\gamma := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}}\left(\mathcal{L}_{\vec{N}}\widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}}(\widehat{g}(\varphi))M^{\mathscr{A}_i\,\gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi;K] := \sum_{N=0}^{\infty} C_{A_1\dots A_N}[\varphi]K^{A_1}\cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

(C1) $$0 = -(\partial_\mu C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1\dots\alpha_K} E^A{}_{\mu,\alpha_1\dots\alpha_K}$$

(C2) $$0 = C_A E^A{}_{\mu:B} - \partial_\gamma\left(C_A E^A{}_{\mu:B}{}^\gamma\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1\dots\alpha_K} E^A{}_{\mu,\alpha_1\dots\alpha_K}$$

(C3) $$0 = -C\delta^\gamma_\mu + \sum_{K=0}^{\infty}(K+1)C_{:A}{}^{\gamma\alpha_1\dots\alpha_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - \sum_{K=0}^{\infty}(K+1)C_{:A}{}^{(\alpha_1\dots\alpha_K|}F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1\dots\alpha_K}$$

(C4) $$0 = -C_A\left(E^A{}_{\mu:B}{}^\gamma + F^A{}_\mu{}^\gamma{}_{:B}\right) + \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{\gamma\alpha_1\dots\alpha_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{(\alpha_1\dots\alpha_K|}F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1\dots\alpha_K}$$

(C5) $$0 = 2\left(\deg P - 1\right)p^{(\mu|\rho}C_{AB}F^A{}_\rho{}^{|\nu)} + \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{\alpha_1\dots\alpha_K(\mu|}M^{A|\nu)}{}_{,\alpha_1\dots\alpha_K} - \sum_{K=0}^{\infty}(-1)^K\binom{K+2}{K}\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu\nu}\right)$$

(C6) $$0 = 2\left(\deg P - 1\right)C_{AB}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty}C_{B:A}{}^{\alpha_1\dots\alpha_K}M^{A\mu}{}_{,\alpha_1\dots\alpha_K} - \sum_{K=0}^{\infty}(-1)^K(K+1)\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu}\right)$$

(C7) $$0 = 2\partial_\mu\left(C_A M^{A[\mu|}{}_{:B}M^{B|\gamma]}\right) - 2\left(\deg P - 1\right)p^{\rho\gamma}\left[C_A E^A{}_\rho + \partial_\mu\left(C_A F^A{}_\rho{}^\mu\right)\right] + \sum_{K=0}^{\infty}C_{:A}{}^{\alpha_1\dots\alpha_K}M^{A\gamma}{}_{,\alpha_1\dots\alpha_K} +$$
$$+\sum_{K=0}^{\infty}\sum_{J=0}^{K}(-1)^J\binom{K}{J}(J+1)\partial^J_{\alpha_1\dots\alpha_J}\left(C_{:A}{}^{\beta_1\dots\beta_{K-J}(\alpha_1\dots\alpha_J|}M^{A|\gamma)}{}_{,\beta_1\dots\beta_{K-J}}\right)$$

(C8) $$0 = 6\left(\deg P - 1\right)C_{AB_1B_2}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - 4C_{A(B_1}M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A}M^{A\mu} - 2C_{B_1B_2:A}{}^\alpha M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta}M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^\mu$$
$$-\sum_{K=0}^{\infty}(-1)^K(K+1)\left(\partial^K_{\alpha_1\dots\alpha_K}C_{B_1:B_2}{}^{\mu\alpha_1\dots\alpha_K}\right)$$

(C9) $$0 = \sum_{K=2}^{\infty}\sum_{J=2}^{K}(-1)^J\binom{K}{J}(J-1)\partial^{J+1}_{\gamma\alpha_1\dots\alpha_J}\left(C_{:A}{}^{\beta_1\dots\beta_{K-J}(\alpha_1\dots\alpha_J|}M^{A|\gamma)}{}_{,\beta_1\dots\beta_{K-J}}\right)$$

$$\vdots$$

⋮

THE SIXTEEN SEQUENCES FOR $N \geq 2$

$$\mathbf{(C10}_{N}\mathbf{)} \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_{\mu}{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}} \right) - C_{:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1}|} F^A{}_{\mu}{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K} \right]$$

$$\mathbf{(C11}_{N}\mathbf{)} \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{B:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_{\mu}{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}} \right) - C_{B:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1}|} F^A{}_{\mu}{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K} \right]$$

$$\mathbf{(C12}_{N}\mathbf{)} \quad 0 = N C_{A(B_1 \dots B_{N-1}} E^A{}_{|\mu| : B_N)} - (\partial_\mu C_{B_1 \dots B_N}) + C_{B_1 \dots B_N : A} E^A{}_\mu + C_{B_1 \dots B_N : A}{}^{\alpha} E^A{}_{\mu,\alpha} + C_{B_1 \dots B_N : A}{}^{\alpha\beta} E^A{}_{\mu,\alpha\beta}$$

$$\mathbf{(C13}_{N}\mathbf{)} \quad 0 = -C_{B_1 \dots B_N} \delta^\gamma_\mu - N C_{A(B_1 \dots B_{N-1}} F^A{}_{|\mu|}{}^{\gamma}{}_{:B_N)} - C_{B_1 \dots B_N : A} F^A{}_\mu{}^\gamma + C_{B_1 \dots B_N : A}{}^{\gamma} E^A{}_\mu - C_{B_1 \dots B_N : A}{}^{\alpha} F^A{}_\mu{}^\gamma{}_{,\alpha} + 2 C_{B_1 \dots B_N : A}{}^{\alpha\gamma} E^A{}_{\mu,\alpha} - C_{B_1 \dots B_N : A}{}^{\alpha\beta} F^A{}_\mu{}^\gamma{}_{,\alpha\beta}$$

$$\mathbf{(C14}_{N}\mathbf{)} \quad 0 = C_{AB_J(B_1 \dots \widehat{B_J} \dots B_{N-1}} E^A{}_{|\mu| : B_N)}{}^{\gamma} - C_{B_1 \dots B_N} \delta^\gamma_\mu \qquad \text{for} \quad J = 1 \dots N$$

$$\mathbf{(C15}_{N}\mathbf{)} \quad 0 = C_{B_1 \dots B_N : A}{}^{\beta_1 \beta_2} E^A{}_\mu - C_{B_1 \dots B_N : A}{}^{(\beta_1|} F^A{}_\mu{}^{|\beta_2)} - 2 C_{B_1 \dots B_N : A}{}^{\alpha(\beta_1|} F^A{}_\mu{}^{|\beta_2)}{}_{,\alpha}$$

$$\mathbf{(C16}_{N}\mathbf{)} \quad 0 = C_{B_1 \dots B_N : A}{}^{(\alpha\beta|} F^A{}_\mu{}^{|\gamma)}$$

$$\mathbf{(C17}_{N}\mathbf{)} \quad 0 = C_{B_1 \dots B_N : A}{}^{(\mu\nu|} M^{A|\gamma)}$$

$$\mathbf{(C18}_{N}\mathbf{)} \quad 0 = C_{AB_1 \dots B_{N-1}} \left(M^{B[\mu|} M^{A|\nu]}{}_{:B} + (\deg P - 1)\, p^{\rho[\mu|} F^A{}_\rho{}^{|\nu]} \right)$$

$$\mathbf{(C19}_{N}\mathbf{)} \quad 0 = C_{B_1 \dots \widehat{B_J} \dots B_{N+1} : B_J}{}^{\mu\nu} - C_{B_1 \dots B_N : B_{N+1}}{}^{\mu\nu} \qquad \text{for} \quad J = 1 \dots N+1$$

$$\mathbf{(C20}_{N}\mathbf{)} \quad 0 = N \cdot (N+1)\, (\deg P - 1)\, C_{AB_1 \dots B_N} p^{\rho(\mu|} F^A{}_\rho{}^{|\nu)} + N C_{B_1 \dots B_N : A}{}^{(\mu|} M^{A|\nu)} + 2N C_{B_1 \dots B_N : A}{}^{\alpha(\mu|} M^{A|\nu)}{}_{,\alpha} + (N-2) C_{B_1 \dots B_{N-1} : B_N}{}^{\mu\nu}$$

$$\begin{aligned} \mathbf{(C21}_{N}\mathbf{)} \quad 0 = {} & (N+2) \cdot (N+1)\, (\deg P - 1)\, C_{AB_1 \dots B_{N+1}} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma \right) - (N+1)^2 C_{A(B_1 \dots B_N} M^{A\mu}{}_{:B_{N+1})} - (N+1) C_{B_1 \dots B_{N+1} : A} M^{A\mu} \\ & - (N+1) C_{B_1 \dots B_{N+1} : A}{}^{\alpha} M^{A\mu}{}_{,\alpha} - (N+1) C_{B_1 \dots B_{N+1} : A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1} C_{B_1 \dots \widehat{B_K} \dots B_{N+1} : B_K}{}^{\mu} + 2 \left(\partial_\gamma C_{B_1 \dots B_N : B_{N+1}}{}^{\gamma\mu} \right) \end{aligned}$$

$$\mathbf{(C22}_{N}\mathbf{)} \quad 0 = C_{A:B}{}^{\mu_1 \dots \mu_N} - \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N}{K} \left(\partial^K_{\alpha_1 \cdot \alpha_K} C_{B:A}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_N} \right)$$

$$\mathbf{(C23}_{N}\mathbf{)} \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu_1 \dots \mu_N|} M^{A|\mu_{N+1})}{}_{,\alpha_1 \dots \alpha_K} + \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N+1}{N+1} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_{N+1}} \right)$$

$$\mathbf{(C24}_{N_{\mathrm{even}}}\mathbf{)} \quad 0 = \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K} \right)$$

$$\mathbf{(C25}_{N_{\mathrm{odd}}}\mathbf{)} \quad 0 = 2 \sum_{K=N-1}^{\infty} \binom{K}{N-1} C_{:A}{}^{\beta_N \dots \beta_K (\mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_N \dots \beta_K} - \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K} \right)$$

Lagrangian for the geometry

$$\mathcal{L}_{\text{gravity}}[G]$$

„Construction equations“

GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^\mu E^A{}_\mu - \partial_\gamma N^\mu F^A{}_\mu{}^\gamma := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}}\left(\mathcal{L}_{\vec{N}}\widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}}(\widehat{g}(\varphi))M^{\mathscr{A}_i\gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi;K] := \sum_{N=0}^{\infty} C_{A_1\dots A_N}[\varphi]K^{A_1}\cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

$$\textbf{(C1)}\quad 0 = -(\partial_\mu C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1\dots\alpha_K} E^A{}_{\mu,\alpha_1\dots\alpha_K}$$

$$\textbf{(C2)}\quad 0 = C_A E^A{}_{\mu:B} - \partial_\gamma\left(C_A E^A{}_{\mu:B}{}^\gamma\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1\dots\alpha_K} E^A{}_{\mu,\alpha_1\dots\alpha_K}$$

$$\textbf{(C3)}\quad 0 = -C\delta^\gamma_\mu + \sum_{K=0}^{\infty}(K+1)C_{:A}{}^{\gamma\alpha_1\dots\alpha_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - \sum_{K=0}^{\infty}(K+1)C_{:A}{}^{(\alpha_1\dots\alpha_K|}F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1\dots\alpha_K}$$

$$\textbf{(C4)}\quad 0 = -C_A\left(E^A{}_{\mu:B}{}^\gamma + F^A{}_\mu{}^\gamma{}_{:B}\right) + \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{\gamma\alpha_1\dots\alpha_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{(\alpha_1\dots\alpha_K|}F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1\dots\alpha_K}$$

$$\textbf{(C5)}\quad 0 = 2(\deg P - 1)\,p^{(\mu|\rho}C_{AB}F^A{}_\rho{}^{|\nu)} + \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{\alpha_1\dots\alpha_K(\mu|}M^{A|\nu)}{}_{,\alpha_1\dots\alpha_K} - \sum_{K=0}^{\infty}(-1)^K\binom{K+2}{K}\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu\nu}\right)$$

$$\textbf{(C6)}\quad 0 = 2(\deg P - 1)\,C_{AB}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty}C_{B:A}{}^{\alpha_1\dots\alpha_K}M^{A\mu}{}_{,\alpha_1\dots\alpha_K} - \sum_{K=0}^{\infty}(-1)^K(K+1)\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu}\right)$$

$$\textbf{(C7)}\quad 0 = 2\partial_\mu\left(C_A M^{A[\mu|}{}_{:B}M^{B|\gamma]}\right) - 2(\deg P - 1)\,p^{\rho\gamma}\left[C_A E^A{}_\rho + \partial_\mu\left(C_A F^A{}_\rho{}^\mu\right)\right] + \sum_{K=0}^{\infty}C_{:A}{}^{\alpha_1\dots\alpha_K}M^{A\gamma}{}_{,\alpha_1\dots\alpha_K} + {}$$
$$+ \sum_{K=0}^{\infty}\sum_{J=0}^{K}(-1)^J\binom{K}{J}(J+1)\partial^J_{\alpha_1\dots\alpha_J}\left(C_{:A}{}^{\beta_1\dots\beta_{K-J}(\alpha_1\dots\alpha_J|}M^{A|\gamma)}{}_{,\beta_1\dots\beta_{K-J}}\right)$$

$$\textbf{(C8)}\quad 0 = 6(\deg P - 1)\,C_{AB_1B_2}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - 4C_{A(B_1}M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A}M^{A\mu} - 2C_{B_1B_2:A}{}^\alpha M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta}M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^\mu$$
$$- \sum_{K=0}^{\infty}(-1)^K(K+1)\left(\partial^K_{\alpha_1\dots\alpha_K}C_{B_1:B_2}{}^{\mu\alpha_1\dots\alpha_K}\right)$$

$$\textbf{(C9)}\quad 0 = \sum_{K=2}^{\infty}\sum_{J=2}^{K}(-1)^J\binom{K}{J}(J-1)\partial^{J+1}_{\gamma\alpha_1\dots\alpha_J}\left(C_{:A}{}^{\beta_1\dots\beta_{K-J}(\alpha_1\dots\alpha_J|}M^{A|\gamma)}{}_{,\beta_1\dots\beta_{K-J}}\right)$$

THE SIXTEEN SEQUENCES FOR $N \geq 2$

$$\textbf{(C10}_N\textbf{)}\quad 0 = \sum_{K=0}^{\infty}\binom{K+N}{K}\left[C_{:A}{}^{\beta_1\dots\beta_N\alpha_1\dots\beta_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - C_{:A}{}^{(\beta_1\dots\beta_N\alpha_1\dots\alpha_{K-1}|}F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1\dots\alpha_K}\right]$$

$$\textbf{(C11}_N\textbf{)}\quad 0 = \sum_{K=0}^{\infty}\binom{K+N}{K}\left[C_{B:A}{}^{\beta_1\dots\beta_N\alpha_1\dots\beta_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - C_{B:A}{}^{(\beta_1\dots\beta_N\alpha_1\dots\alpha_{K-1}|}F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1\dots\alpha_K}\right]$$

$$\textbf{(C12}_N\textbf{)}\quad 0 = NC_{A(B_1\dots B_{N-1}}E^A{}_{|\mu|:B_N)} - (\partial_\mu C_{B_1\dots B_N}) + C_{B_1\dots B_N:A}E^A{}_\mu + C_{B_1\dots B_N:A}{}^\alpha E^A{}_{\mu,\alpha} + C_{B_1\dots B_N:A}{}^{\alpha\beta}E^A{}_{\mu,\alpha\beta}$$

$$\textbf{(C13}_N\textbf{)}\quad 0 = -C_{B_1\dots B_N}\delta^\gamma_\mu - NC_{A(B_1\dots B_{N-1}}F^A{}_{|\mu|}{}^\gamma{}_{:B_N)} - C_{B_1\dots B_N:A}F^A{}_\mu{}^\gamma + C_{B_1\dots B_N:A}{}^\gamma E^A{}_\mu - C_{B_1\dots B_N:A}{}^\alpha F^A{}_\mu{}^\gamma{}_{,\alpha} + 2C_{B_1\dots B_N:A}{}^{\alpha\gamma}E^A{}_{\mu,\alpha} - C_{B_1\dots B_N:A}{}^{\alpha\beta}F^A{}_\mu{}^\gamma{}_{,\alpha\beta}$$

$$\textbf{(C14}_N\textbf{)}\quad 0 = C_{AB_J(B_1\dots\widehat{B_J}\dots B_{N-1}}E^A{}_{|\mu|:B_N)}{}^\gamma - C_{B_1\dots B_N}\delta^\gamma_\mu \qquad \text{for} \quad J = 1\dots N$$

$$\textbf{(C15}_N\textbf{)}\quad 0 = C_{B_1\dots B_N:A}{}^{\beta_1\beta_2}E^A{}_\mu - C_{B_1\dots B_N:A}{}^{(\beta_1|}F^A{}_\mu{}^{|\beta_2)} - 2C_{B_1\dots B_N:A}{}^{\alpha(\beta_1|}F^A{}_\mu{}^{|\beta_2)}{}_{,\alpha}$$

$$\textbf{(C16}_N\textbf{)}\quad 0 = C_{B_1\dots B_N:A}{}^{(\alpha\beta|}F^A{}_\mu{}^{|\gamma)}$$

$$\textbf{(C17}_N\textbf{)}\quad 0 = C_{B_1\dots B_N:A}{}^{(\mu\nu|}M^{A|\gamma)}$$

$$\textbf{(C18}_N\textbf{)}\quad 0 = C_{AB_1\dots B_{N-1}}\left(M^{B[\mu|}M^{A|\nu]}{}_{:B} + (\deg P - 1)\,p^{\rho[\mu|}F^A{}_\rho{}^{|\nu]}\right)$$

$$\textbf{(C19}_N\textbf{)}\quad 0 = C_{B_1\dots\widehat{B_J}\dots B_{N+1}:B_J}{}^{\mu\nu} - C_{B_1\dots B_N:B_{N+1}}{}^{\mu\nu} \qquad \text{for} \quad J = 1\dots N+1$$

$$\textbf{(C20}_N\textbf{)}\quad 0 = N\cdot(N+1)(\deg P - 1)\,C_{AB_1\dots B_N}p^{\rho(\mu|}F^A{}_\rho{}^{|\nu)} + NC_{B_1\dots B_N:A}{}^{(\mu|}M^{A|\nu)} + 2NC_{B_1\dots B_N:A}{}^{\alpha(\mu|}M^{A|\nu)}{}_{,\alpha} + (N-2)C_{B_1\dots B_{N-1}:B_N}{}^{\mu\nu}$$

$$\textbf{(C21}_N\textbf{)}\quad 0 = (N+2)\cdot(N+1)(\deg P - 1)\,C_{AB_1\dots B_{N+1}}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - (N+1)^2C_{A(B_1\dots B_N}M^{A\mu}{}_{:B_{N+1})} - (N+1)C_{B_1\dots B_{N+1}:A}M^{A\mu}$$
$$-(N+1)C_{B_1\dots B_{N+1}:A}{}^\alpha M^{A\mu}{}_{,\alpha} - (N+1)C_{B_1\dots B_{N+1}:A}{}^{\alpha\beta}M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1}C_{B_1\dots\widehat{B_K}\dots B_{N+1}:B_K}{}^\mu + 2\left(\partial_\gamma C_{B_1\dots B_N:B_{N+1}}{}^{\gamma\mu}\right)$$

$$\textbf{(C22}_N\textbf{)}\quad 0 = C_{A:B}{}^{\mu_1\dots\mu_N} - \sum_{K=0}^{\infty}(-1)^{K+N}\binom{K+N}{K}\left(\partial^K_{\alpha_1\cdot\alpha_K}C_{B:A}{}^{\alpha_1\dots\alpha_K\mu_1\dots\mu_N}\right)$$

$$\textbf{(C23}_N\textbf{)}\quad 0 = \sum_{K=0}^{\infty}\binom{K+N}{K}C_{B:A}{}^{\alpha_1\dots\alpha_K(\mu_1\dots\mu_N|}M^{A|\mu_{N+1})}{}_{,\alpha_1\dots\alpha_K} + \sum_{K=0}^{\infty}(-1)^{K+N}\binom{K+N+1}{N+1}\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu_1\dots\mu_{N+1}}\right)$$

$$\textbf{(C24}_{N\text{even}}\textbf{)}\quad 0 = \sum_{K=N}^{\infty}\sum_{J=N+1}^{K+1}(-1)^J\binom{K}{J-1}\binom{J}{N}\partial^{J-N}_{\alpha_1\dots\alpha_{J-N}}\left(C_{:A}{}^{\beta_J\dots\beta_K(\alpha_1\dots\alpha_{J-N}\mu_1\dots\mu_{N-1}|}M^{A|\mu_N)}{}_{,\beta_J\dots\beta_K}\right)$$

$$\textbf{(C25}_{N\text{odd}}\textbf{)}\quad 0 = 2\sum_{K=N-1}^{\infty}\binom{K}{N-1}C_{:A}{}^{\beta_N\dots\beta_K(\mu_1\dots\mu_{N-1}|}M^{A|\mu_N)}{}_{,\beta_N\dots\beta_K} - \sum_{K=N}^{\infty}\sum_{J=N+1}^{K+1}(-1)^J\binom{K}{J-1}\binom{J}{N}\partial^{J-N}_{\alpha_1\dots\alpha_{J-N}}\left(C_{:A}{}^{\beta_J\dots\beta_K(\alpha_1\dots\alpha_{J-N}\mu_1\dots\mu_{N-1}|}M^{A|\mu_N)}{}_{,\beta_J\dots\beta_K}\right)$$

DSSW ’16

hyperbolic principal polynomial

hyperbolic dual polynomial

$$P_G(k, k, \dots, k)$$

canonically

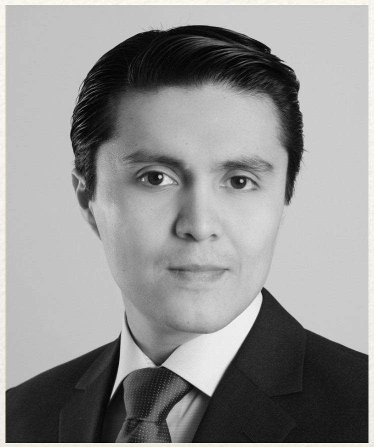
quantizable

$$\mathcal{L}_{\text{matter}}[\Phi; G)$$

Lagrangian for the geometry

$\mathcal{L}_{\text{gravity}}[G]$

Theory



Rivera

Wohlfarth

Witte

Wolz

GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^\mu E^A{}_\mu - \partial_\gamma N^\mu F^A{}_\mu{}^\gamma := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}} \left(\mathcal{L}_{\vec{N}} \widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}} (\widehat{g}(\varphi)) M^{\mathscr{A}_i \gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi; K] := \sum_{N=0}^{\infty} C_{A_1 \dots A_N}[\varphi] K^{A_1} \cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

(C1) $0 = -(\partial_\mu C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \dots \alpha_K} E^A{}_{\mu,\alpha_1 \dots \alpha_K}$

(C2) $0 = C_A E^A{}_{\mu:B} - \partial_\gamma \left(C_A E^A{}_{\mu:B}{}^\gamma\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \dots \alpha_K} E^A{}_{\mu,\alpha_1 \dots \alpha_K}$

(C3) $0 = -C\delta^\gamma_\mu + \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{\gamma\alpha_1 \dots \alpha_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{(\alpha_1 \dots \alpha_K|} F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1 \dots \alpha_K}$

(C4) $0 = -C_A \left(E^A{}_{\mu:B}{}^\gamma + F^A{}_\mu{}^\gamma{}_{:B}\right) + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\gamma\alpha_1 \dots \alpha_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{(\alpha_1 \dots \alpha_K|} F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1 \dots \alpha_K}$

(C5) $0 = 2(\deg P - 1)\, p^{(\mu|\rho} C_{AB} F^A{}_\rho{}^{|\nu)} + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu|} M^{A|\nu)}{}_{,\alpha_1 \dots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K \binom{K+2}{K} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu\nu}\right)$

(C6) $0 = 2(\deg P - 1)\, C_{AB} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \dots \alpha_K} M^{A\mu}{}_{,\alpha_1 \dots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu}\right)$

(C7) $0 = 2\partial_\mu \left(C_A M^{A[\mu|}{}_{:B} M^{B|\gamma]}\right) - 2(\deg P - 1)\, p^{\rho\gamma} \left[C_A E^A{}_\rho + \partial_\mu \left(C_A F^A{}_\rho{}^\mu\right)\right] + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \dots \alpha_K} M^{A\gamma}{}_{,\alpha_1 \dots \alpha_K} +$

$+ \sum_{K=0}^{\infty} \sum_{J=0}^{K} (-1)^J \binom{K}{J} (J+1) \partial^J_{\alpha_1 \dots \alpha_J} \left(C_{:A}{}^{\beta_1 \dots \beta_{K-J} (\alpha_1 \dots \alpha_J|} M^{A|\gamma)}{}_{,\beta_1 \dots \beta_{K-J}}\right)$

(C8) $0 = 6(\deg P - 1)\, C_{AB_1B_2} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - 4C_{A(B_1} M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A} M^{A\mu} - 2C_{B_1B_2:A}{}^\alpha M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^\mu$

$- \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{B_1:B_2}{}^{\mu\alpha_1 \dots \alpha_K}\right)$

(C9) $0 = \sum_{K=2}^{\infty} \sum_{J=2}^{K} (-1)^J \binom{K}{J} (J-1) \partial^{J+1}_{\gamma\alpha_1 \dots \alpha_J} \left(C_{:A}{}^{\beta_1 \dots \beta_{K-J} (\alpha_1 \dots \alpha_J|} M^{A|\gamma)}{}_{,\beta_1 \dots \beta_{K-J}}\right)$

THE SIXTEEN SEQUENCES FOR $N \geq 2$

($C10_N$) $0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - C_{:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1}|} F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K}\right]$

($C11_N$) $0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{B:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - C_{B:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1}|} F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K}\right]$

($C12_N$) $0 = NC_{A(B_1 \dots B_{N-1}} E^A{}_{|\mu|:B_N)} - (\partial_\mu C_{B_1 \dots B_N}) + C_{B_1 \dots B_N:A} E^A{}_\mu + C_{B_1 \dots B_N:A}{}^\alpha E^A{}_{\mu,\alpha} + C_{B_1 \dots B_N:A}{}^{\alpha\beta} E^A{}_{\mu,\alpha\beta}$

($C13_N$) $0 = -C_{B_1 \dots B_N} \delta^\gamma_\mu - NC_{A(B_1 \dots B_{N-1}} F^A{}_{|\mu|}{}^\gamma{}_{:B_N)} - C_{B_1 \dots B_N:A} F^A{}_\mu{}^\gamma + C_{B_1 \dots B_N:A}{}^\gamma E^A{}_\mu - C_{B_1 \dots B_N:A}{}^\alpha F^A{}_\mu{}^\gamma{}_{,\alpha} + 2C_{B_1 \dots B_N:A}{}^{\alpha\gamma} E^A{}_{\mu,\alpha} - C_{B_1 \dots B_N:A}{}^{\alpha\beta} F^A{}_\mu{}^\gamma{}_{,\alpha\beta}$

($C14_N$) $0 = C_{AB_J(B_1 \dots \widehat{B_J} \dots B_{N-1}} E^A{}_{|\mu|:B_N)}{}^\gamma - C_{B_1 \dots B_N} \delta^\gamma_\mu \quad \text{for} \quad J = 1 \dots N$

($C15_N$) $0 = C_{B_1 \dots B_N:A}{}^{\beta_1\beta_2} E^A{}_\mu - C_{B_1 \dots B_N:A}{}^{(\beta_1|} F^A{}_\mu{}^{|\beta_2)} - 2C_{B_1 \dots B_N:A}{}^{\alpha(\beta_1|} F^A{}_\mu{}^{|\beta_2)}{}_{,\alpha}$

($C16_N$) $0 = C_{B_1 \dots B_N:A}{}^{(\alpha\beta|} F^A{}_\mu{}^{|\gamma)}$

($C17_N$) $0 = C_{B_1 \dots B_N:A}{}^{(\mu\nu|} M^{A|\gamma)}$

($C18_N$) $0 = C_{AB_1 \dots B_{N-1}} \left(M^{B[\mu|} M^{A|\nu]}{}_{:B} + (\deg P - 1)\, p^{\rho[\mu|} F^A{}_\rho{}^{|\nu]}\right)$

($C19_N$) $0 = C_{B_1 \dots \widehat{B_J} \dots B_{N+1}:B_J}{}^{\mu\nu} - C_{B_1 \dots B_N:B_{N+1}}{}^{\mu\nu} \quad \text{for} \quad J = 1 \dots N+1$

($C20_N$) $0 = N \cdot (N+1)(\deg P - 1)\, C_{AB_1 \dots B_N} p^{\rho(\mu|} F^A{}_\rho{}^{|\nu)} + NC_{B_1 \dots B_N:A}{}^{(\mu|} M^{A|\nu)} + 2NC_{B_1 \dots B_N:A}{}^{\alpha(\mu|} M^{A|\nu)}{}_{,\alpha} + (N-2) C_{B_1 \dots B_{N-1}:B_N}{}^{\mu\nu}$

($C21_N$) $0 = (N+2) \cdot (N+1)(\deg P - 1)\, C_{AB_1 \dots B_{N+1}} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - (N+1)^2 C_{A(B_1 \dots B_N} M^{A\mu}{}_{:B_{N+1})} - (N+1) C_{B_1 \dots B_{N+1}:A} M^{A\mu}$

$-(N+1) C_{B_1 \dots B_{N+1}:A}{}^\alpha M^{A\mu}{}_{,\alpha} - (N+1) C_{B_1 \dots B_{N+1}:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1} C_{B_1 \dots \widehat{B_K} \dots B_{N+1}:B_K}{}^\mu + 2\left(\partial_\gamma C_{B_1 \dots B_N:B_{N+1}}{}^{\gamma\mu}\right)$

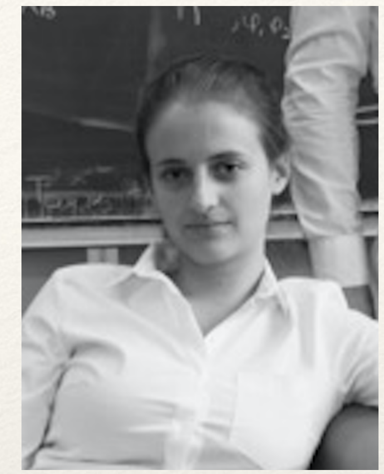
($C22_N$) $0 = C_{A:B}{}^{\mu_1 \dots \mu_N} - \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N}{K} \left(\partial^K_{\alpha_1 \cdot \alpha_K} C_{B:A}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_N}\right)$

($C23_N$) $0 = \sum_{K=0}^{\infty} \binom{K+N}{K} C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu_1 \dots \mu_N|} M^{A|\mu_{N+1})}{}_{,\alpha_1 \dots \alpha_K} + \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N+1}{N+1} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_{N+1}}\right)$

($C24_{N\text{even}}$) $0 = \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K}\right)$

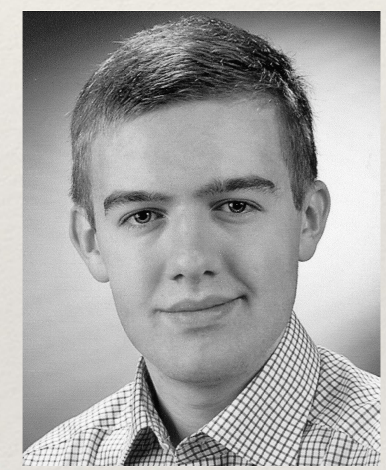
($C25_{N\text{odd}}$) $0 = 2 \sum_{K=N-1}^{\infty} \binom{K}{N-1} C_{:A}{}^{\beta_N \dots \beta_K (\mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_N \dots \beta_K} - \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K}\right)$

Solutions



Stritzelberger

Schneider



Düll

Fischer

DSSW '16

$P_G(k, k, \dots, k)$

$\mathcal{L}_{\text{matter}}[\Phi; G]$

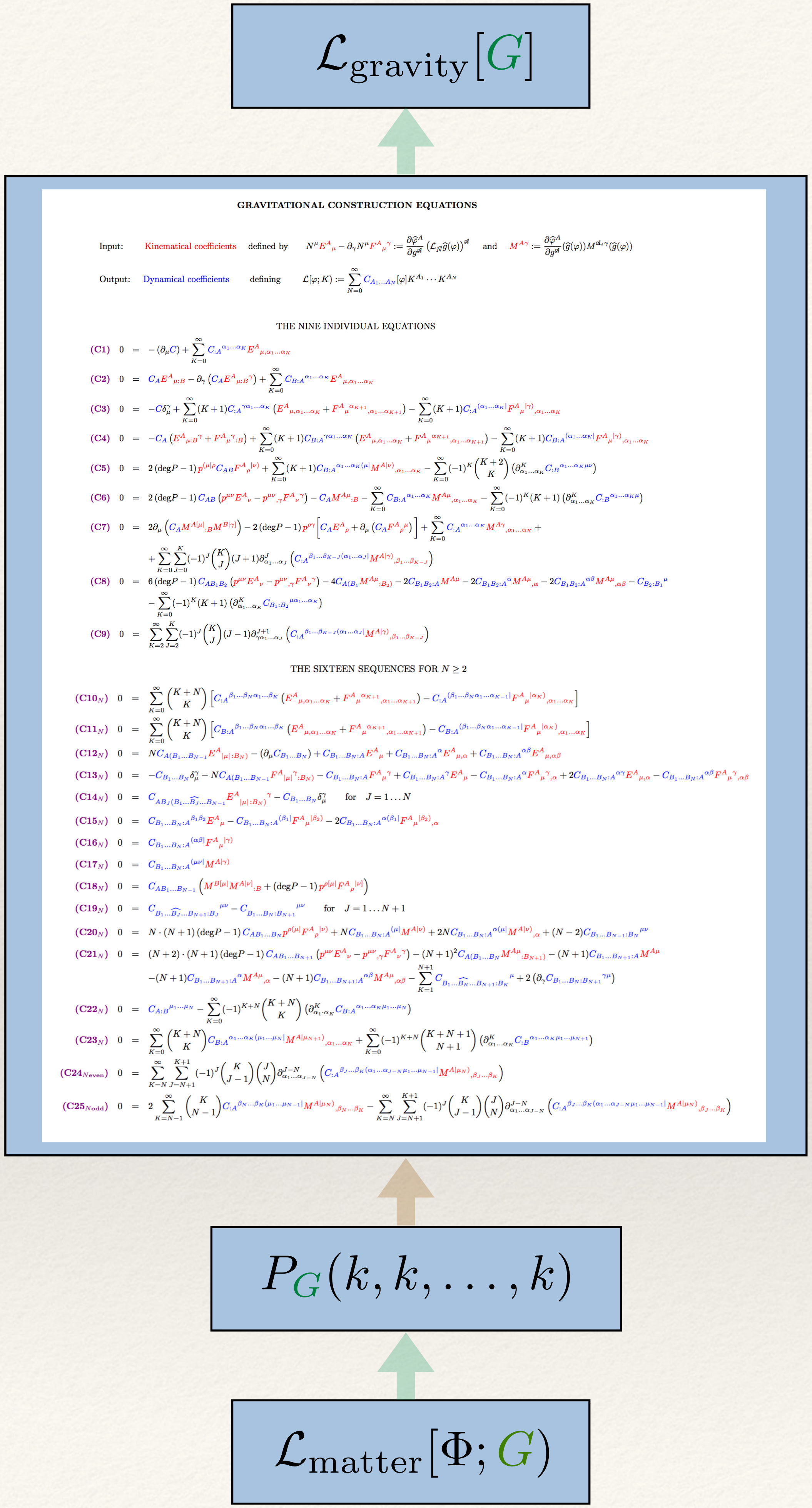
hyperbolic principal polynomial

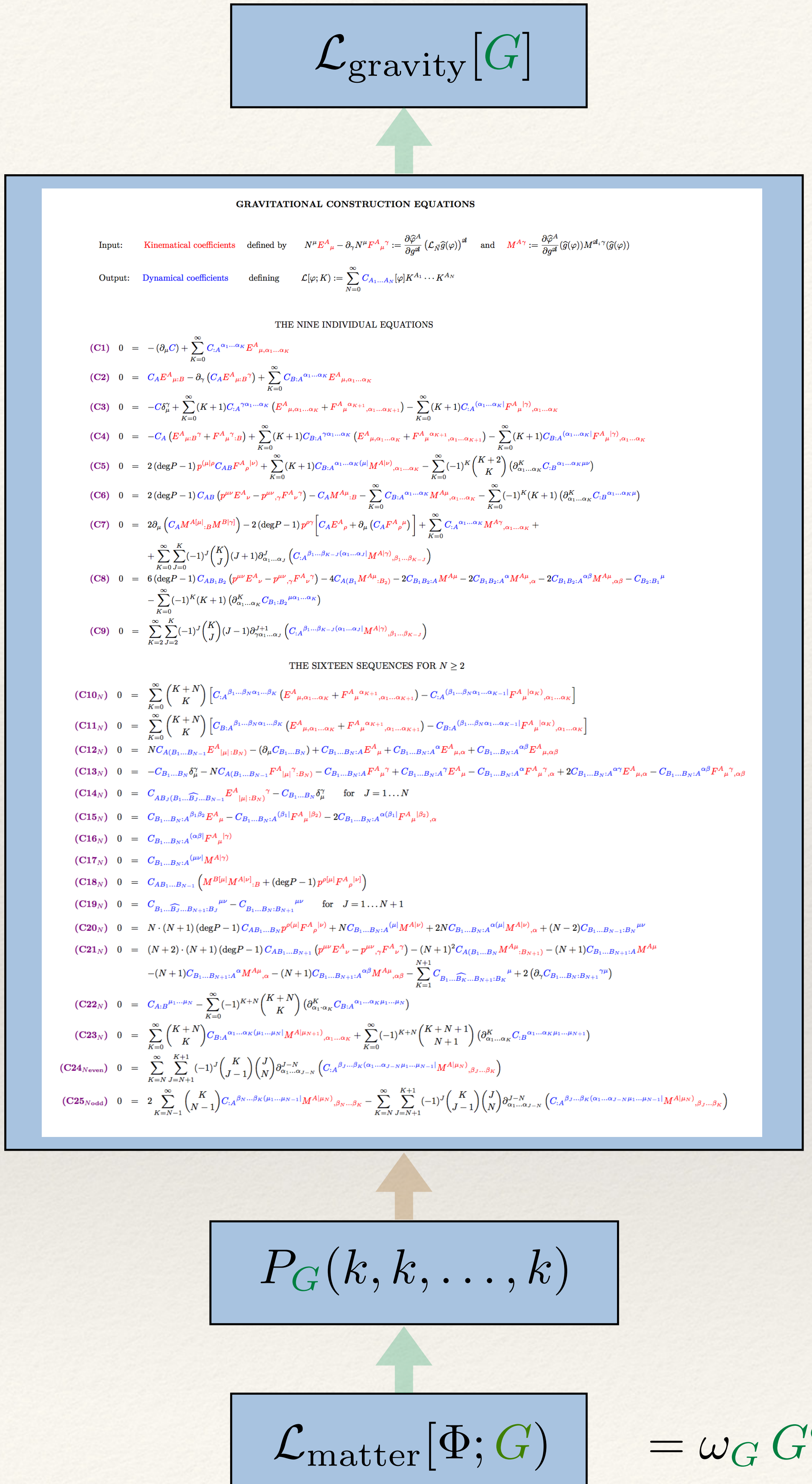
hyperbolic dual polynomial

canonically

quantizable

Case study


$\mathcal{L}_{\text{gravity}}[G]$
GRAVITATIONAL CONSTRUCTION EQUATIONS
Input: Kinematical coefficients defined by
Output: Dynamical coefficients defining
THE NINE INDIVIDUAL EQUATIONS
THE SIXTEEN SEQUENCES FOR $N \geq 2$
$P_G(k, k, \ldots, k)$
$\mathcal{L}_{\text{matter}}[\Phi; G]$



$$= \omega_G \, G^{abcd} F_{ab} F_{cd}$$

$$\mathcal{L}_{\text{gravity}}[G]$$

GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^\mu E^A{}_\mu - \partial_\gamma N^\mu F^A{}_\mu{}^\gamma := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}} \left(\mathcal{L}_{\vec{N}} \widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}}(\widehat{g}(\varphi)) M^{\mathscr{A}_i \gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi; K] := \sum_{N=0}^{\infty} C_{A_1 \dots A_N}[\varphi] K^{A_1} \cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

(C1) $0 = -(\partial_\mu C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \dots \alpha_K} E^A{}_{\mu,\alpha_1 \dots \alpha_K}$

(C2) $0 = C_A E^A{}_{\mu:B} - \partial_\gamma \left(C_A E^A{}_{\mu:B}{}^\gamma\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \dots \alpha_K} E^A{}_{\mu,\alpha_1 \dots \alpha_K}$

(C3) $0 = -C\delta^\gamma_\mu + \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{\gamma\alpha_1 \dots \alpha_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{(\alpha_1 \dots \alpha_K|} F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1 \dots \alpha_K}$

(C4) $0 = -C_A \left(E^A{}_{\mu:B}{}^\gamma + F^A{}_\mu{}^\gamma{}_{:B}\right) + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\gamma\alpha_1 \dots \alpha_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{(\alpha_1 \dots \alpha_K|} F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1 \dots \alpha_K}$

(C5) $0 = 2\,(\deg P - 1)\, p^{(\mu|\rho} C_{AB} F^A{}_\rho{}^{|\nu)} + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu|} M^{A|\nu)}{}_{,\alpha_1 \dots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K \binom{K+2}{K} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu\nu}\right)$

(C6) $0 = 2\,(\deg P - 1)\, C_{AB} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \dots \alpha_K} M^{A\mu}{}_{,\alpha_1 \dots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu}\right)$

(C7) $0 = 2\partial_\mu \left(C_A M^{A[\mu|}{}_{:B} M^{B|\gamma]}\right) - 2\,(\deg P - 1)\, p^{\rho\gamma} \left[C_A E^A{}_\rho + \partial_\mu \left(C_A F^A{}_\rho{}^\mu\right)\right] + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \dots \alpha_K} M^{A\gamma}{}_{,\alpha_1 \dots \alpha_K} +$
$+ \sum_{K=0}^{\infty} \sum_{J=0}^{K} (-1)^J \binom{K}{J} (J+1) \partial^J_{\alpha_1 \dots \alpha_J} \left(C_{:A}{}^{\beta_1 \dots \beta_{K-J} (\alpha_1 \dots \alpha_J|} M^{A|\gamma)}{}_{,\beta_1 \dots \beta_{K-J}}\right)$

(C8) $0 = 6\,(\deg P - 1)\, C_{AB_1B_2} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - 4C_{A(B_1} M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A} M^{A\mu} - 2C_{B_1B_2:A}{}^\alpha M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^\mu$
$- \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{B_1:B_2}{}^{\mu\alpha_1 \dots \alpha_K}\right)$

(C9) $0 = \sum_{K=2}^{\infty} \sum_{J=2}^{K} (-1)^J \binom{K}{J} (J-1) \partial^{J+1}_{\gamma\alpha_1 \dots \alpha_J} \left(C_{:A}{}^{\beta_1 \dots \beta_{K-J} (\alpha_1 \dots \alpha_J|} M^{A|\gamma)}{}_{,\beta_1 \dots \beta_{K-J}}\right)$

THE SIXTEEN SEQUENCES FOR $N \geq 2$

(C10$_N$) $0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - C_{:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1}|} F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K}\right]$

(C11$_N$) $0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{B:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - C_{B:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1}|} F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K}\right]$

(C12$_N$) $0 = NC_{A(B_1 \dots B_{N-1}} E^A{}_{|\mu|:B_N)} - (\partial_\mu C_{B_1 \dots B_N}) + C_{B_1 \dots B_N:A} E^A{}_\mu + C_{B_1 \dots B_N:A}{}^\alpha E^A{}_{\mu,\alpha} + C_{B_1 \dots B_N:A}{}^{\alpha\beta} E^A{}_{\mu,\alpha\beta}$

(C13$_N$) $0 = -C_{B_1 \dots B_N} \delta^\gamma_\mu - NC_{A(B_1 \dots B_{N-1}} F^A{}_{|\mu|}{}^\gamma{}_{:B_N)} - C_{B_1 \dots B_N:A} F^A{}_\mu{}^\gamma + C_{B_1 \dots B_N:A}{}^\gamma E^A{}_\mu - C_{B_1 \dots B_N:A}{}^\alpha F^A{}_\mu{}^\gamma{}_{,\alpha} + 2C_{B_1 \dots B_N:A}{}^{\alpha\gamma} E^A{}_{\mu,\alpha} - C_{B_1 \dots B_N:A}{}^{\alpha\beta} F^A{}_\mu{}^\gamma{}_{,\alpha\beta}$

(C14$_N$) $0 = C_{AB_J(B_1 \dots \widehat{B_J} \dots B_{N-1}} E^A{}_{|\mu|:B_N)}{}^\gamma - C_{B_1 \dots B_N} \delta^\gamma_\mu$ for $J = 1 \dots N$

(C15$_N$) $0 = C_{B_1 \dots B_N:A}{}^{\beta_1\beta_2} E^A{}_\mu - C_{B_1 \dots B_N:A}{}^{(\beta_1|} F^A{}_\mu{}^{|\beta_2)} - 2C_{B_1 \dots B_N:A}{}^{\alpha(\beta_1|} F^A{}_\mu{}^{|\beta_2)}{}_{,\alpha}$

(C16$_N$) $0 = C_{B_1 \dots B_N:A}{}^{(\alpha\beta|} F^A{}_\mu{}^{|\gamma)}$

(C17$_N$) $0 = C_{B_1 \dots B_N:A}{}^{(\mu\nu|} M^{A|\gamma)}$

(C18$_N$) $0 = C_{AB_1 \dots B_{N-1}} \left(M^{B[\mu|} M^{A|\nu]}{}_{:B} + (\deg P - 1)\, p^{\rho[\mu|} F^A{}_\rho{}^{|\nu]}\right)$

(C19$_N$) $0 = C_{B_1 \dots \widehat{B_J} \dots B_{N+1}:B_J}{}^{\mu\nu} - C_{B_1 \dots B_N:B_{N+1}}{}^{\mu\nu}$ for $J = 1 \dots N+1$

(C20$_N$) $0 = N \cdot (N+1)\,(\deg P - 1)\, C_{AB_1 \dots B_N} p^{\rho(\mu|} F^A{}_\rho{}^{|\nu)} + NC_{B_1 \dots B_N:A}{}^{(\mu|} M^{A|\nu)} + 2NC_{B_1 \dots B_N:A}{}^{\alpha(\mu|} M^{A|\nu)}{}_{,\alpha} + (N-2) C_{B_1 \dots B_{N-1}:B_N}{}^{\mu\nu}$

(C21$_N$) $0 = (N+2) \cdot (N+1)\,(\deg P - 1)\, C_{AB_1 \dots B_{N+1}} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - (N+1)^2 C_{A(B_1 \dots B_N} M^{A\mu}{}_{:B_{N+1})} - (N+1) C_{B_1 \dots B_{N+1}:A} M^{A\mu}$
$-(N+1) C_{B_1 \dots B_{N+1}:A}{}^\alpha M^{A\mu}{}_{,\alpha} - (N+1) C_{B_1 \dots B_{N+1}:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1} C_{B_1 \dots \widehat{B_K} \dots B_{N+1}:B_K}{}^\mu + 2\left(\partial_\gamma C_{B_1 \dots B_N:B_{N+1}}{}^{\gamma\mu}\right)$

(C22$_N$) $0 = C_{A:B}{}^{\mu_1 \dots \mu_N} - \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N}{K} \left(\partial^K_{\alpha_1 \cdot \alpha_K} C_{B:A}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_N}\right)$

(C23$_N$) $0 = \sum_{K=0}^{\infty} \binom{K+N}{K} C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu_1 \dots \mu_N|} M^{A|\mu_{N+1})}{}_{,\alpha_1 \dots \alpha_K} + \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N+1}{N+1} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_{N+1}}\right)$

(C24$_{N\text{even}}$) $0 = \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K}\right)$

(C25$_{N\text{odd}}$) $0 = 2 \sum_{K=N-1}^{\infty} \binom{K}{N-1} C_{:A}{}^{\beta_N \dots \beta_K (\mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_N \dots \beta_K} - \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1}|} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K}\right)$

$$P_G(k, k, \dots, k)$$

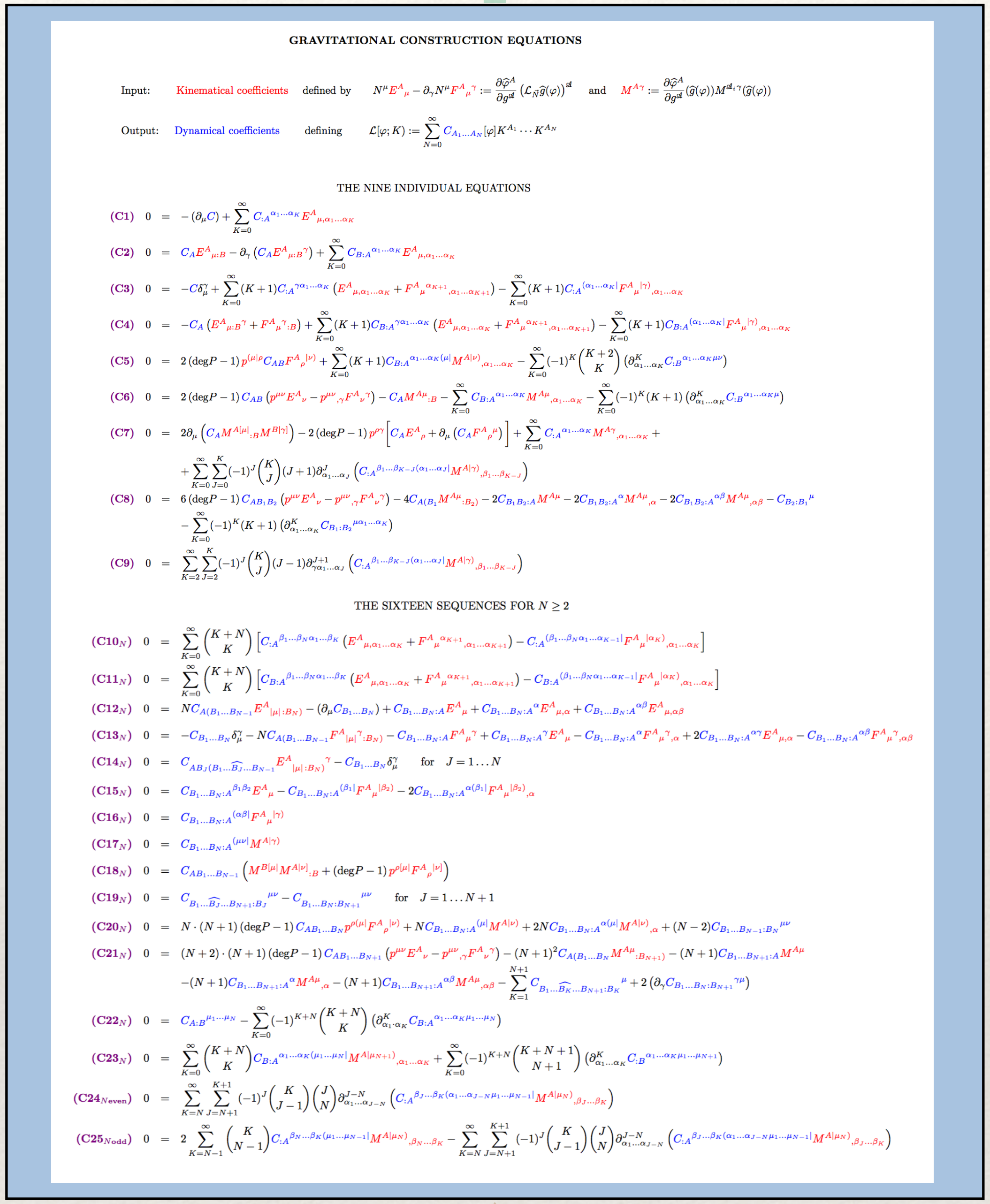
$$\mathcal{L}_{\text{matter}}[\Phi; G) \quad = \omega_G\, G^{abcd} F_{ab} F_{cd}$$

alternatively:

supposedly simple bi-metric theory

$$\sqrt{-G}\, G^{ab}\, \partial_a \varphi \partial_b \varphi + \sqrt{-H}\, H^{ab}\, \partial_a \psi \partial_b \psi$$

$$\mathcal{L}_{\text{gravity}}[G]$$



GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^\mu E^A{}_\mu - \partial_\gamma N^\mu F^A{}_\mu{}^\gamma := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}} \left(\mathcal{L}_{\vec{N}} \widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial \widehat{\varphi}^A}{\partial g^{\mathscr{A}}} (\widehat{g}(\varphi)) M^{\mathscr{A}_i \gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi; K] := \sum_{N=0}^{\infty} C_{A_1 \dots A_N}[\varphi] K^{A_1} \cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

$$\text{(C1)} \quad 0 = -(\partial_\mu C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \dots \alpha_K} E^A{}_{\mu,\alpha_1 \dots \alpha_K}$$

$$\text{(C2)} \quad 0 = C_A E^A{}_{\mu:B} - \partial_\gamma \left(C_A E^A{}_{\mu:B}{}^\gamma\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \dots \alpha_K} E^A{}_{\mu,\alpha_1 \dots \alpha_K}$$

$$\text{(C3)} \quad 0 = -C\delta^\gamma_\mu + \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{\gamma\alpha_1 \dots \alpha_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{(\alpha_1 \dots \alpha_K |} F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1 \dots \alpha_K}$$

$$\text{(C4)} \quad 0 = -C_A \left(E^A{}_{\mu:B}{}^\gamma + F^A{}_\mu{}^\gamma{}_{:B}\right) + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\gamma\alpha_1 \dots \alpha_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{(\alpha_1 \dots \alpha_K |} F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1 \dots \alpha_K}$$

$$\text{(C5)} \quad 0 = 2\,(\deg P - 1)\, p^{(\mu|\rho} C_{AB} F^A{}_\rho{}^{|\nu)} + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu|} M^{A|\nu)}{}_{,\alpha_1 \dots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K \binom{K+2}{K} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu\nu}\right)$$

$$\text{(C6)} \quad 0 = 2\,(\deg P - 1)\, C_{AB} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \dots \alpha_K} M^{A\mu}{}_{,\alpha_1 \dots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu}\right)$$

$$\text{(C7)} \quad 0 = 2\partial_\mu \left(C_A M^{A[\mu|}{}_{:B} M^{B|\gamma]}\right) - 2\,(\deg P - 1)\, p^{\rho\gamma} \left[C_A E^A{}_\rho + \partial_\mu \left(C_A F^A{}_\rho{}^\mu\right)\right] + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \dots \alpha_K} M^{A\gamma}{}_{,\alpha_1 \dots \alpha_K} + \\ + \sum_{K=0}^{\infty} \sum_{J=0}^{K} (-1)^J \binom{K}{J} (J+1) \partial^J_{\alpha_1 \dots \alpha_J} \left(C_{:A}{}^{\beta_1 \dots \beta_{K-J} (\alpha_1 \dots \alpha_J |} M^{A|\gamma)}{}_{,\beta_1 \dots \beta_{K-J}}\right)$$

$$\text{(C8)} \quad 0 = 6\,(\deg P - 1)\, C_{AB_1B_2} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - 4C_{A(B_1} M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A} M^{A\mu} - 2C_{B_1B_2:A}{}^\alpha M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^\mu \\ - \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{B_1:B_2}{}^{\mu\alpha_1 \dots \alpha_K}\right)$$

$$\text{(C9)} \quad 0 = \sum_{K=2}^{\infty} \sum_{J=2}^{K} (-1)^J \binom{K}{J} (J-1) \partial^{J+1}_{\gamma\alpha_1 \dots \alpha_J} \left(C_{:A}{}^{\beta_1 \dots \beta_{K-J} (\alpha_1 \dots \alpha_J |} M^{A|\gamma)}{}_{,\beta_1 \dots \beta_{K-J}}\right)$$

THE SIXTEEN SEQUENCES FOR $N \geq 2$

$$\text{(C10}_N\text{)} \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - C_{:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1} |} F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K}\right]$$

$$\text{(C11}_N\text{)} \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{B:A}{}^{\beta_1 \dots \beta_N \alpha_1 \dots \beta_K} \left(E^A{}_{\mu,\alpha_1 \dots \alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1 \dots \alpha_{K+1}}\right) - C_{B:A}{}^{(\beta_1 \dots \beta_N \alpha_1 \dots \alpha_{K-1} |} F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1 \dots \alpha_K}\right]$$

$$\text{(C12}_N\text{)} \quad 0 = NC_{A(B_1 \dots B_{N-1}} E^A{}_{|\mu|:B_N)} - (\partial_\mu C_{B_1 \dots B_N}) + C_{B_1 \dots B_N:A} E^A{}_\mu + C_{B_1 \dots B_N:A}{}^\alpha E^A{}_{\mu,\alpha} + C_{B_1 \dots B_N:A}{}^{\alpha\beta} E^A{}_{\mu,\alpha\beta}$$

$$\text{(C13}_N\text{)} \quad 0 = -C_{B_1 \dots B_N} \delta^\gamma_\mu - NC_{A(B_1 \dots B_{N-1}} F^A{}_{|\mu|}{}^\gamma{}_{:B_N)} - C_{B_1 \dots B_N:A} F^A{}_\mu{}^\gamma + C_{B_1 \dots B_N:A}{}^\gamma E^A{}_\mu - C_{B_1 \dots B_N:A}{}^\alpha F^A{}_\mu{}^\gamma{}_{,\alpha} + 2C_{B_1 \dots B_N:A}{}^{\alpha\gamma} E^A{}_{\mu,\alpha} - C_{B_1 \dots B_N:A}{}^{\alpha\beta} F^A{}_\mu{}^\gamma{}_{,\alpha\beta}$$

$$\text{(C14}_N\text{)} \quad 0 = C_{AB_J(B_1 \dots \widehat{B_J} \dots B_{N-1}} E^A{}_{|\mu|:B_N)}{}^\gamma - C_{B_1 \dots B_N} \delta^\gamma_\mu \qquad \text{for} \quad J = 1 \dots N$$

$$\text{(C15}_N\text{)} \quad 0 = C_{B_1 \dots B_N:A}{}^{\beta_1\beta_2} E^A{}_\mu - C_{B_1 \dots B_N:A}{}^{(\beta_1|} F^A{}_\mu{}^{|\beta_2)} - 2C_{B_1 \dots B_N:A}{}^{\alpha(\beta_1|} F^A{}_\mu{}^{|\beta_2)}{}_{,\alpha}$$

$$\text{(C16}_N\text{)} \quad 0 = C_{B_1 \dots B_N:A}{}^{(\alpha\beta|} F^A{}_\mu{}^{|\gamma)}$$

$$\text{(C17}_N\text{)} \quad 0 = C_{B_1 \dots B_N:A}{}^{(\mu\nu|} M^{A|\gamma)}$$

$$\text{(C18}_N\text{)} \quad 0 = C_{AB_1 \dots B_{N-1}} \left(M^{B[\mu|} M^{A|\nu]}{}_{:B} + (\deg P - 1)\, p^{\rho[\mu|} F^A{}_\rho{}^{|\nu]}\right)$$

$$\text{(C19}_N\text{)} \quad 0 = C_{B_1 \dots \widehat{B_J} \dots B_{N+1}:B_J}{}^{\mu\nu} - C_{B_1 \dots B_N:B_{N+1}}{}^{\mu\nu} \qquad \text{for} \quad J = 1 \dots N+1$$

$$\text{(C20}_N\text{)} \quad 0 = N \cdot (N+1)\,(\deg P - 1)\, C_{AB_1 \dots B_N} p^{\rho(\mu|} F^A{}_\rho{}^{|\nu)} + NC_{B_1 \dots B_N:A}{}^{(\mu|} M^{A|\nu)} + 2NC_{B_1 \dots B_N:A}{}^{\alpha(\mu|} M^{A|\nu)}{}_{,\alpha} + (N-2) C_{B_1 \dots B_{N-1}:B_N}{}^{\mu\nu}$$

$$\text{(C21}_N\text{)} \quad 0 = (N+2) \cdot (N+1)\,(\deg P - 1)\, C_{AB_1 \dots B_{N+1}} \left(p^{\mu\nu} E^A{}_\nu - p^{\mu\nu}{}_{,\gamma} F^A{}_\nu{}^\gamma\right) - (N+1)^2 C_{A(B_1 \dots B_N} M^{A\mu}{}_{:B_{N+1})} - (N+1) C_{B_1 \dots B_{N+1}:A} M^{A\mu} \\ -(N+1) C_{B_1 \dots B_{N+1}:A}{}^\alpha M^{A\mu}{}_{,\alpha} - (N+1) C_{B_1 \dots B_{N+1}:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1} C_{B_1 \dots \widehat{B_K} \dots B_{N+1}:B_K}{}^\mu + 2\left(\partial_\gamma C_{B_1 \dots B_N:B_{N+1}}{}^{\gamma\mu}\right)$$

$$\text{(C22}_N\text{)} \quad 0 = C_{A:B}{}^{\mu_1 \dots \mu_N} - \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N}{K} \left(\partial^K_{\alpha_1 \cdot \alpha_K} C_{B:A}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_N}\right)$$

$$\text{(C23}_N\text{)} \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} C_{B:A}{}^{\alpha_1 \dots \alpha_K (\mu_1 \dots \mu_N |} M^{A|\mu_{N+1})}{}_{,\alpha_1 \dots \alpha_K} + \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N+1}{N+1} \left(\partial^K_{\alpha_1 \dots \alpha_K} C_{:B}{}^{\alpha_1 \dots \alpha_K \mu_1 \dots \mu_{N+1}}\right)$$

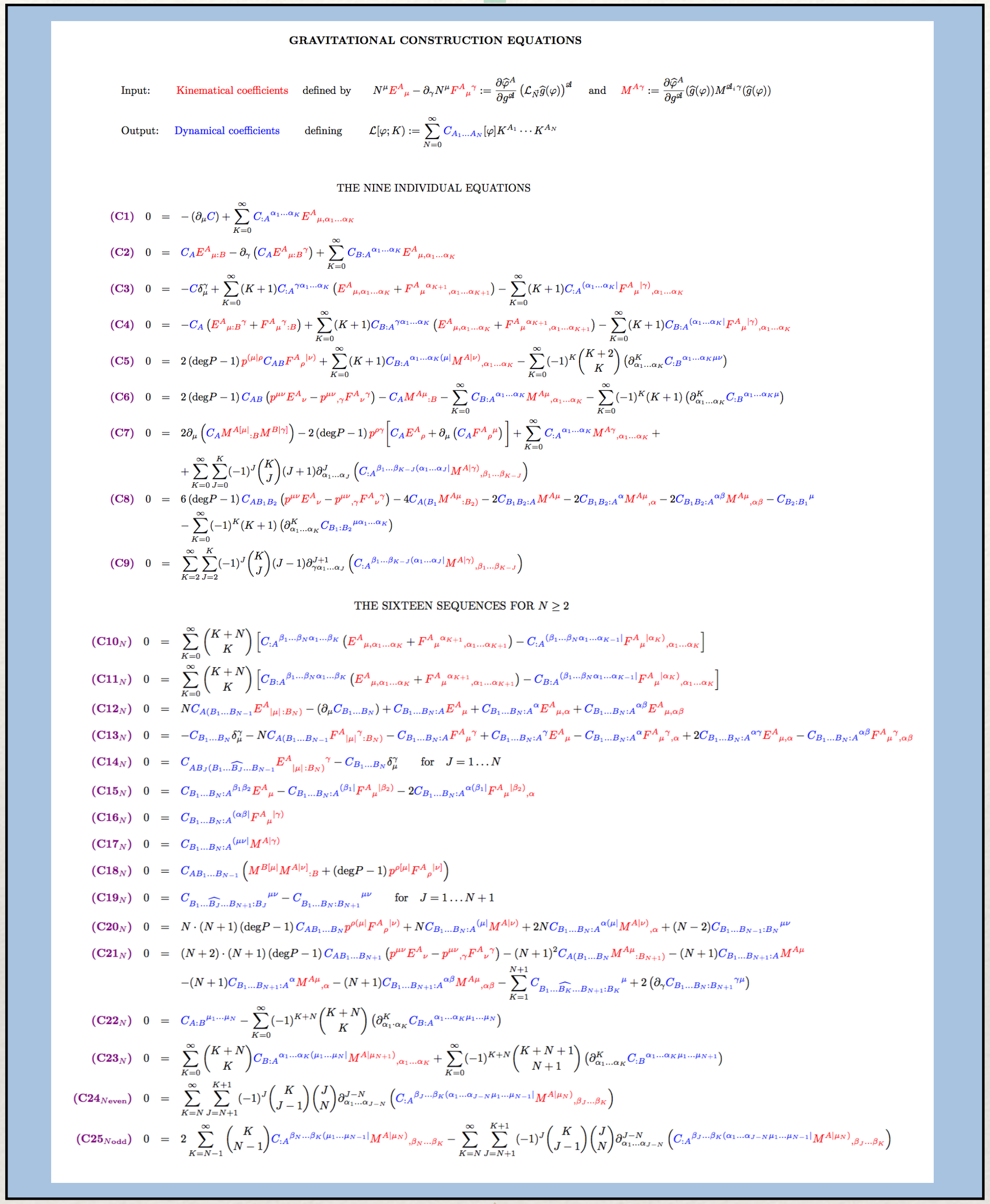
$$\text{(C24}_{N\text{even}}\text{)} \quad 0 = \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1} |} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K}\right)$$

$$\text{(C25}_{N\text{odd}}\text{)} \quad 0 = 2 \sum_{K=N-1}^{\infty} \binom{K}{N-1} C_{:A}{}^{\beta_N \dots \beta_K (\mu_1 \dots \mu_{N-1} |} M^{A|\mu_N)}{}_{,\beta_N \dots \beta_K} - \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \dots \alpha_{J-N}} \left(C_{:A}{}^{\beta_J \dots \beta_K (\alpha_1 \dots \alpha_{J-N} \mu_1 \dots \mu_{N-1} |} M^{A|\mu_N)}{}_{,\beta_J \dots \beta_K}\right)$$

$$P_G(k, k, \dots, k)$$

$$\mathcal{L}_{\text{matter}}[\Phi; G] \qquad = \omega_G\, G^{abcd} F_{ab} F_{cd}$$

$$\mathcal{L}_{\text{gravity}}[G]$$



GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^{\mu} E^{A}{}_{\mu} - \partial_{\gamma} N^{\mu} F^{A}{}_{\mu}{}^{\gamma} := \frac{\partial \widehat{\varphi}^{A}}{\partial g^{\mathscr{A}}} \left(\mathcal{L}_{\vec{N}} \widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial \widehat{\varphi}^{A}}{\partial g^{\mathscr{A}}}(\widehat{g}(\varphi)) M^{\mathscr{A}_{i}\gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi; K] := \sum_{N=0}^{\infty} C_{A_1 \ldots A_N}[\varphi] K^{A_1} \cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

$$(\mathrm{C1}) \quad 0 = -(\partial_{\mu} C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \ldots \alpha_K} E^{A}{}_{\mu, \alpha_1 \ldots \alpha_K}$$

$$(\mathrm{C2}) \quad 0 = C_A E^{A}{}_{\mu:B} - \partial_{\gamma}\left(C_A E^{A}{}_{\mu:B}{}^{\gamma}\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \ldots \alpha_K} E^{A}{}_{\mu, \alpha_1 \ldots \alpha_K}$$

$$(\mathrm{C3}) \quad 0 = -C \delta_{\mu}^{\gamma} + \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{\gamma \alpha_1 \ldots \alpha_K} \left(E^{A}{}_{\mu, \alpha_1 \ldots \alpha_K} + F^{A}{}_{\mu}{}^{\alpha_{K+1}}{}_{, \alpha_1 \ldots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{:A}{}^{(\alpha_1 \ldots \alpha_K |} F^{A}{}_{\mu}{}^{| \gamma)}{}_{, \alpha_1 \ldots \alpha_K}$$

$$(\mathrm{C4}) \quad 0 = -C_A\left(E^{A}{}_{\mu:B}{}^{\gamma} + F^{A}{}_{\mu}{}^{\gamma}{}_{:B}\right) + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\gamma \alpha_1 \ldots \alpha_K} \left(E^{A}{}_{\mu, \alpha_1 \ldots \alpha_K} + F^{A}{}_{\mu}{}^{\alpha_{K+1}}{}_{, \alpha_1 \ldots \alpha_{K+1}}\right) - \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{(\alpha_1 \ldots \alpha_K |} F^{A}{}_{\mu}{}^{| \gamma)}{}_{, \alpha_1 \ldots \alpha_K}$$

$$(\mathrm{C5}) \quad 0 = 2(\deg P - 1) p^{(\mu | \rho} C_{AB} F^{A}{}_{\rho}{}^{| \nu)} + \sum_{K=0}^{\infty} (K+1) C_{B:A}{}^{\alpha_1 \ldots \alpha_K (\mu |} M^{A | \nu)}{}_{, \alpha_1 \ldots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K \binom{K+2}{K} \left(\partial^K_{\alpha_1 \ldots \alpha_K} C_{:B}{}^{\alpha_1 \ldots \alpha_K \mu \nu}\right)$$

$$(\mathrm{C6}) \quad 0 = 2(\deg P - 1) C_{AB}\left(p^{\mu\nu} E^{A}{}_{\nu} - p^{\mu\nu}{}_{,\gamma} F^{A}{}_{\nu}{}^{\gamma}\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1 \ldots \alpha_K} M^{A\mu}{}_{, \alpha_1 \ldots \alpha_K} - \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \ldots \alpha_K} C_{:B}{}^{\alpha_1 \ldots \alpha_K \mu}\right)$$

$$(\mathrm{C7}) \quad 0 = 2\partial_{\mu}\left(C_A M^{A[\mu|}{}_{:B} M^{B|\gamma]}\right) - 2(\deg P - 1) p^{\rho\gamma}\left[C_A E^{A}{}_{\rho} + \partial_{\mu}\left(C_A F^{A}{}_{\rho}{}^{\mu}\right)\right] + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1 \ldots \alpha_K} M^{A\gamma}{}_{, \alpha_1 \ldots \alpha_K} + \\ + \sum_{K=0}^{\infty} \sum_{J=0}^{K} (-1)^J \binom{K}{J} (J+1) \partial^J_{\alpha_1 \ldots \alpha_J}\left(C_{:A}{}^{\beta_1 \ldots \beta_{K-J} (\alpha_1 \ldots \alpha_J |} M^{A | \gamma)}{}_{, \beta_1 \ldots \beta_{K-J}}\right)$$

$$(\mathrm{C8}) \quad 0 = 6(\deg P - 1) C_{AB_1B_2}\left(p^{\mu\nu} E^{A}{}_{\nu} - p^{\mu\nu}{}_{,\gamma} F^{A}{}_{\nu}{}^{\gamma}\right) - 4C_{A(B_1} M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A} M^{A\mu} - 2C_{B_1B_2:A}{}^{\alpha} M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^{\mu} \\ - \sum_{K=0}^{\infty} (-1)^K (K+1) \left(\partial^K_{\alpha_1 \ldots \alpha_K} C_{B_1:B_2}{}^{\mu \alpha_1 \ldots \alpha_K}\right)$$

$$(\mathrm{C9}) \quad 0 = \sum_{K=2}^{\infty} \sum_{J=2}^{K} (-1)^J \binom{K}{J} (J-1) \partial^{J+1}_{\gamma \alpha_1 \ldots \alpha_J}\left(C_{:A}{}^{\beta_1 \ldots \beta_{K-J} (\alpha_1 \ldots \alpha_J |} M^{A | \gamma)}{}_{, \beta_1 \ldots \beta_{K-J}}\right)$$

THE SIXTEEN SEQUENCES FOR $N \geq 2$

$$(\mathrm{C10}_N) \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{:A}{}^{\beta_1 \ldots \beta_N \alpha_1 \ldots \beta_K}\left(E^{A}{}_{\mu, \alpha_1 \ldots \alpha_K} + F^{A}{}_{\mu}{}^{\alpha_{K+1}}{}_{, \alpha_1 \ldots \alpha_{K+1}}\right) - C_{:A}{}^{(\beta_1 \ldots \beta_N \alpha_1 \ldots \alpha_{K-1} |} F^{A}{}_{\mu}{}^{| \alpha_K)}{}_{, \alpha_1 \ldots \alpha_K}\right]$$

$$(\mathrm{C11}_N) \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} \left[C_{B:A}{}^{\beta_1 \ldots \beta_N \alpha_1 \ldots \beta_K}\left(E^{A}{}_{\mu, \alpha_1 \ldots \alpha_K} + F^{A}{}_{\mu}{}^{\alpha_{K+1}}{}_{, \alpha_1 \ldots \alpha_{K+1}}\right) - C_{B:A}{}^{(\beta_1 \ldots \beta_N \alpha_1 \ldots \alpha_{K-1} |} F^{A}{}_{\mu}{}^{| \alpha_K)}{}_{, \alpha_1 \ldots \alpha_K}\right]$$

$$(\mathrm{C12}_N) \quad 0 = NC_{A(B_1 \ldots B_{N-1}} E^{A}{}_{|\mu| : B_N)} - (\partial_{\mu} C_{B_1 \ldots B_N}) + C_{B_1 \ldots B_N : A} E^{A}{}_{\mu} + C_{B_1 \ldots B_N : A}{}^{\alpha} E^{A}{}_{\mu, \alpha} + C_{B_1 \ldots B_N : A}{}^{\alpha\beta} E^{A}{}_{\mu, \alpha\beta}$$

$$(\mathrm{C13}_N) \quad 0 = -C_{B_1 \ldots B_N} \delta^{\gamma}_{\mu} - NC_{A(B_1 \ldots B_{N-1}} F^{A}{}_{|\mu|}{}^{\gamma}{}_{: B_N)} - C_{B_1 \ldots B_N : A} F^{A}{}_{\mu}{}^{\gamma} + C_{B_1 \ldots B_N : A}{}^{\gamma} E^{A}{}_{\mu} - C_{B_1 \ldots B_N : A}{}^{\alpha} F^{A}{}_{\mu}{}^{\gamma}{}_{,\alpha} + 2C_{B_1 \ldots B_N : A}{}^{\alpha\gamma} E^{A}{}_{\mu,\alpha} - C_{B_1 \ldots B_N : A}{}^{\alpha\beta} F^{A}{}_{\mu}{}^{\gamma}{}_{,\alpha\beta}$$

$$(\mathrm{C14}_N) \quad 0 = C_{AB_J(B_1 \ldots \widehat{B_J} \ldots B_{N-1}} E^{A}{}_{|\mu| : B_N)}{}^{\gamma} - C_{B_1 \ldots B_N} \delta^{\gamma}_{\mu} \qquad \text{for} \quad J = 1 \ldots N$$

$$(\mathrm{C15}_N) \quad 0 = C_{B_1 \ldots B_N : A}{}^{\beta_1 \beta_2} E^{A}{}_{\mu} - C_{B_1 \ldots B_N : A}{}^{(\beta_1 |} F^{A}{}_{\mu}{}^{| \beta_2)} - 2C_{B_1 \ldots B_N : A}{}^{\alpha(\beta_1 |} F^{A}{}_{\mu}{}^{| \beta_2)}{}_{,\alpha}$$

$$(\mathrm{C16}_N) \quad 0 = C_{B_1 \ldots B_N : A}{}^{(\alpha\beta |} F^{A}{}_{\mu}{}^{| \gamma)}$$

$$(\mathrm{C17}_N) \quad 0 = C_{B_1 \ldots B_N : A}{}^{(\mu\nu |} M^{A | \gamma)}$$

$$(\mathrm{C18}_N) \quad 0 = C_{AB_1 \ldots B_{N-1}}\left(M^{B[\mu|} M^{A|\nu]}{}_{:B} + (\deg P - 1) p^{\rho[\mu|} F^{A}{}_{\rho}{}^{|\nu]}\right)$$

$$(\mathrm{C19}_N) \quad 0 = C_{B_1 \ldots \widehat{B_J} \ldots B_{N+1} : B_J}{}^{\mu\nu} - C_{B_1 \ldots B_N : B_{N+1}}{}^{\mu\nu} \qquad \text{for} \quad J = 1 \ldots N+1$$

$$(\mathrm{C20}_N) \quad 0 = N \cdot (N+1)(\deg P - 1) C_{AB_1 \ldots B_N} p^{\rho(\mu|} F^{A}{}_{\rho}{}^{|\nu)} + NC_{B_1 \ldots B_N : A}{}^{(\mu|} M^{A|\nu)} + 2NC_{B_1 \ldots B_N : A}{}^{\alpha(\mu|} M^{A|\nu)}{}_{,\alpha} + (N-2) C_{B_1 \ldots B_{N-1} : B_N}{}^{\mu\nu}$$

$$(\mathrm{C21}_N) \quad 0 = (N+2) \cdot (N+1)(\deg P - 1) C_{AB_1 \ldots B_{N+1}}\left(p^{\mu\nu} E^{A}{}_{\nu} - p^{\mu\nu}{}_{,\gamma} F^{A}{}_{\nu}{}^{\gamma}\right) - (N+1)^2 C_{A(B_1 \ldots B_N} M^{A\mu}{}_{:B_{N+1})} - (N+1) C_{B_1 \ldots B_{N+1} : A} M^{A\mu} \\ -(N+1) C_{B_1 \ldots B_{N+1} : A}{}^{\alpha} M^{A\mu}{}_{,\alpha} - (N+1) C_{B_1 \ldots B_{N+1} : A}{}^{\alpha\beta} M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1} C_{B_1 \ldots \widehat{B_K} \ldots B_{N+1} : B_K}{}^{\mu} + 2\left(\partial_{\gamma} C_{B_1 \ldots B_N : B_{N+1}}{}^{\gamma\mu}\right)$$

$$(\mathrm{C22}_N) \quad 0 = C_{A:B}{}^{\mu_1 \ldots \mu_N} - \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N}{K} \left(\partial^K_{\alpha_1 \cdot \alpha_K} C_{B:A}{}^{\alpha_1 \ldots \alpha_K \mu_1 \ldots \mu_N}\right)$$

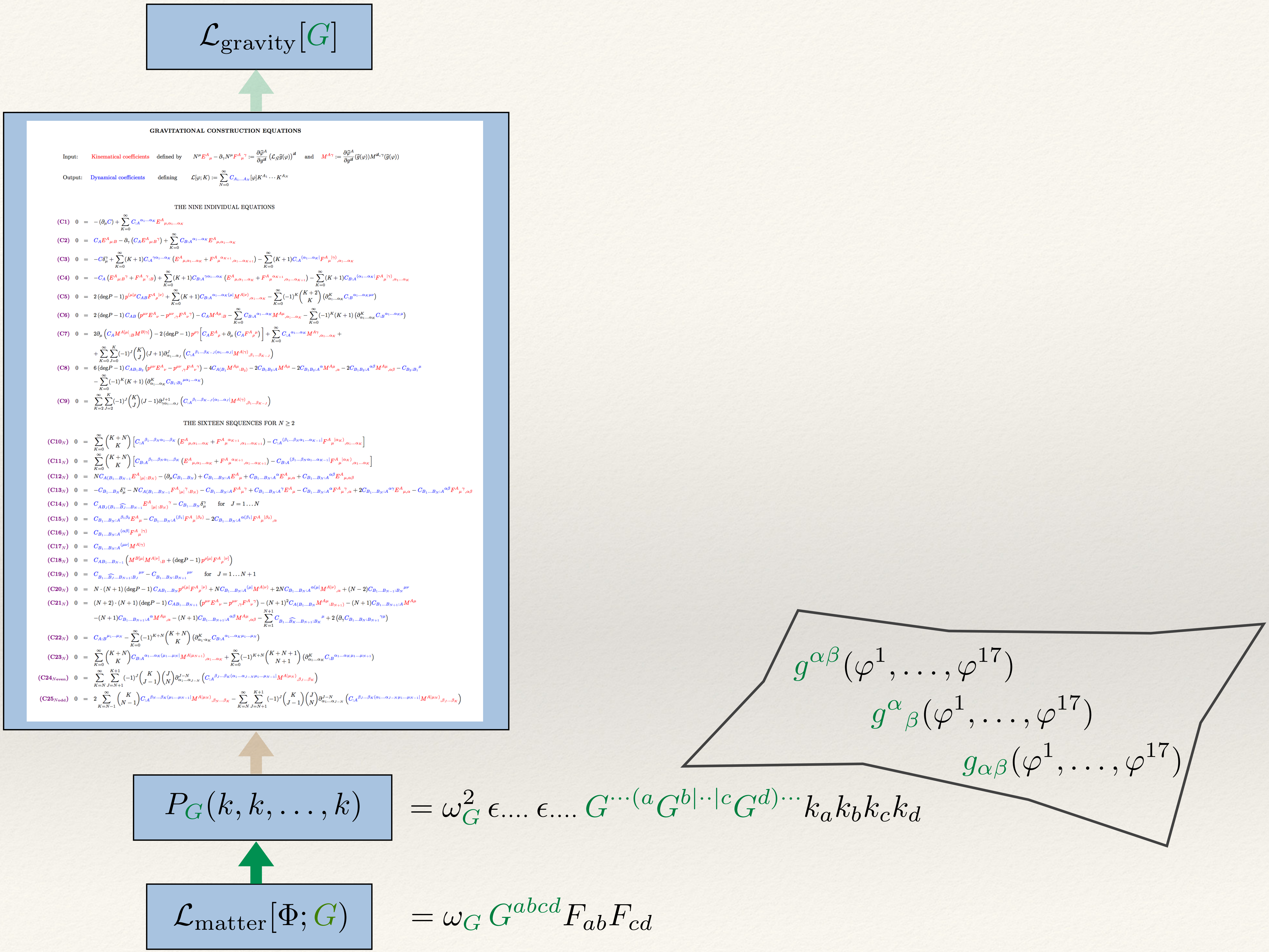
$$(\mathrm{C23}_N) \quad 0 = \sum_{K=0}^{\infty} \binom{K+N}{K} C_{B:A}{}^{\alpha_1 \ldots \alpha_K (\mu_1 \ldots \mu_N |} M^{A | \mu_{N+1})}{}_{, \alpha_1 \ldots \alpha_K} + \sum_{K=0}^{\infty} (-1)^{K+N} \binom{K+N+1}{N+1} \left(\partial^K_{\alpha_1 \ldots \alpha_K} C_{:B}{}^{\alpha_1 \ldots \alpha_K \mu_1 \ldots \mu_{N+1}}\right)$$

$$(\mathrm{C24}_{N_{\mathrm{even}}}) \quad 0 = \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \ldots \alpha_{J-N}}\left(C_{:A}{}^{\beta_J \ldots \beta_K (\alpha_1 \ldots \alpha_{J-N} \mu_1 \ldots \mu_{N-1} |} M^{A | \mu_N)}{}_{, \beta_J \ldots \beta_K}\right)$$

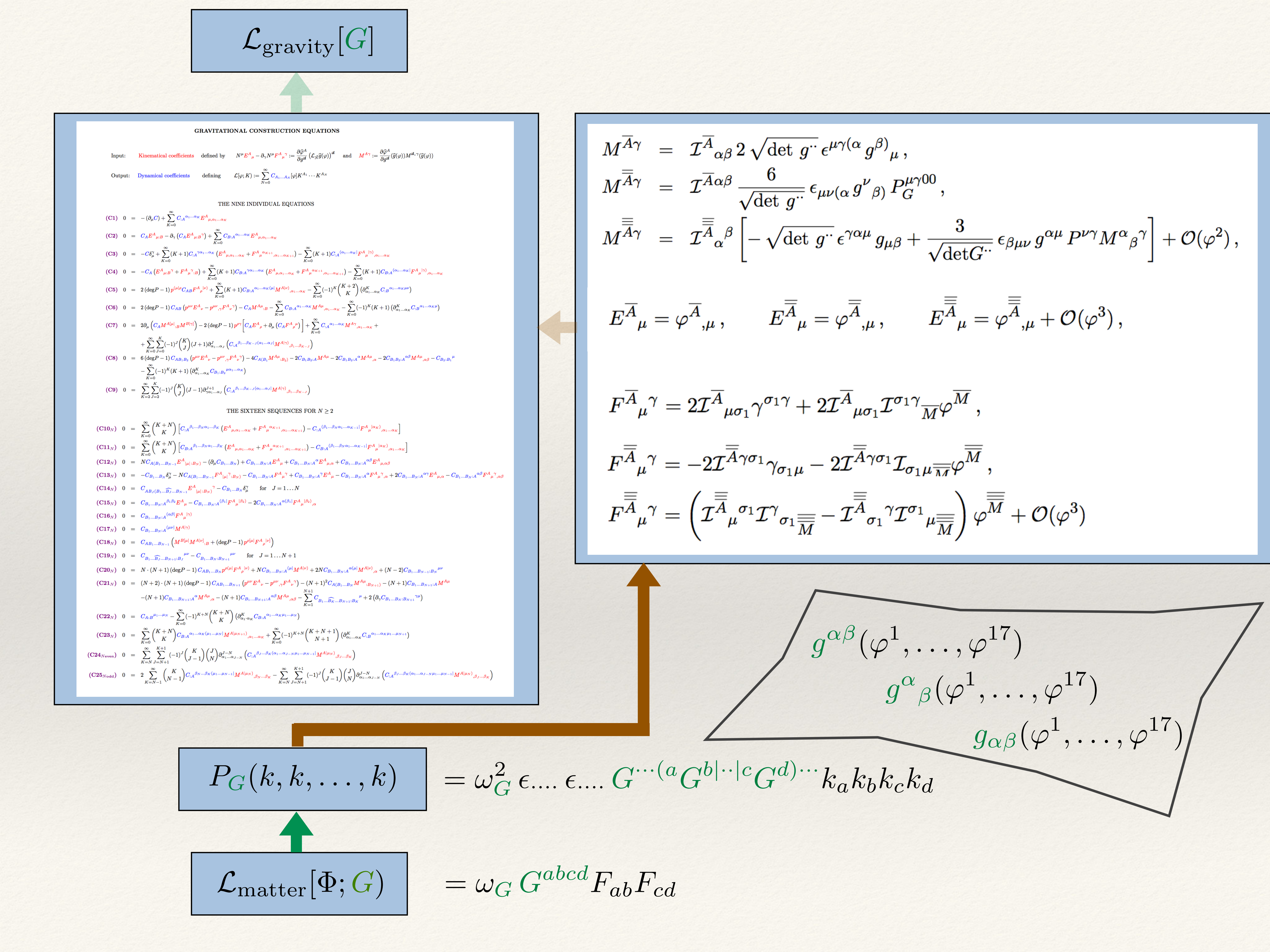
$$(\mathrm{C25}_{N_{\mathrm{odd}}}) \quad 0 = 2 \sum_{K=N-1}^{\infty} \binom{K}{N-1} C_{:A}{}^{\beta_N \ldots \beta_K (\mu_1 \ldots \mu_{N-1} |} M^{A | \mu_N)}{}_{, \beta_N \ldots \beta_K} - \sum_{K=N}^{\infty} \sum_{J=N+1}^{K+1} (-1)^J \binom{K}{J-1} \binom{J}{N} \partial^{J-N}_{\alpha_1 \ldots \alpha_{J-N}}\left(C_{:A}{}^{\beta_J \ldots \beta_K (\alpha_1 \ldots \alpha_{J-N} \mu_1 \ldots \mu_{N-1} |} M^{A | \mu_N)}{}_{, \beta_J \ldots \beta_K}\right)$$

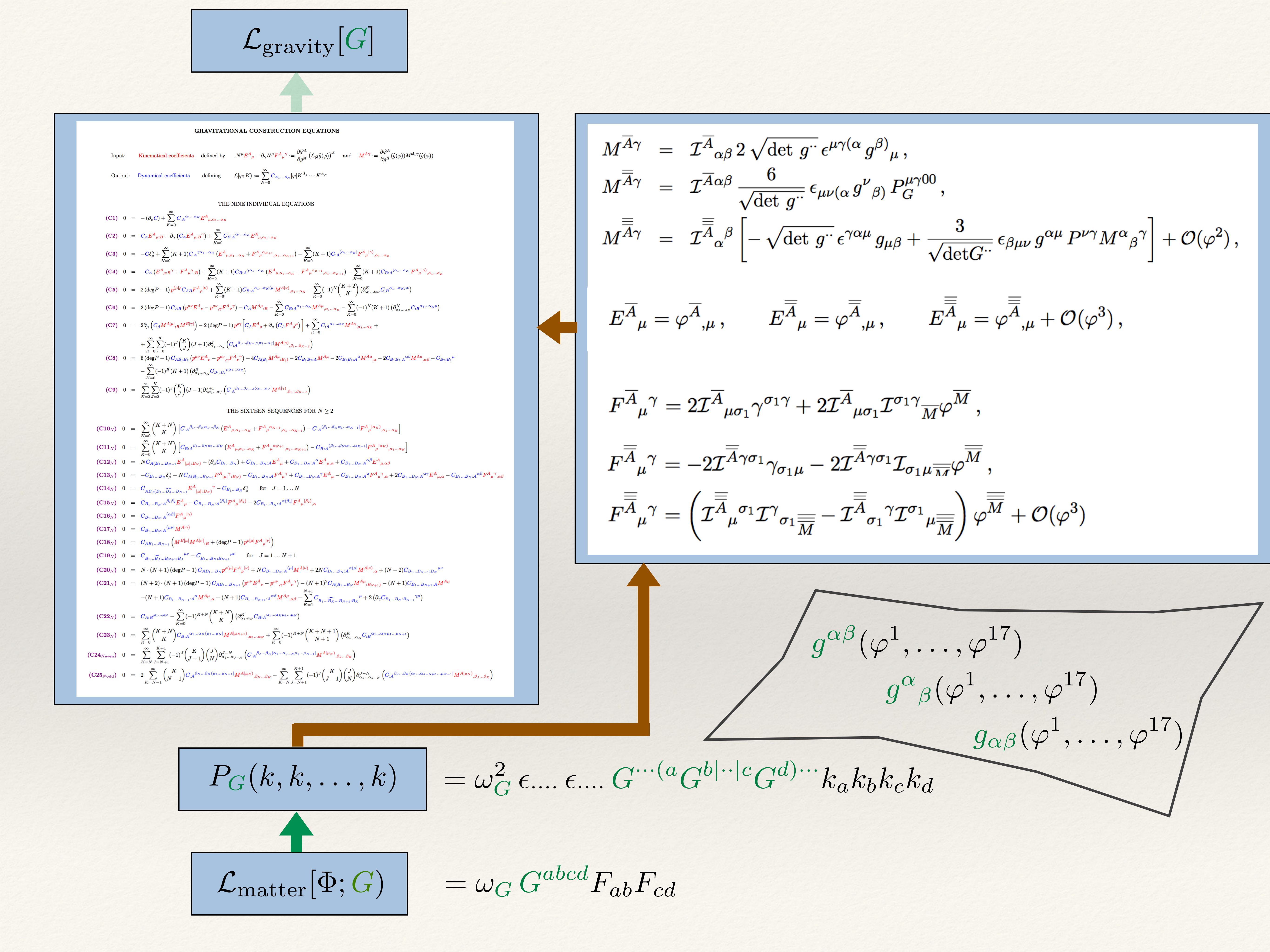
$$P_G(k, k, \ldots, k) \quad = \omega_G^2 \, \epsilon_{\ldots.} \, \epsilon_{\ldots.} \, G^{\cdots(a} G^{b|\cdot\cdot|c} G^{d)\cdots} k_a k_b k_c k_d$$

$$\mathcal{L}_{\text{matter}}[\Phi; G] \quad = \omega_G \, G^{abcd} F_{ab} F_{cd}$$



$\mathcal{L}_{\mathrm{gravity}}[G]$
GRAVITATIONAL CONSTRUCTION EQUATIONS
Input: Kinematical coefficients defined by
Output: Dynamical coefficients defining
THE NINE INDIVIDUAL EQUATIONS
THE SIXTEEN SEQUENCES FOR $N \geq 2$
$g^{\alpha\beta}(\varphi^1, \ldots, \varphi^{17})$
$g^{\alpha}{}_{\beta}(\varphi^1, \ldots, \varphi^{17})$
$g_{\alpha\beta}(\varphi^1, \ldots, \varphi^{17})$
$P_G(k, k, \ldots, k)$
$= \omega_G^2\, \epsilon_{\ldots.}\, \epsilon_{\ldots.}\, G^{\cdots(a}G^{b|\cdot\cdot|c}G^{d)\cdots} k_a k_b k_c k_d$
$\mathcal{L}_{\mathrm{matter}}[\Phi; G)$
$= \omega_G\, G^{abcd} F_{ab} F_{cd}$


$\mathcal{L}_{\text{gravity}}[G]$
GRAVITATIONAL CONSTRUCTION EQUATIONS
THE NINE INDIVIDUAL EQUATIONS
THE SIXTEEN SEQUENCES FOR $N \geq 2$
$M^{\bar{A}\gamma} = \mathcal{I}^{\bar{A}}{}_{\alpha\beta}\, 2\sqrt{\det g^{\cdot\cdot}}\, \epsilon^{\mu\gamma(\alpha}\, g^{\beta)}{}_{\mu}\,,$
$M^{\bar{\bar{A}}\gamma} = \mathcal{I}^{\bar{\bar{A}}\alpha\beta}\, \frac{6}{\sqrt{\det g^{\cdot\cdot}}}\, \epsilon_{\mu\nu(\alpha}\, g^{\nu}{}_{\beta)}\, P_G^{\mu\gamma 00}\,,$
$M^{\bar{\bar{\bar{A}}}\gamma} = \mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\alpha}{}^{\beta} \left[-\sqrt{\det g^{\cdot\cdot}}\, \epsilon^{\gamma\alpha\mu}\, g_{\mu\beta} + \frac{3}{\sqrt{\det G^{\cdot\cdot}}}\, \epsilon_{\beta\mu\nu}\, g^{\alpha\mu}\, P^{\nu\gamma} M^{\alpha}{}_{\beta}{}^{\gamma} \right] + \mathcal{O}(\varphi^2)\,,$
$E^{\bar{A}}{}_{\mu} = \varphi^{\bar{A}}{}_{,\mu}\,, \qquad E^{\bar{\bar{A}}}{}_{\mu} = \varphi^{\bar{\bar{A}}}{}_{,\mu}\,, \qquad E^{\bar{\bar{\bar{A}}}}{}_{\mu} = \varphi^{\bar{\bar{\bar{A}}}}{}_{,\mu} + \mathcal{O}(\varphi^3)\,,$
$F^{\bar{A}}{}_{\mu}{}^{\gamma} = 2\mathcal{I}^{\bar{A}}{}_{\mu\sigma_1}\gamma^{\sigma_1\gamma} + 2\mathcal{I}^{\bar{A}}{}_{\mu\sigma_1}\mathcal{I}^{\sigma_1\gamma}{}_{\bar{M}}\varphi^{\bar{M}}\,,$
$F^{\bar{\bar{A}}}{}_{\mu}{}^{\gamma} = -2\mathcal{I}^{\bar{\bar{A}}\gamma\sigma_1}\gamma_{\sigma_1\mu} - 2\mathcal{I}^{\bar{\bar{A}}\gamma\sigma_1}\mathcal{I}_{\sigma_1\mu\bar{\bar{M}}}\varphi^{\bar{\bar{M}}}\,,$
$F^{\bar{\bar{\bar{A}}}}{}_{\mu}{}^{\gamma} = \left(\mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\mu}{}^{\sigma_1}\mathcal{I}^{\gamma}{}_{\sigma_1\bar{\bar{\bar{M}}}} - \mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\sigma_1}{}^{\gamma}\mathcal{I}^{\sigma_1}{}_{\mu\bar{\bar{\bar{M}}}} \right) \varphi^{\bar{\bar{\bar{M}}}} + \mathcal{O}(\varphi^3)$
$g^{\alpha\beta}(\varphi^1, \ldots, \varphi^{17})$
$g^{\alpha}{}_{\beta}(\varphi^1, \ldots, \varphi^{17})$
$g_{\alpha\beta}(\varphi^1, \ldots, \varphi^{17})$
$P_G(k, k, \ldots, k) = \omega_G^2\, \epsilon_{\ldots}\, \epsilon_{\ldots}\, G^{\cdots(a} G^{b|\cdot\cdot|c} G^{d)\cdots} k_a k_b k_c k_d$
$\mathcal{L}_{\text{matter}}[\Phi; G] = \omega_G\, G^{abcd} F_{ab} F_{cd}$



$\mathcal{L}_{\text{gravity}}[G]$
GRAVITATIONAL CONSTRUCTION EQUATIONS
Input: Kinematical coefficients defined by
Output: Dynamical coefficients defining
THE NINE INDIVIDUAL EQUATIONS
THE SIXTEEN SEQUENCES FOR $N \geq 2$
$M^{\bar{A}\gamma} = \mathcal{I}^{\bar{A}}{}_{\alpha\beta}\, 2\sqrt{\det g^{\cdot\cdot}}\, \epsilon^{\mu\gamma(\alpha}\, g^{\beta)}{}_{\mu}\,,$
$M^{\bar{\bar{A}}\gamma} = \mathcal{I}^{\bar{\bar{A}}\alpha\beta}\, \frac{6}{\sqrt{\det g^{\cdot\cdot}}}\, \epsilon_{\mu\nu(\alpha}\, g^{\nu}{}_{\beta)}\, P_G^{\mu\gamma 00}\,,$
$M^{\bar{\bar{\bar{A}}}\gamma} = \mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\alpha}{}^{\beta}\left[-\sqrt{\det g^{\cdot\cdot}}\, \epsilon^{\gamma\alpha\mu}\, g_{\mu\beta} + \frac{3}{\sqrt{\det G^{\cdot\cdot}}}\, \epsilon_{\beta\mu\nu}\, g^{\alpha\mu}\, P^{\nu\gamma} M^{\alpha}{}_{\beta}{}^{\gamma}\right] + \mathcal{O}(\varphi^2)\,,$
$E^{\bar{A}}{}_{\mu} = \varphi^{\bar{A}}{}_{,\mu}\,, \qquad E^{\bar{\bar{A}}}{}_{\mu} = \varphi^{\bar{\bar{A}}}{}_{,\mu}\,, \qquad E^{\bar{\bar{\bar{A}}}}{}_{\mu} = \varphi^{\bar{\bar{\bar{A}}}}{}_{,\mu} + \mathcal{O}(\varphi^3)\,,$
$F^{\bar{A}}{}_{\mu}{}^{\gamma} = 2\mathcal{I}^{\bar{A}}{}_{\mu\sigma_1}\gamma^{\sigma_1\gamma} + 2\mathcal{I}^{\bar{A}}{}_{\mu\sigma_1}\mathcal{I}^{\sigma_1\gamma}{}_{\bar{M}}\varphi^{\bar{M}}\,,$
$F^{\bar{\bar{A}}}{}_{\mu}{}^{\gamma} = -2\mathcal{I}^{\bar{\bar{A}}\gamma\sigma_1}\gamma_{\sigma_1\mu} - 2\mathcal{I}^{\bar{\bar{A}}\gamma\sigma_1}\mathcal{I}_{\sigma_1\mu\bar{\bar{M}}}\varphi^{\bar{\bar{M}}}\,,$
$F^{\bar{\bar{\bar{A}}}}{}_{\mu}{}^{\gamma} = \left(\mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\mu}{}^{\sigma_1}\mathcal{I}^{\gamma}{}_{\sigma_1\bar{\bar{\bar{M}}}} - \mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\sigma_1}{}^{\gamma}\mathcal{I}^{\sigma_1}{}_{\mu\bar{\bar{\bar{M}}}}\right)\varphi^{\bar{\bar{\bar{M}}}} + \mathcal{O}(\varphi^3)$
$g^{\alpha\beta}(\varphi^1, \ldots, \varphi^{17})$
$g^{\alpha}{}_{\beta}(\varphi^1, \ldots, \varphi^{17})$
$g_{\alpha\beta}(\varphi^1, \ldots, \varphi^{17})$
$P_G(k, k, \ldots, k)$
$= \omega_G^2\, \epsilon_{\cdots\cdot}\, \epsilon_{\cdots\cdot}\, G^{\cdots(a}G^{b|\cdot\cdot|c}G^{d)\cdots}k_a k_b k_c k_d$
$\mathcal{L}_{\text{matter}}[\Phi; G)$
$= \omega_G\, G^{abcd} F_{ab} F_{cd}$

$$\mathcal{L}_{\text{gravity}}[G]$$

exact ← **hard** (wait for worthy matter action)

perturbative ← **hinges** on involutivity of {H,H}=D

symmetry ← **feasible**, but quite involved (M. Düll)

GRAVITATIONAL CONSTRUCTION EQUATIONS

Input: Kinematical coefficients defined by $N^\mu E^A{}_\mu - \partial_\gamma N^\mu F^A{}_\mu{}^\gamma := \frac{\partial\widehat{\varphi}^A}{\partial g^{\mathscr{A}}}\left(\mathcal{L}_{\vec N}\widehat{g}(\varphi)\right)^{\mathscr{A}}$ and $M^{A\gamma} := \frac{\partial\widehat{\varphi}^A}{\partial g^{\mathscr{A}}}(\widehat{g}(\varphi))M^{\mathscr{A}_i\gamma}(\widehat{g}(\varphi))$

Output: Dynamical coefficients defining $\mathcal{L}[\varphi;K] := \sum_{N=0}^{\infty} C_{A_1\dots A_N}[\varphi]K^{A_1}\cdots K^{A_N}$

THE NINE INDIVIDUAL EQUATIONS

$$\textbf{(C1)}\quad 0 = -(\partial_\mu C) + \sum_{K=0}^{\infty} C_{:A}{}^{\alpha_1\dots\alpha_K} E^A{}_{\mu,\alpha_1\dots\alpha_K}$$

$$\textbf{(C2)}\quad 0 = C_A E^A{}_{\mu:B} - \partial_\gamma\left(C_A E^A{}_{\mu:B}{}^\gamma\right) + \sum_{K=0}^{\infty} C_{B:A}{}^{\alpha_1\dots\alpha_K} E^A{}_{\mu,\alpha_1\dots\alpha_K}$$

$$\textbf{(C3)}\quad 0 = -C\delta^\gamma_\mu + \sum_{K=0}^{\infty}(K+1)C_{:A}{}^{\gamma\alpha_1\dots\alpha_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - \sum_{K=0}^{\infty}(K+1)C_{:A}{}^{(\alpha_1\dots\alpha_K|}F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1\dots\alpha_K}$$

$$\textbf{(C4)}\quad 0 = -C_A\left(E^A{}_{\mu:B}{}^\gamma + F^A{}_\mu{}^\gamma{}_{:B}\right) + \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{\gamma\alpha_1\dots\alpha_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{(\alpha_1\dots\alpha_K|}F^A{}_\mu{}^{|\gamma)}{}_{,\alpha_1\dots\alpha_K}$$

$$\textbf{(C5)}\quad 0 = 2(\deg P - 1)\,p^{(\mu|\rho}C_{AB}F^A{}_\rho{}^{|\nu)} + \sum_{K=0}^{\infty}(K+1)C_{B:A}{}^{\alpha_1\dots\alpha_K(\mu|}M^{A|\nu)}{}_{,\alpha_1\dots\alpha_K} - \sum_{K=0}^{\infty}(-1)^K\binom{K+2}{K}\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu\nu}\right)$$

$$\textbf{(C6)}\quad 0 = 2(\deg P - 1)\,C_{AB}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - C_A M^{A\mu}{}_{:B} - \sum_{K=0}^{\infty}C_{B:A}{}^{\alpha_1\dots\alpha_K}M^{A\mu}{}_{,\alpha_1\dots\alpha_K} - \sum_{K=0}^{\infty}(-1)^K(K+1)\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu}\right)$$

$$\textbf{(C7)}\quad 0 = 2\partial_\mu\left(C_A M^{A[\mu|}{}_{:B}M^{B|\gamma]}\right) - 2(\deg P - 1)\,p^{\rho\gamma}\left[C_A E^A{}_\rho + \partial_\mu\left(C_A F^A{}_\rho{}^\mu\right)\right] + \sum_{K=0}^{\infty}C_{:A}{}^{\alpha_1\dots\alpha_K}M^{A\gamma}{}_{,\alpha_1\dots\alpha_K} + \sum_{K=0}^{\infty}\sum_{J=0}^{K}(-1)^J\binom{K}{J}(J+1)\partial^J_{\alpha_1\dots\alpha_J}\left(C_{:A}{}^{\beta_1\dots\beta_{K-J}(\alpha_1\dots\alpha_J|}M^{A|\gamma)}{}_{,\beta_1\dots\beta_{K-J}}\right)$$

$$\textbf{(C8)}\quad 0 = 6(\deg P - 1)\,C_{AB_1B_2}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - 4C_{A(B_1}M^{A\mu}{}_{:B_2)} - 2C_{B_1B_2:A}M^{A\mu} - 2C_{B_1B_2:A}{}^\alpha M^{A\mu}{}_{,\alpha} - 2C_{B_1B_2:A}{}^{\alpha\beta}M^{A\mu}{}_{,\alpha\beta} - C_{B_2:B_1}{}^\mu - \sum_{K=0}^{\infty}(-1)^K(K+1)\left(\partial^K_{\alpha_1\dots\alpha_K}C_{B_1:B_2}{}^{\mu\alpha_1\dots\alpha_K}\right)$$

$$\textbf{(C9)}\quad 0 = \sum_{K=2}^{\infty}\sum_{J=2}^{K}(-1)^J\binom{K}{J}(J-1)\partial^{J+1}_{\gamma\alpha_1\dots\alpha_J}\left(C_{:A}{}^{\beta_1\dots\beta_{K-J}(\alpha_1\dots\alpha_J|}M^{A|\gamma)}{}_{,\beta_1\dots\beta_{K-J}}\right)$$

THE SIXTEEN SEQUENCES FOR $N \geq 2$

$$\textbf{(C10}_N\textbf{)}\quad 0 = \sum_{K=0}^{\infty}\binom{K+N}{K}\left[C_{:A}{}^{\beta_1\dots\beta_N\alpha_1\dots\beta_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - C_{:A}{}^{(\beta_1\dots\beta_N\alpha_1\dots\alpha_{K-1}|}F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1\dots\alpha_K}\right]$$

$$\textbf{(C11}_N\textbf{)}\quad 0 = \sum_{K=0}^{\infty}\binom{K+N}{K}\left[C_{B:A}{}^{\beta_1\dots\beta_N\alpha_1\dots\beta_K}\left(E^A{}_{\mu,\alpha_1\dots\alpha_K} + F^A{}_\mu{}^{\alpha_{K+1}}{}_{,\alpha_1\dots\alpha_{K+1}}\right) - C_{B:A}{}^{(\beta_1\dots\beta_N\alpha_1\dots\alpha_{K-1}|}F^A{}_\mu{}^{|\alpha_K)}{}_{,\alpha_1\dots\alpha_K}\right]$$

$$\textbf{(C12}_N\textbf{)}\quad 0 = NC_{A(B_1\dots B_{N-1}}E^A{}_{|\mu|:B_N)} - (\partial_\mu C_{B_1\dots B_N}) + C_{B_1\dots B_N:A}E^A{}_\mu + C_{B_1\dots B_N:A}{}^\alpha E^A{}_{\mu,\alpha} + C_{B_1\dots B_N:A}{}^{\alpha\beta}E^A{}_{\mu,\alpha\beta}$$

$$\textbf{(C13}_N\textbf{)}\quad 0 = -C_{B_1\dots B_N}\delta^\gamma_\mu - NC_{A(B_1\dots B_{N-1}}F^A{}_{|\mu|}{}^\gamma{}_{:B_N)} - C_{B_1\dots B_N:A}F^A{}_\mu{}^\gamma + C_{B_1\dots B_N:A}{}^\gamma E^A{}_\mu - C_{B_1\dots B_N:A}{}^\alpha F^A{}_\mu{}^\gamma{}_{,\alpha} + 2C_{B_1\dots B_N:A}{}^{\alpha\gamma}E^A{}_{\mu,\alpha} - C_{B_1\dots B_N:A}{}^{\alpha\beta}F^A{}_\mu{}^\gamma{}_{,\alpha\beta}$$

$$\textbf{(C14}_N\textbf{)}\quad 0 = C_{AB_J(B_1\dots\widehat{B_J}\dots B_{N-1}}E^A{}_{|\mu|:B_N)}{}^\gamma - C_{B_1\dots B_N}\delta^\gamma_\mu \qquad \text{for} \quad J = 1\dots N$$

$$\textbf{(C15}_N\textbf{)}\quad 0 = C_{B_1\dots B_N:A}{}^{\beta_1\beta_2}E^A{}_\mu - C_{B_1\dots B_N:A}{}^{(\beta_1|}F^A{}_\mu{}^{|\beta_2)} - 2C_{B_1\dots B_N:A}{}^{\alpha(\beta_1|}F^A{}_\mu{}^{|\beta_2)}{}_{,\alpha}$$

$$\textbf{(C16}_N\textbf{)}\quad 0 = C_{B_1\dots B_N:A}{}^{(\alpha\beta|}F^A{}_\mu{}^{|\gamma)}$$

$$\textbf{(C17}_N\textbf{)}\quad 0 = C_{B_1\dots B_N:A}{}^{(\mu\nu|}M^{A|\gamma)}$$

$$\textbf{(C18}_N\textbf{)}\quad 0 = C_{AB_1\dots B_{N-1}}\left(M^{B[\mu|}M^{A|\nu]}{}_{:B} + (\deg P - 1)\,p^{\rho[\mu|}F^A{}_\rho{}^{|\nu]}\right)$$

$$\textbf{(C19}_N\textbf{)}\quad 0 = C_{B_1\dots\widehat{B_J}\dots B_{N+1}:B_J}{}^{\mu\nu} - C_{B_1\dots B_N:B_{N+1}}{}^{\mu\nu} \qquad \text{for} \quad J = 1\dots N+1$$

$$\textbf{(C20}_N\textbf{)}\quad 0 = N\cdot(N+1)(\deg P - 1)\,C_{AB_1\dots B_N}p^{\rho(\mu|}F^A{}_\rho{}^{|\nu)} + NC_{B_1\dots B_N:A}{}^{(\mu|}M^{A|\nu)} + 2NC_{B_1\dots B_N:A}{}^{\alpha(\mu|}M^{A|\nu)}{}_{,\alpha} + (N-2)C_{B_1\dots B_{N-1}:B_N}{}^{\mu\nu}$$

$$\textbf{(C21}_N\textbf{)}\quad 0 = (N+2)\cdot(N+1)(\deg P - 1)\,C_{AB_1\dots B_{N+1}}\left(p^{\mu\nu}E^A{}_\nu - p^{\mu\nu}{}_{,\gamma}F^A{}_\nu{}^\gamma\right) - (N+1)^2C_{A(B_1\dots B_N}M^{A\mu}{}_{:B_{N+1})} - (N+1)C_{B_1\dots B_{N+1}:A}M^{A\mu} - (N+1)C_{B_1\dots B_{N+1}:A}{}^\alpha M^{A\mu}{}_{,\alpha} - (N+1)C_{B_1\dots B_{N+1}:A}{}^{\alpha\beta}M^{A\mu}{}_{,\alpha\beta} - \sum_{K=1}^{N+1}C_{B_1\dots\widehat{B_K}\dots B_{N+1}:B_K}{}^\mu + 2\left(\partial_\gamma C_{B_1\dots B_N:B_{N+1}}{}^{\gamma\mu}\right)$$

$$\textbf{(C22}_N\textbf{)}\quad 0 = C_{A:B}{}^{\mu_1\dots\mu_N} - \sum_{K=0}^{\infty}(-1)^{K+N}\binom{K+N}{K}\left(\partial^K_{\alpha_1\cdot\alpha_K}C_{B:A}{}^{\alpha_1\dots\alpha_K\mu_1\dots\mu_N}\right)$$

$$\textbf{(C23}_N\textbf{)}\quad 0 = \sum_{K=0}^{\infty}\binom{K+N}{K}C_{B:A}{}^{\alpha_1\dots\alpha_K(\mu_1\dots\mu_N|}M^{A|\mu_{N+1})}{}_{,\alpha_1\dots\alpha_K} + \sum_{K=0}^{\infty}(-1)^{K+N}\binom{K+N+1}{N+1}\left(\partial^K_{\alpha_1\dots\alpha_K}C_{:B}{}^{\alpha_1\dots\alpha_K\mu_1\dots\mu_{N+1}}\right)$$

$$\textbf{(C24}_{N\text{even}}\textbf{)}\quad 0 = \sum_{K=N}^{\infty}\sum_{J=N+1}^{K+1}(-1)^J\binom{K}{J-1}\binom{J}{N}\partial^{J-N}_{\alpha_1\dots\alpha_{J-N}}\left(C_{:A}{}^{\beta_J\dots\beta_K(\alpha_1\dots\alpha_{J-N}\mu_1\dots\mu_{N-1}|}M^{A|\mu_N)}{}_{,\beta_J\dots\beta_K}\right)$$

$$\textbf{(C25}_{N\text{odd}}\textbf{)}\quad 0 = 2\sum_{K=N-1}^{\infty}\binom{K}{N-1}C_{:A}{}^{\beta_N\dots\beta_K(\mu_1\dots\mu_{N-1}|}M^{A|\mu_N)}{}_{,\beta_N\dots\beta_K} - \sum_{K=N}^{\infty}\sum_{J=N+1}^{K+1}(-1)^J\binom{K}{J-1}\binom{J}{N}\partial^{J-N}_{\alpha_1\dots\alpha_{J-N}}\left(C_{:A}{}^{\beta_J\dots\beta_K(\alpha_1\dots\alpha_{J-N}\mu_1\dots\mu_{N-1}|}M^{A|\mu_N)}{}_{,\beta_J\dots\beta_K}\right)$$

$$M^{\bar{A}\gamma} = \mathcal{I}^{\bar{A}}{}_{\alpha\beta}\,2\sqrt{\det g^{\cdot\cdot}}\,\epsilon^{\mu\gamma(\alpha}g^{\beta)}{}_\mu\,,$$

$$M^{\bar{\bar{A}}\gamma} = \mathcal{I}^{\bar{\bar{A}}\alpha\beta}\,\frac{6}{\sqrt{\det g^{\cdot\cdot}}}\,\epsilon_{\mu\nu(\alpha}g^\nu{}_{\beta)}\,P_G^{\mu\gamma00}\,,$$

$$M^{\bar{\bar{\bar{A}}}\gamma} = \mathcal{I}^{\bar{\bar{\bar{A}}}}{}_\alpha{}^\beta\left[-\sqrt{\det g^{\cdot\cdot}}\,\epsilon^{\gamma\alpha\mu}g_{\mu\beta} + \frac{3}{\sqrt{\det G^{\cdot\cdot}}}\,\epsilon_{\beta\mu\nu}\,g^{\alpha\mu}\,P^{\nu\gamma}M^\alpha{}_\beta{}^\gamma\right] + \mathcal{O}(\varphi^2)\,,$$

$$E^{\bar{A}}{}_\mu = \varphi^{\bar{A}}{}_{,\mu}\,, \qquad E^{\bar{\bar{A}}}{}_\mu = \varphi^{\bar{\bar{A}}}{}_{,\mu}\,, \qquad E^{\bar{\bar{\bar{A}}}}{}_\mu = \varphi^{\bar{\bar{\bar{A}}}}{}_{,\mu} + \mathcal{O}(\varphi^3)\,,$$

$$F^{\bar{A}}{}_\mu{}^\gamma = 2\mathcal{I}^{\bar{A}}{}_{\mu\sigma_1}\gamma^{\sigma_1\gamma} + 2\mathcal{I}^{\bar{A}}{}_{\mu\sigma_1}\mathcal{I}^{\sigma_1\gamma}{}_{\overline{M}}\varphi^{\overline{M}}\,,$$

$$F^{\bar{\bar{A}}}{}_\mu{}^\gamma = -2\mathcal{I}^{\bar{\bar{A}}\gamma\sigma_1}\gamma_{\sigma_1\mu} - 2\mathcal{I}^{\bar{\bar{A}}\gamma\sigma_1}\mathcal{I}_{\sigma_1\mu\overline{\overline{M}}}\varphi^{\overline{\overline{M}}}\,,$$

$$F^{\bar{\bar{\bar{A}}}}{}_\mu{}^\gamma = \left(\mathcal{I}^{\bar{\bar{\bar{A}}}}{}_\mu{}^{\sigma_1}\mathcal{I}^\gamma{}_{\sigma_1\overline{\overline{\overline{M}}}} - \mathcal{I}^{\bar{\bar{\bar{A}}}}{}_{\sigma_1}{}^\gamma\mathcal{I}^{\sigma_1}{}_{\mu\overline{\overline{\overline{M}}}}\right)\varphi^{\overline{\overline{\overline{M}}}} + \mathcal{O}(\varphi^3)$$

$$g^{\alpha\beta}(\varphi^1,\dots,\varphi^{17})$$

$$g^\alpha{}_\beta(\varphi^1,\dots,\varphi^{17})$$

$$g_{\alpha\beta}(\varphi^1,\dots,\varphi^{17})$$

$$P_G(k,k,\dots,k) = \omega_G^2\,\epsilon_{\dots}\,\epsilon_{\dots}\,G^{\dots(a}G^{b|\cdot\cdot|c}G^{d)\dots}k_ak_bk_ck_d$$

$$\mathcal{L}_{\text{matter}}[\Phi;G] = \omega_G\,G^{abcd}F_{ab}F_{cd}$$

$$
\begin{aligned}
L(\varphi,\partial\varphi,\partial\partial\varphi)(K) \quad = \quad & \lambda+2\left(g_{10}-g_{9}\right)\gamma_{\alpha\beta}\overline{\varphi}^{\alpha\beta}+2\left(g_{8}-g_{7}\right)\gamma^{\alpha\beta}\overline{\overline{\varphi}}_{\alpha\beta}+\\
& \left[\left(\frac{22}{3}g_{12}-18g_{13}-18g_{14}-\frac{8}{3}g_{16}-2g_{17}-\frac{1}{2}g_{19}\right)\gamma_{\alpha\beta}\gamma^{\mu\nu}+\left(-8g_{12}+16g_{13}+24g_{14}-2g_{15}+4g_{16}\right)\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta}\right]\overline{\varphi}^{\alpha\beta}{}_{,\mu\nu}+\\
& \left[\left(2g_{12}-2g_{13}+14g_{14}-4g_{15}+4g_{16}-2g_{17}-\frac{1}{2}g_{19}\right)\gamma^{\alpha\beta}\gamma^{\mu\nu}+\left(-\frac{8}{2}g_{12}-8g_{14}+2g_{15}-\frac{8}{3}g_{16}\right)\gamma^{\mu\alpha}\gamma^{\beta\nu}\right]\overline{\overline{\varphi}}_{\alpha\beta,\mu\nu}+\\
& \left[\frac{1}{2}g_{9}\gamma_{\alpha\beta}\gamma_{\mu\nu}+\left(g_{8}+g_{9}-g_{10}\right)\gamma_{\alpha\mu}\gamma_{\nu\beta}\right]\overline{\varphi}^{\alpha\beta}\overline{\varphi}^{\mu\nu}+\left(g_{10}\gamma_{\alpha\beta}\gamma^{\mu\nu}+2g_{8}\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta}\right)\overline{\varphi}^{\alpha\beta}\overline{\overline{\varphi}}_{\mu\nu}+g_{11}\gamma_{\mu\alpha}\delta^{\nu}_{\beta}\overline{\varphi}^{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu}+\\
& \left[\frac{1}{2}\left(-g_{7}+g_{8}+g_{10}\right)\gamma^{\alpha\beta}\gamma^{\mu\nu}+g_{7}\gamma^{\alpha\mu}\gamma^{\nu\beta}\right]\overline{\overline{\varphi}}_{\alpha\beta}\overline{\overline{\varphi}}_{\mu\nu}+g_{11}\gamma^{\nu\alpha}\delta^{\beta}_{\mu}\overline{\overline{\varphi}}_{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu}+2\left(g_{8}-g_{7}\right)\overline{\overline{\overline{\varphi}}}^{\alpha}{}_{\beta}\overline{\overline{\overline{\varphi}}}^{\beta}{}_{\alpha}+\\
& 2\left(g_{6}-g_{5}\right)\epsilon_{\alpha\mu}{}^{\lambda}\gamma_{\beta\nu}\overline{\varphi}^{\alpha\beta}\overline{\varphi}^{\mu\nu}{}_{,\lambda}+2\left(g_{6}-g_{5}\right)\epsilon_{\alpha}{}^{\mu\lambda}\delta^{\nu}_{\beta}\overline{\varphi}^{\alpha\beta}\overline{\overline{\varphi}}_{\mu\nu,\lambda}+2\left(g_{6}-g_{5}\right)\epsilon^{\alpha}{}_{\mu}{}^{\lambda}\delta^{\beta}_{\nu}\overline{\overline{\varphi}}_{\alpha\beta}\overline{\varphi}^{\mu\nu}{}_{,\lambda}+2\left(g_{6}-g_{5}\right)\epsilon^{\alpha\mu\lambda}\gamma^{\beta\nu}\overline{\overline{\varphi}}_{\alpha\beta}\overline{\overline{\varphi}}_{\mu\nu,\lambda}+8\left(g_{6}-g_{5}\right)\epsilon_{\alpha\mu}{}^{\lambda}\gamma^{\beta\nu}\overline{\overline{\overline{\varphi}}}^{\alpha}{}_{\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\lambda}+\\
& \left[\left(-\frac{3}{2}g_{12}+\frac{7}{2}g_{13}-\frac{5}{2}g_{14}+\frac{3}{2}g_{15}-g_{16}+\frac{3}{2}g_{17}+\frac{1}{2}g_{18}+\frac{3}{8}g_{19}+g_{20}\right)\gamma_{\alpha\beta}\gamma_{\mu\nu}\gamma^{\lambda\kappa}+\left(\frac{5}{3}g_{12}+g_{22}-3g_{13}+g_{14}-g_{15}+\frac{2}{3}g_{16}-g_{17}-\frac{1}{4}g_{19}\right)\gamma_{\alpha\beta}\delta^{\lambda}_{\mu}\delta^{\kappa}_{\nu}+\right.\\
& \left(-8g_{12}+2g_{21}+20g_{13}+32g_{14}-3g_{15}+6g_{16}+g_{17}\right)\gamma_{\alpha\mu}\gamma_{\nu\beta}\gamma^{\lambda\kappa}+\left(4g_{12}-8g_{13}-18g_{14}+2g_{15}-4g_{16}\right)\gamma_{\alpha\mu}\delta^{\lambda}_{\nu}\delta^{\kappa}_{\beta}+\\
& \left.\left(\frac{11}{2}g_{12}+g_{22}-\frac{23}{2}g_{13}-\frac{19}{2}g_{14}-\frac{1}{2}g_{15}-g_{16}-\frac{3}{2}g_{17}-\frac{3}{8}g_{19}\right)\gamma_{\mu\nu}\delta^{\lambda}_{\alpha}\delta^{\kappa}_{\beta}\right]\overline{\varphi}^{\alpha\beta}\overline{\varphi}^{\mu\nu}{}_{,\lambda\kappa}+\\
& \left[\left(-\frac{11}{6}g_{12}+\frac{5}{2}g_{13}+\frac{1}{2}g_{14}+\frac{1}{2}g_{15}-\frac{1}{3}g_{16}+\frac{1}{2}g_{17}+\frac{1}{2}g_{18}+\frac{1}{8}g_{19}+g_{20}\right)\gamma_{\alpha\beta}\gamma^{\mu\nu}\gamma^{\lambda\kappa}+\left(2g_{12}+g_{22}-2g_{13}-2g_{14}\right)\gamma_{\alpha\beta}\gamma^{\mu\lambda}\gamma^{\kappa\nu}+\right.\\
& \left(\frac{14}{3}g_{12}+2g_{21}-6g_{13}+6g_{14}-3g_{15}+\frac{8}{3}g_{16}-g_{17}-\frac{1}{2}g_{19}\right)\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta}\gamma^{\lambda\kappa}+\left(-\frac{20}{3}g_{12}+8g_{13}-2g_{14}+2g_{15}-\frac{8}{3}g_{16}\right)\delta^{\mu}_{\alpha}\gamma^{\nu\lambda}\delta^{\kappa}_{\beta}+\\
& \left.\left(\frac{17}{6}g_{12}+g_{22}-\frac{7}{2}g_{13}+\frac{13}{2}g_{14}-\frac{5}{2}g_{15}+\frac{7}{3}g_{16}-\frac{3}{2}g_{17}-\frac{3}{8}g_{19}\right)\gamma^{\mu\nu}\delta^{\lambda}_{\alpha}\delta^{\kappa}_{\beta}\right]\overline{\varphi}^{\alpha\beta}\overline{\overline{\varphi}}_{\mu\nu,\lambda\kappa}+\\
& 2\left(g_{2}\gamma_{\alpha\beta}\gamma^{\nu\lambda}\delta^{\kappa}_{\mu}+g_{3}\gamma_{\alpha\mu}\gamma^{\lambda\kappa}\delta^{\nu}_{\beta}+4g_{4}\gamma_{\alpha\mu}\gamma^{\nu\lambda}\delta^{\kappa}_{\beta}\right)\overline{\varphi}^{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\lambda\kappa}\\
& \left[\left(\frac{13}{6}g_{12}-\frac{11}{2}g_{13}-\frac{23}{2}g_{14}+\frac{3}{2}g_{15}-\frac{7}{3}g_{16}+\frac{1}{2}g_{17}+\frac{1}{2}g_{18}+\frac{1}{8}g_{19}+g_{20}\right)\gamma^{\alpha\beta}\gamma_{\mu\nu}\gamma^{\lambda\kappa}+\right.\\
& \left(-\frac{7}{3}g_{12}+g_{22}+5g_{13}+13g_{14}-2g_{15}+\frac{8}{3}g_{16}-g_{17}-\frac{1}{4}g_{19}\right)\gamma^{\alpha\beta}\delta^{\lambda}_{\mu}\delta^{\kappa}_{\nu}+\left(-\frac{2}{3}g_{12}+2g_{21}+2g_{13}+14g_{14}-3g_{15}+\frac{10}{3}g_{16}-g_{17}-\frac{1}{2}g_{19}\right)\delta^{\alpha}_{\mu}\delta^{\beta}_{\nu}\gamma^{\lambda\kappa}+\\
& \left.\left(4g_{12}-8g_{13}-18g_{14}+2g_{15}-4g_{16}\right)\gamma^{\alpha\lambda}\delta^{\kappa}_{\mu}\delta^{\beta}_{\nu}+\left(-\frac{11}{6}g_{12}+g_{22}+\frac{13}{2}g_{13}+\frac{17}{2}g_{14}-\frac{1}{2}g_{15}+\frac{5}{3}g_{16}+\frac{1}{2}g_{17}+\frac{1}{8}g_{19}\right)\gamma^{\alpha\lambda}\gamma_{\mu\nu}\gamma^{\kappa\beta}\right]\overline{\overline{\varphi}}_{\alpha\beta}\overline{\varphi}^{\mu\nu}{}_{,\lambda\kappa}\\
& \left[\left(-\frac{5}{6}g_{12}+\frac{3}{2}g_{13}+\frac{15}{2}g_{14}-\frac{3}{2}g_{15}+\frac{5}{3}g_{16}-\frac{1}{2}g_{17}+\frac{1}{2}g_{18}-\frac{1}{8}g_{19}+g_{20}\right)\gamma^{\alpha\beta}\gamma^{\mu\nu}\gamma^{\lambda\kappa}+\left(\frac{2}{3}g_{12}+g_{22}-2g_{13}-6g_{14}+g_{15}-\frac{4}{3}g_{16}\right)\gamma^{\alpha\beta}\gamma^{\mu\lambda}\gamma^{\kappa\nu}+\right.\\
& \left(\frac{8}{3}g_{12}+2g_{21}-4g_{13}-8g_{14}+g_{15}-\frac{4}{3}g_{16}+g_{17}\right)\gamma^{\lambda\kappa}\gamma^{\alpha\mu}\gamma^{\nu\beta}+\left(-\frac{4}{3}g_{12}+8g_{13}+14g_{14}-2g_{15}+\frac{8}{3}g_{16}\right)\gamma^{\lambda\alpha}\gamma^{\beta\mu}\gamma^{\nu\kappa}+\\
& \left.\left(\frac{5}{6}g_{12}+g_{22}-\frac{3}{2}g_{13}-\frac{15}{2}g_{14}+\frac{3}{2}g_{15}-\frac{5}{3}g_{16}+\frac{1}{2}g_{17}+\frac{1}{8}g_{19}\right)\gamma^{\mu\nu}\gamma^{\alpha\lambda}\gamma^{\kappa\beta}\right]\overline{\overline{\varphi}}_{\alpha\beta}\overline{\overline{\varphi}}_{\mu\nu,\lambda\kappa}+\\
& \qquad\qquad\qquad\qquad\vdots
\end{aligned}
$$

$$\vdots$$

$$2\Big(g_2\gamma^{\alpha\beta}\gamma^{\nu\lambda}\delta^{\kappa}_{\mu}+g_3\gamma^{\lambda\kappa}\gamma^{\nu\alpha}\delta^{\beta}_{\mu}+4g_4\gamma^{\alpha\lambda}\gamma^{\kappa\nu}\delta^{\beta}_{\mu}\Big)\overline{\overline{\varphi}}_{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\lambda\kappa}+2\Big(g_2\gamma_{\mu\nu}\gamma^{\beta\lambda}\delta^{\kappa}_{\alpha}+g_3\gamma_{\alpha\mu}\gamma^{\lambda\kappa}\delta^{\beta}_{\nu}+4g_4\gamma_{\alpha\mu}\gamma^{\beta\lambda}\delta^{\kappa}_{\nu}\Big)\overline{\overline{\overline{\varphi}}}^{\alpha}{}_{\beta}\overline{\varphi}^{\mu\nu}{}_{,\lambda\kappa}+$$

$$2\Big(g_2\gamma^{\mu\nu}\gamma^{\beta\lambda}\delta^{\kappa}_{\alpha}+g_3\gamma^{\beta\mu}\gamma^{\lambda\kappa}\delta^{\nu}_{\alpha}+4g_4\gamma^{\beta\lambda}\gamma^{\kappa\mu}\delta^{\nu}_{\alpha}\Big)\overline{\overline{\overline{\varphi}}}^{\alpha}{}_{\beta}\overline{\overline{\varphi}}_{\mu\nu,\lambda\kappa}+$$

$$\Big[\Big(-4g_{12}-8g_{21}+4g_{13}-12g_{14}+4g_{15}-4g_{16}+g_{19}\Big)\gamma_{\alpha\mu}\gamma^{\beta\nu}\gamma^{\lambda\kappa}+\Big(24g_{14}-4g_{15}+8g_{16}\Big)\gamma_{\alpha\mu}\gamma^{\beta\lambda}\gamma^{\kappa\nu}\Big]\overline{\overline{\overline{\varphi}}}^{\alpha}{}_{\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\lambda\kappa}+$$

$$\Big[\Big(5g_{12}+g_{22}-8g_{13}-5g_{14}-g_{15}-g_{17}-\frac{1}{4}g_{19}\Big)\left(\gamma_{\alpha\beta}\delta^{\lambda}_{\mu}\delta^{\kappa}_{\nu}+\gamma_{\mu\nu}\delta^{\lambda}_{\alpha}\delta^{\kappa}_{\beta}\right)+\Big(-\frac{7}{3}g_{12}+5g_{13}+g_{14}+g_{15}-\frac{1}{3}g_{16}+g_{17}+\frac{1}{2}g_{18}+\frac{1}{4}g_{19}\Big)\gamma_{\alpha\beta}\gamma_{\mu\nu}\gamma^{\lambda\kappa}+$$

$$\Big(-\frac{2}{3}g_{12}-8g_{14}+2g_{15}-\frac{8}{3}g_{16}\Big)\gamma_{\alpha\mu}\delta^{\lambda}_{\beta}\delta^{\kappa}_{\nu}+\Big(-8g_{12}+20g_{13}+36g_{14}-4g_{15}+7g_{16}+g_{17}\Big)\gamma_{\alpha\mu}\gamma_{\nu\beta}\gamma^{\lambda\kappa}+$$

$$\Big(\frac{8}{3}g_{12}-6g_{13}-16g_{14}+2g_{15}-\frac{10}{3}g_{16}\Big)\gamma_{\alpha\mu}\delta^{\lambda}_{\nu}\delta^{\kappa}_{\beta}\Big]\overline{\varphi}^{\alpha\beta}{}_{,\lambda}\overline{\varphi}^{\mu\nu}{}_{,\kappa}+$$

$$\Big[\Big(-g_{12}+g_{13}-7g_{14}+2g_{15}-2g_{16}+g_{17}+g_{18}+\frac{1}{4}g_{19}\Big)\gamma_{\alpha\beta}\gamma^{\mu\nu}\gamma^{\lambda\kappa}+\Big(\frac{8}{3}g_{12}+2g_{22}+8g_{14}-2g_{15}+\frac{8}{3}g_{16}\Big)\gamma_{\alpha\beta}\gamma^{\mu\lambda}\gamma^{\kappa\nu}+$$

$$\Big(\frac{8}{3}g_{12}-4g_{13}+24g_{14}-7g_{15}+\frac{20}{3}g_{16}-2g_{17}-g_{19}\Big)\gamma^{\lambda\kappa}\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta}+\Big(2g_{12}+2g_{22}-2g_{13}+14g_{14}-4g_{15}+4g_{16}-2g_{17}-\frac{1}{2}g_{19}\Big)\gamma^{\mu\nu}\delta^{\lambda}_{\alpha}\delta^{\kappa}_{\beta}+$$

$$\Big(-\frac{8}{3}g_{12}+4g_{13}-8g_{14}+2g_{15}-\frac{8}{3}g_{16}\Big)\gamma^{\mu\lambda}\delta^{\kappa}_{\alpha}\delta^{\nu}_{\beta}+\Big(-\frac{4}{3}g_{12}-16g_{14}+4g_{15}-\frac{16}{3}g_{16}\Big)\gamma^{\mu\kappa}\delta^{\lambda}_{\alpha}\delta^{\nu}_{\beta}\Big]\overline{\varphi}^{\alpha\beta}{}_{,\lambda}\overline{\overline{\varphi}}_{\mu\nu,\kappa}$$

$$4\Big[g_2\gamma_{\alpha\beta}\gamma^{\nu\lambda}\delta^{\kappa}_{\mu}+2g_4\left(\gamma_{\alpha\mu}\gamma^{\nu\lambda}\delta^{\kappa}_{\beta}+\gamma_{\alpha\mu}\gamma^{\nu\kappa}\delta^{\lambda}_{\beta}\right)+(g_3-g_1)\gamma_{\alpha\mu}\gamma^{\lambda\kappa}\delta^{\nu}_{\beta}\Big]\overline{\varphi}^{\alpha\beta}{}_{,\lambda}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\kappa}$$

$$\Big(g_{22}\left(\gamma^{\alpha\beta}\gamma^{\lambda\mu}\gamma^{\nu\kappa}+\gamma^{\mu\nu}\gamma^{\lambda\alpha}\gamma^{\beta\kappa}\right)+\frac{1}{2}g_{18}\gamma^{\alpha\beta}\gamma^{\mu\nu}\gamma^{\lambda\kappa}+2g_{12}\gamma^{\lambda\alpha}\gamma^{\beta\mu}\gamma^{\nu\kappa}+g_{17}\gamma^{\alpha\mu}\gamma^{\nu\beta}\gamma^{\kappa\lambda}+2g_{13}\gamma^{\lambda\mu}\gamma^{\nu\alpha}\gamma^{\beta\kappa}\Big)\overline{\overline{\varphi}}_{\alpha\beta,\lambda}\overline{\overline{\varphi}}_{\mu\nu,\kappa}$$

$$4\Big[g_2\gamma^{\alpha\beta}\gamma^{\nu(\kappa}\delta^{\lambda)}_{\mu}+2g_4\left(\gamma^{\alpha\kappa}\gamma^{\lambda\nu}\delta^{\beta}_{\mu}+\gamma^{\nu\kappa}\gamma^{\lambda\alpha}\delta^{\beta}_{\mu}\right)+(g_3-g_1)\gamma^{\lambda\kappa}\gamma^{\alpha\nu}\delta^{\beta}_{\mu}\Big]\overline{\overline{\varphi}}_{\alpha\beta,\lambda}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\kappa}+$$

$$\Big(2g_{16}\gamma^{\nu\kappa}\delta^{\lambda}_{\alpha}\delta^{\beta}_{\mu}+g_{19}\gamma_{\alpha\mu}\gamma^{\beta\nu}\gamma^{\lambda\kappa}+2g_{15}\gamma_{\alpha\mu}\gamma^{\beta\kappa}\gamma^{\lambda\nu}\Big)\overline{\overline{\overline{\varphi}}}^{\alpha}{}_{\beta,\lambda}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\kappa}$$

$$2g_5\gamma_{\alpha\mu}\delta^{\nu}_{\beta}\overline{k}^{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu}+2g_5\gamma^{\alpha\nu}\delta^{\beta}_{\mu}\overline{\overline{k}}_{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu}+2g_6\gamma_{\mu\alpha}\delta^{\beta}_{\nu}\overline{\overline{\overline{k}}}^{\alpha}{}_{\beta}\overline{\varphi}^{\mu\nu}+2g_6\gamma^{\mu\beta}\delta^{\nu}_{\alpha}\overline{\overline{\overline{k}}}^{\alpha}{}_{\beta}\overline{\overline{\varphi}}_{\mu\nu}+$$

$$\Big(-\frac{16}{3}g_{12}+8g_{13}+4g_{14}+2g_{15}-\frac{4}{3}g_{16}\Big)\epsilon_{\alpha\mu}{}^{\lambda}\delta^{\nu}_{\beta}\overline{k}^{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\lambda}+\Big(\frac{16}{3}g_{12}-8g_{13}-12g_{14}+2g_{15}-\frac{8}{3}g_{16}\Big)\epsilon^{\alpha}{}_{\mu}{}^{\lambda}\gamma^{\beta\nu}\overline{\overline{k}}_{\alpha\beta}\overline{\overline{\overline{\varphi}}}^{\mu}{}_{\nu,\lambda}+4g_{14}\epsilon_{\alpha\mu}{}^{\lambda}\delta^{\beta}_{\nu}\overline{\overline{\overline{k}}}^{\alpha}{}_{\beta}\overline{\varphi}^{\mu\nu}{}_{,\lambda}+4g_{14}\epsilon_{\alpha}{}^{\mu\lambda}\gamma^{\beta\nu}\overline{\overline{\overline{k}}}^{\alpha}{}_{\beta}\overline{\overline{\varphi}}_{\mu\nu,\lambda}+$$

$$\Big[\Big(-\frac{7}{3}g_{12}+5g_{13}+g_{14}+g_{15}-\frac{1}{3}g_{16}+g_{17}+\frac{1}{4}g_{19}+g_{20}\Big)\gamma_{\alpha\beta}\gamma_{\mu\nu}+\Big(\frac{8}{3}g_{12}+2g_{21}-4g_{13}-4g_{14}-\frac{1}{3}g_{16}\Big)\gamma_{\alpha\mu}\gamma_{\nu\beta}\Big]\overline{k}^{\alpha\beta}\overline{k}^{\mu\nu}+$$

$$\Big[\Big(-g_{12}+g_{13}-7g_{14}+2g_{15}-2g_{16}+g_{17}+\frac{1}{4}g_{19}+2g_{20}\Big)\gamma_{\alpha\beta}\gamma^{\mu\nu}+\Big(\frac{4}{3}g_{12}+4g_{21}+4g_{14}-g_{15}+\frac{4}{3}g_{16}\Big)\delta^{\mu}_{\alpha}\delta^{\nu}_{\beta}\Big]\overline{k}^{\alpha\beta}\overline{\overline{k}}_{\mu\nu}+$$

$$4g_1\gamma_{\alpha\mu}\delta^{\nu}_{\beta}\overline{k}^{\alpha\beta}\overline{\overline{\overline{k}}}^{\mu}{}_{\nu}+\Big(g_{20}\gamma^{\alpha\beta}\gamma^{\mu\nu}+2g_{21}\gamma^{\alpha\mu}\gamma^{\nu\beta}\Big)\overline{\overline{k}}_{\alpha\beta}\overline{\overline{k}}_{\mu\nu}+4g_1\gamma^{\alpha\nu}\delta^{\beta}_{\mu}\overline{\overline{k}}_{\alpha\beta}\overline{\overline{\overline{k}}}^{\mu}{}_{\nu}+4\Big(-g_{12}-2g_{21}+g_{13}+g_{14}\Big)\overline{\overline{\overline{k}}}^{\alpha}{}_{\beta}\overline{\overline{\overline{k}}}^{\beta}{}_{\alpha}$$

$$+\mathcal{O}(3)$$

scalar modes $\widetilde{F}-\widetilde{E}$, $\widetilde{F}+\widetilde{E}$, E, C, and a:

$$\left[\frac{\delta H}{\delta a}\right] = \Delta\left\{\kappa_1\,(\widetilde{F}-\widetilde{E}) + \kappa_2\,(\widetilde{F}+\widetilde{E}) + (\frac{2}{3}\kappa_1 + 2\kappa_3)\,\Delta E\right\}$$

$$\left[\frac{\delta H}{\delta N^a}\right]^{(\mathrm{S})} = \partial_a \frac{\mathrm{d}}{\mathrm{d}\,t}\left\{\kappa_1\,(\widetilde{F}-\widetilde{E}) + \kappa_2\,(\widetilde{F}+\widetilde{E}) + (\frac{2}{3}\kappa_1 + 2\kappa_3)\,\Delta E\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} - \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STR})} = \frac{1}{3}\gamma_{ab}\left\{\frac{3}{2}\kappa_1\,(\ddot{\widetilde{F}}-\ddot{\widetilde{E}}) - \frac{1}{2}\kappa_1\,\Delta(\widetilde{F}-\widetilde{E}) + \frac{3}{2}\kappa_2\,(\ddot{\widetilde{F}}+\ddot{\widetilde{E}}) + (\frac{3}{2}\kappa_1 - \kappa_2)\,\Delta(\widetilde{F}+\widetilde{E}) + \kappa_3\,\Delta\Delta E + 2\kappa_1\,\Delta a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} + \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STR})} = \frac{1}{3}\gamma_{ab}\left\{\frac{3}{2}\kappa_2\,(\ddot{\widetilde{F}}-\ddot{\widetilde{E}}) + (\frac{3}{2}\kappa_1 - \kappa_2)\,\Delta(\widetilde{F}-\widetilde{E}) + \kappa_4\,(\ddot{\widetilde{F}}+\ddot{\widetilde{E}}) + (3\kappa_2 - \kappa_4)\,\Delta(\widetilde{F}+\widetilde{E}) + \kappa_5\,(\widetilde{F}+\widetilde{E}) + (\kappa_1 - \frac{1}{3}\kappa_2 + 3\kappa_3)\,\Delta\Delta E + 2\kappa_2\,\Delta a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} - \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STF})} = \Delta_{ab}\left\{-\frac{1}{2}\kappa_3\,\Delta E + \frac{1}{4}\kappa_1\,(\widetilde{F}-\widetilde{E}) + (-\frac{3}{4}\kappa_1 + \frac{1}{2}\kappa_2)\,(\widetilde{F}+\widetilde{E}) - \kappa_1\,a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} + \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STF})} = \Delta_{ab}\left\{\kappa_6\,\ddot{E} + (\frac{1}{3}\kappa_1 + \frac{1}{2}\kappa_3 - \kappa_6)\,\Delta E + \kappa_7\,\Box C + \kappa_8\,\dot{C} + \kappa_9\,E + \kappa_{10}\,C + (\frac{1}{4}\kappa_1 + \frac{3}{2}\kappa_3)\,(\widetilde{F}-\widetilde{E}) + (\frac{3}{4}\kappa_1 + \frac{9}{2}\kappa_3)\,(\widetilde{F}+\widetilde{E}) + (\kappa_1 + 6\kappa_3)\,a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{STF})} = \Delta_{ab}\left\{\frac{1}{2}\kappa_7\,\Box E + \kappa_{11}\,\Box C - \frac{1}{2}\kappa_8\,\dot{E} + \frac{1}{2}\kappa_{10}\,E - 2\kappa_9\,C\right\}$$

vector modes F_a, C_a, and N_a:

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} - \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{V})} = \partial_{(b}\left\{3\kappa_3\,\ddot{F}_{a)} - (\kappa_1 + 6\kappa_3)\,\epsilon_{a)}{}^{c\mu}\dot{C}_{c,\mu} - \kappa_1\,\dot{N}_{a)}\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} + \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{V})} = \partial_{(b}\left\{(3\kappa_3 + 2\kappa_6)\,\ddot{F}_{a)} + \kappa_{11}\,\Delta F_{a)} + 2\kappa_7\,\Box C_{a)} + (\kappa_1 - 4\kappa_6 - 2\kappa_{11})\,\epsilon_{a)}{}^{c\mu}\dot{C}_{c,\mu} + 2\kappa_8\,\epsilon_{a)}{}^{c\mu}F_{c,\mu} + (\kappa_1 + 6\kappa_3)\,\dot{N}_{a)} + 2\kappa_9\,F_{a)} + 2\kappa_{10}\,C_{a)}\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{V})} = \partial_{(b}\left\{\kappa_7\,\Box F_{a)} + 2\kappa_{11}\,\ddot{C}_{a)} + (-\kappa_1 + 4\kappa_6)\,\Delta C_{a)} + (3\kappa_3 + 2\kappa_6 + \kappa_{11})\,\epsilon_{a)}{}^{c\mu}\dot{F}_{c,\mu} + 4\kappa_8\,\epsilon_{a)}{}^{c\mu}C_{c,\mu} + (\kappa_1 + 6\kappa_3)\,\epsilon_{a)}{}^{c\mu}N_{c,\mu} - \kappa_8\,\dot{F}_{a)} + \kappa_{10}\,F_{a)} - 4\kappa_9\,C_{a)}\right\}$$

$$\left[\frac{\delta H}{\delta N^a}\right]^{(\mathrm{V})} = \Delta\left\{3\kappa_3\,\dot{F}_a - (\kappa_1 + 6\kappa_3)\,\epsilon_a{}^{c\mu}C_{c,\mu} - \kappa_1\,N_a\right\}$$

tensor modes $F^{ab}-E^{ab}$, $F^{ab}+E^{ab}$, and C^{ab}:

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} - \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{TT})} = -\frac{1}{4}\kappa_1\,\Box(F_{ab} - E_{ab}) + (\frac{1}{4}\kappa_1 + \frac{3}{2}\kappa_3)\,\left[(\ddot{F}_{ab} + \ddot{E}_{ab}) + \Delta(F_{ab} + E_{ab})\right] - (\kappa_1 + 6\kappa_3)\,\epsilon_{(a}{}^{c\mu}\dot{C}_{b)c,\mu}$$

$$\begin{aligned}\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}} + \frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{TT})} &= (\frac{1}{4}\kappa_1 + \frac{3}{2}\kappa_3)\,\left[(\ddot{F}_{ab} - \ddot{E}_{ab}) + \Delta(F_{ab} - E_{ab})\right] + (-\frac{1}{4}\kappa_1 + \kappa_6)\,(\ddot{F}_{ab} + \ddot{E}_{ab}) + (-\frac{3}{4}\kappa_1 + 3\kappa_6 + 2\kappa_{11})\,\Delta(F_{ab} + E_{ab}) \\ &\quad + \kappa_7\,\Box C_{ab} + (\kappa_1 - 4\kappa_6 - 2\kappa_{11})\,\epsilon_{(a}{}^{c\mu}\dot{C}_{b)c,\mu} + 2\kappa_8\,\epsilon_{(a}{}^{c\mu}(F_{b)c,\mu} + E_{b)c,\mu}) + \kappa_8\,\dot{C}_{ab} + \kappa_9\,(F_{ab} + E_{ab}) + \kappa_{10}\,C_{ab}\end{aligned}$$

$$\begin{aligned}\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{TT})} &= \frac{1}{2}\kappa_7\,\Box(F_{ab} + E_{ab}) + \kappa_{11}\,\ddot{C}_{ab} + (-2\kappa_1 + 8\kappa_6 + 3\kappa_{11})\,\Delta C_{ab} + (\frac{1}{2}\kappa_1 + 3\kappa_3)\,\epsilon_{(a}{}^{c\mu}(\dot{F}_{b)c,\mu} - \dot{E}_{b)c,\mu}) + (-\frac{1}{2}\kappa_1 + 2\kappa_6 + \kappa_{11})\,\epsilon_{(a}{}^{c\mu}(\dot{F}_{b)c,\mu} + \dot{E}_{b)c,\mu}) \\ &\quad + 4\kappa_8\,\epsilon_{(a}{}^{c\mu}C_{b)c,\mu} - \frac{1}{2}\kappa_8\,(\dot{F}_{ab} + \dot{E}_{ab}) + \frac{1}{2}\kappa_{10}\,(F_{ab} + E_{ab}) - 2\kappa_9\,C_{ab}\end{aligned}$$

EQUATIONS OF MOTION FOR SCALAR, VECTOR, AND TENSOR MODES

scalar modes $\widetilde{F}-\widetilde{E}$, $\widetilde{F}+\widetilde{E}$, E, C, and a:

$$
\begin{aligned}
\left[\frac{\delta H}{\delta a}\right] &= \Delta\left\{\kappa_1\,(\widetilde{F}-\widetilde{E})+\kappa_2\,(\widetilde{F}+\widetilde{E})+(\frac{2}{3}\kappa_1+2\kappa_3)\,\Delta E\right\}\\
\left[\frac{\delta H}{\delta N^a}\right]^{(\mathrm{S})} &= \partial_a\frac{\mathrm{d}}{\mathrm{d}\,t}\left\{\kappa_1\,(\widetilde{F}-\widetilde{E})+\kappa_2\,(\widetilde{F}+\widetilde{E})+(\frac{2}{3}\kappa_1+2\kappa_3)\,\Delta E\right\}\\
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STR})} &= \frac{1}{3}\gamma_{ab}\left\{\frac{3}{2}\kappa_1\,(\ddot{\widetilde{F}}-\ddot{\widetilde{E}})-\frac{1}{2}\kappa_1\,\Delta(\widetilde{F}-\widetilde{E})+\frac{3}{2}\kappa_2\,(\ddot{\widetilde{F}}+\ddot{\widetilde{E}})+(\frac{3}{2}\kappa_1-\kappa_2)\,\Delta(\widetilde{F}+\widetilde{E})+\kappa_3\,\Delta\Delta E+2\kappa_1\,\Delta a\right\}\\
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STR})} &= \frac{1}{3}\gamma_{ab}\left\{\frac{3}{2}\kappa_2\,(\ddot{\widetilde{F}}-\ddot{\widetilde{E}})+(\frac{3}{2}\kappa_1-\kappa_2)\,\Delta(\widetilde{F}-\widetilde{E})+\kappa_4\,(\ddot{\widetilde{F}}+\ddot{\widetilde{E}})+(3\kappa_2-\kappa_4)\,\Delta(\widetilde{F}+\widetilde{E})+\kappa_5\,(\widetilde{F}+\widetilde{E})+(\kappa_1-\frac{1}{3}\kappa_2+3\kappa_3)\,\Delta\Delta E+2\kappa_2\,\Delta a\right\}\\
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STF})} &= \Delta_{ab}\left\{-\frac{1}{2}\kappa_3\,\Delta E+\frac{1}{4}\kappa_1\,(\widetilde{F}-\widetilde{E})+(-\frac{3}{4}\kappa_1+\frac{1}{2}\kappa_2)\,(\widetilde{F}+\widetilde{E})-\kappa_1\,a\right\}\\
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STF})} &= \Delta_{ab}\left\{\kappa_6\,\ddot{E}+(\frac{1}{3}\kappa_1+\frac{1}{2}\kappa_3-\kappa_6)\,\Delta E+\kappa_7\,\Box C+\kappa_8\,\dot{C}+\kappa_9\,E+\kappa_{10}\,C+(\frac{1}{4}\kappa_1+\frac{3}{2}\kappa_3)\,(\widetilde{F}-\widetilde{E})+(\frac{3}{4}\kappa_1+\frac{9}{2}\kappa_3)\,(\widetilde{F}+\widetilde{E})+(\kappa_1+6\kappa_3)\,a\right\}\\
\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{STF})} &= \Delta_{ab}\left\{\frac{1}{2}\kappa_7\,\Box E+\kappa_{11}\,\Box C-\frac{1}{2}\kappa_8\,\dot{E}+\frac{1}{2}\kappa_{10}\,E-2\kappa_9\,C\right\}
\end{aligned}
$$

vector modes F_a, C_a, and N_a:

$$
\begin{aligned}
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{V})} &= \partial_{(b}\left\{3\kappa_3\,\ddot{F}_{a)}-(\kappa_1+6\kappa_3)\,\epsilon_{a)}{}^{c\mu}\dot{C}_{c,\mu}-\kappa_1\,\dot{N}_{a)}\right\}\\
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{V})} &= \partial_{(b}\left\{(3\kappa_3+2\kappa_6)\,\ddot{F}_{a)}+\kappa_{11}\,\Delta F_{a)}+2\kappa_7\,\Box C_{a)}+(\kappa_1-4\kappa_6-2\kappa_{11})\,\epsilon_{a)}{}^{c\mu}\dot{C}_{c,\mu}+2\kappa_8\,\epsilon_{a)}{}^{c\mu}F_{c,\mu}+(\kappa_1+6\kappa_3)\,\dot{N}_{a)}+2\kappa_9\,F_{a)}+2\kappa_{10}\,C_{a)}\right\}\\
\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{V})} &= \partial_{(b}\left\{\kappa_7\,\Box F_{a)}+2\kappa_{11}\,\ddot{C}_{a)}+(-\kappa_1+4\kappa_6)\,\Delta C_{a)}+(3\kappa_3+2\kappa_6+\kappa_{11})\,\epsilon_{a)}{}^{c\mu}\dot{F}_{c,\mu}+4\kappa_8\,\epsilon_{a)}{}^{c\mu}C_{c,\mu}+(\kappa_1+6\kappa_3)\,\epsilon_{a)}{}^{c\mu}N_{c,\mu}-\kappa_8\,\dot{F}_{a)}+\kappa_{10}\,F_{a)}-4\kappa_9\,C_{a)}\right\}\\
\left[\frac{\delta H}{\delta N^a}\right]^{(\mathrm{V})} &= \Delta\left\{3\kappa_3\,\dot{F}_a-(\kappa_1+6\kappa_3)\,\epsilon_a{}^{c\mu}C_{c,\mu}-\kappa_1\,N_a\right\}
\end{aligned}
$$

tensor modes $F^{ab}-E^{ab}$, $F^{ab}+E^{ab}$, and C^{ab}:

$$
\begin{aligned}
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{TT})} &= -\frac{1}{4}\kappa_1\,\Box(F_{ab}-E_{ab})+(\frac{1}{4}\kappa_1+\frac{3}{2}\kappa_3)\,\left[(\ddot{F}_{ab}+\ddot{E}_{ab})+\Delta(F_{ab}+E_{ab})\right]-(\kappa_1+6\kappa_3)\,\epsilon_{(a}{}^{c\mu}\dot{C}_{b)c,\mu}\\
\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{TT})} &= (\frac{1}{4}\kappa_1+\frac{3}{2}\kappa_3)\,\left[(\ddot{F}_{ab}-\ddot{E}_{ab})+\Delta(F_{ab}-E_{ab})\right]+(-\frac{1}{4}\kappa_1+\kappa_6)\,(\ddot{F}_{ab}+\ddot{E}_{ab})+(-\frac{3}{4}\kappa_1+3\kappa_6+2\kappa_{11})\,\Delta(F_{ab}+E_{ab})\\
&\quad+\kappa_7\,\Box C_{ab}+(\kappa_1-4\kappa_6-2\kappa_{11})\,\epsilon_{(a}{}^{c\mu}\dot{C}_{b)c,\mu}+2\kappa_8\,\epsilon_{(a}{}^{c\mu}(F_{b)c,\mu}+E_{b)c,\mu})+\kappa_8\,\dot{C}_{ab}+\kappa_9\,(F_{ab}+E_{ab})+\kappa_{10}\,C_{ab}\\
\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{TT})} &= \frac{1}{2}\kappa_7\,\Box(F_{ab}+E_{ab})+\kappa_{11}\,\ddot{C}_{ab}+(-2\kappa_1+8\kappa_6+3\kappa_{11})\,\Delta C_{ab}+(\frac{1}{2}\kappa_1+3\kappa_3)\,\epsilon_{(a}{}^{c\mu}(\dot{F}_{b)c,\mu}-\dot{E}_{b)c,\mu})+(-\frac{1}{2}\kappa_1+2\kappa_6+\kappa_{11})\,\epsilon_{(a}{}^{c\mu}(\dot{F}_{b)c,\mu}+\dot{E}_{b)c,\mu})\\
&\quad+4\kappa_8\,\epsilon_{(a}{}^{c\mu}C_{b)c,\mu}-\frac{1}{2}\kappa_8\,(\dot{F}_{ab}+\dot{E}_{ab})+\frac{1}{2}\kappa_{10}\,(F_{ab}+E_{ab})-2\kappa_9\,C_{ab}
\end{aligned}
$$

Only 9 gravitational constants $\kappa_1,\ldots,\kappa_9$ left to be determined by experiment

EQUATIONS OF MOTION FOR SCALAR, VECTOR, AND TENSOR MODES

scalar modes $\widetilde{F}-\widetilde{E}$, $\widetilde{F}+\widetilde{E}$, E, C, and a:

$$\left[\frac{\delta H}{\delta a}\right] = \Delta\left\{\kappa_1\,(\widetilde{F}-\widetilde{E})+\kappa_2\,(\widetilde{F}+\widetilde{E})+(\frac{2}{3}\kappa_1+2\kappa_3)\,\Delta E\right\}$$

$$\left[\frac{\delta H}{\delta N^a}\right]^{(\mathrm{S})} = \partial_a\frac{\mathrm{d}}{\mathrm{d}\,t}\left\{\kappa_1\,(\widetilde{F}-\widetilde{E})+\kappa_2\,(\widetilde{F}+\widetilde{E})+(\frac{2}{3}\kappa_1+2\kappa_3)\,\Delta E\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STR})} = \frac{1}{3}\gamma_{ab}\left\{\frac{3}{2}\kappa_1\,(\ddot{\widetilde{F}}-\ddot{\widetilde{E}})-\frac{1}{2}\kappa_1\,\Delta(\widetilde{F}-\widetilde{E})+\frac{3}{2}\kappa_2\,(\ddot{\widetilde{F}}+\ddot{\widetilde{E}})+(\frac{3}{2}\kappa_1-\kappa_2)\,\Delta(\widetilde{F}+\widetilde{E})+\kappa_3\,\Delta\Delta E+2\kappa_1\,\Delta a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STR})} = \frac{1}{3}\gamma_{ab}\left\{\frac{3}{2}\kappa_2\,(\ddot{\widetilde{F}}-\ddot{\widetilde{E}})+(\frac{3}{2}\kappa_1-\kappa_2)\,\Delta(\widetilde{F}-\widetilde{E})+\kappa_4\,(\ddot{\widetilde{F}}+\ddot{\widetilde{E}})+(3\kappa_2-\kappa_4)\,\Delta(\widetilde{F}+\widetilde{E})+\kappa_5\,(\widetilde{F}+\widetilde{E})+(\kappa_1-\frac{1}{3}\kappa_2+3\kappa_3)\,\Delta\Delta E+2\kappa_2\,\Delta a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STF})} = \Delta_{ab}\left\{-\frac{1}{2}\kappa_3\,\Delta E+\frac{1}{4}\kappa_1\,(\widetilde{F}-\widetilde{E})+(-\frac{3}{4}\kappa_1+\frac{1}{2}\kappa_2)\,(\widetilde{F}+\widetilde{E})-\kappa_1\,a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{STF})} = \Delta_{ab}\left\{\kappa_6\,\ddot{E}+(\frac{1}{3}\kappa_1+\frac{1}{2}\kappa_3-\kappa_6)\,\Delta E+\kappa_7\,\Box C+\kappa_8\,\dot{C}+\kappa_9\,E+\kappa_{10}\,C+(\frac{1}{4}\kappa_1+\frac{3}{2}\kappa_3)\,(\widetilde{F}-\widetilde{E})+(\frac{3}{4}\kappa_1+\frac{9}{2}\kappa_3)\,(\widetilde{F}+\widetilde{E})+(\kappa_1+6\kappa_3)\,a\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{STF})} = \Delta_{ab}\left\{\frac{1}{2}\kappa_7\,\Box E+\kappa_{11}\,\Box C-\frac{1}{2}\kappa_8\,\dot{E}+\frac{1}{2}\kappa_{10}\,E-2\kappa_9\,C\right\}$$

vector modes F_a, C_a, and N_a:

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}-\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{V})} = \partial_{(b}\left\{3\kappa_3\,\ddot{F}_{a)}-(\kappa_1+6\kappa_3)\,\epsilon_{a)}{}^{c\mu}\dot{C}_{c,\mu}-\kappa_1\,\dot{N}_{a)}\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{V})} = \partial_{(b}\left\{(3\kappa_3+2\kappa_6)\,\ddot{F}_{a)}+\kappa_{11}\,\Delta F_{a)}+2\kappa_7\,\Box C_{a)}+(\kappa_1-4\kappa_6-2\kappa_{11})\,\epsilon_{a)}{}^{c\mu}\dot{C}_{c,\mu}+2\kappa_8\,\epsilon_{a)}{}^{c\mu}F_{c,\mu}+(\kappa_1+6\kappa_3)\,\dot{N}_{a)}+2\kappa_9\,F_{a)}+2\kappa_{10}\,C_{a)}\right\}$$

$$\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{V})} = \partial_{(b}\left\{\kappa_7\,\Box F_{a)}+2\kappa_{11}\,\ddot{C}_{a)}+(-\kappa_1+4\kappa_6)\,\Delta C_{a)}+(3\kappa_3+2\kappa_6+\kappa_{11})\,\epsilon_{a)}{}^{c\mu}\dot{F}_{c,\mu}+4\kappa_8\,\epsilon_{a)}{}^{c\mu}C_{c,\mu}+(\kappa_1+6\kappa_3)\,\epsilon_{a)}{}^{c\mu}N_{c,\mu}-\kappa_8\,\dot{F}_{a)}+\kappa_{10}\,F_{a)}-4\kappa_9\,C_{a)}\right\}$$

$$\left[\frac{\delta H}{\delta N^a}\right]^{(\mathrm{V})} = \Delta\left\{3\kappa_3\,\dot{F}_a-(\kappa_1+6\kappa_3)\,\epsilon_a{}^{c\mu}C_{c,\mu}-\kappa_1\,N_a\right\}$$

tensor modes $F^{ab}-E^{ab}$, $F^{ab}+E^{ab}$, and C^{ab}:

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$$\begin{aligned}\left[\frac{\delta H}{\delta\bar{\varphi}^{ab}}+\frac{\delta H}{\delta\bar{\bar{\varphi}}^{ab}}\right]^{(\mathrm{TT})} &= (\frac{1}{4}\kappa_1+\frac{3}{2}\kappa_3)\,\left[(\ddot{F}_{ab}-\ddot{E}_{ab})+\Delta(F_{ab}-E_{ab})\right]+(-\frac{1}{4}\kappa_1+\kappa_6)\,(\ddot{F}_{ab}+\ddot{E}_{ab})+(-\frac{3}{4}\kappa_1+3\kappa_6+2\kappa_{11})\,\Delta(F_{ab}+E_{ab})\\&\quad+\kappa_7\,\Box C_{ab}+(\kappa_1-4\kappa_6-2\kappa_{11})\,\epsilon_{(a}{}^{c\mu}\dot{C}_{b)c,\mu}+2\kappa_8\,\epsilon_{(a}{}^{c\mu}(F_{b)c,\mu}+E_{b)c,\mu})+\kappa_8\,\dot{C}_{ab}+\kappa_9\,(F_{ab}+E_{ab})+\kappa_{10}\,C_{ab}\end{aligned}$$

$$\begin{aligned}\left[\frac{\delta H}{\delta\bar{\bar{\bar{\varphi}}}^{ab}}\right]^{(\mathrm{TT})} &= \frac{1}{2}\kappa_7\,\Box(F_{ab}+E_{ab})+\kappa_{11}\,\ddot{C}_{ab}+(-2\kappa_1+8\kappa_6+3\kappa_{11})\,\Delta C_{ab}+(\frac{1}{2}\kappa_1+3\kappa_3)\,\epsilon_{(a}{}^{c\mu}(\dot{F}_{b)c,\mu}-\dot{E}_{b)c,\mu})+(-\frac{1}{2}\kappa_1+2\kappa_6+\kappa_{11})\,\epsilon_{(a}{}^{c\mu}(\dot{F}_{b)c,\mu}+\dot{E}_{b)c,\mu})\\&\quad+4\kappa_8\,\epsilon_{(a}{}^{c\mu}C_{b)c,\mu}-\frac{1}{2}\kappa_8\,(\dot{F}_{ab}+\dot{E}_{ab})+\frac{1}{2}\kappa_{10}\,(F_{ab}+E_{ab})-2\kappa_9\,C_{ab}\end{aligned}$$

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Exact solution hard (gapless solution for standard model matter without hindsight: F. Wolz)

Perturbative/symmetric solutions require case-by-case analysis (caveat: involutivity)

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General Theory and Perturbative Solutions

F. Wolz

PhD thesis Gottfried Wilhelm Leibniz University Hannover (2022)

Measurement of constants

κ_1

κ_2

κ_3

κ_4

κ_5

κ_6

κ_7

κ_8

κ_9

Measurement of constants

Measurement of constants

C
L
A
S
S
I
C
A
L

Ray optical limit

$$P^{abcd} = g^{(ab}g^{cd)} \ + \ \text{second order}$$

Q
U
A
N
T
U
M

κ_1

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Modified energy transport

from next order WKB, feels ray-optically invisible non-metricities

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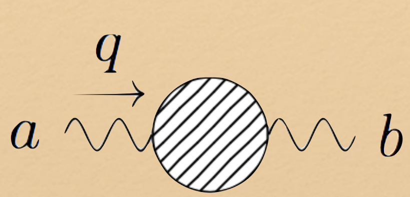
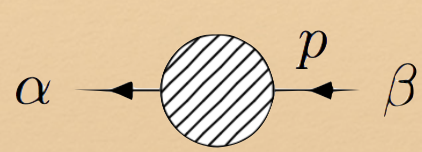
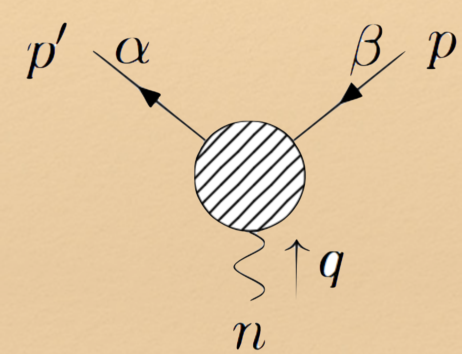
QUANTUM

QED renormalizable and gauge invariant to any loop order







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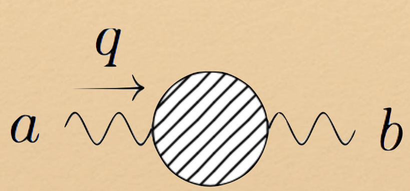

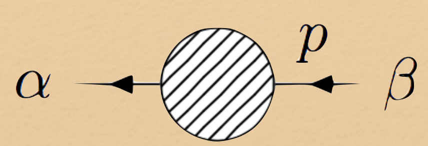

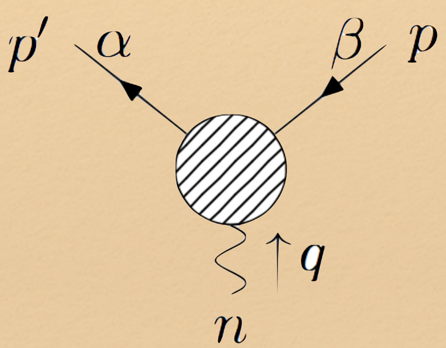

Anomalous magnetic moment

$$\alpha \left(1 - \frac{E^{ij}{}_{ij}}{12}\right) = 1/137.035\,999\,2(2)$$

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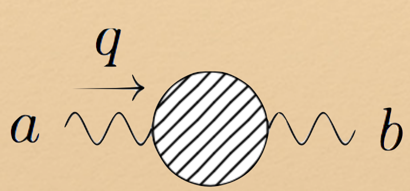

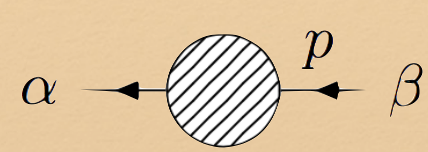

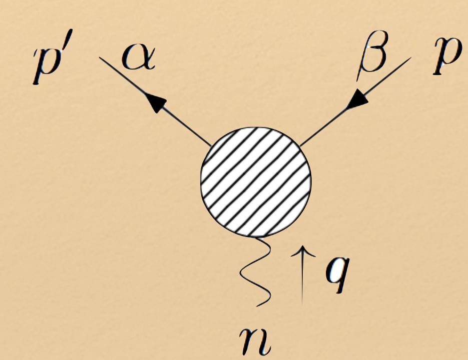

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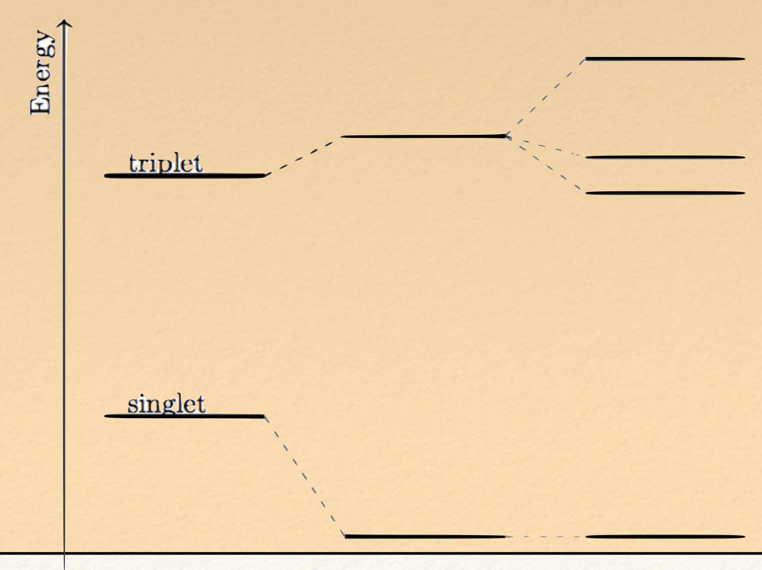
Hyperfine structure of hydrogen



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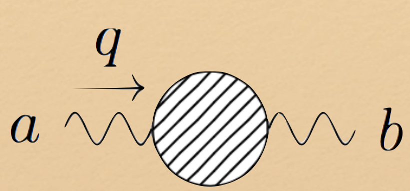

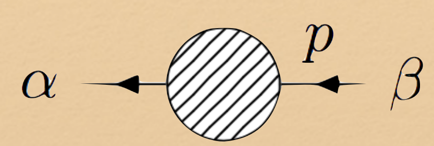

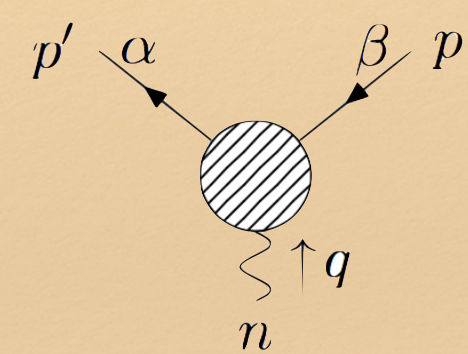

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Second highly precisely measured quantity

$$Q(\alpha, E^{ij}{}_{ij}) = \;\ldots$$

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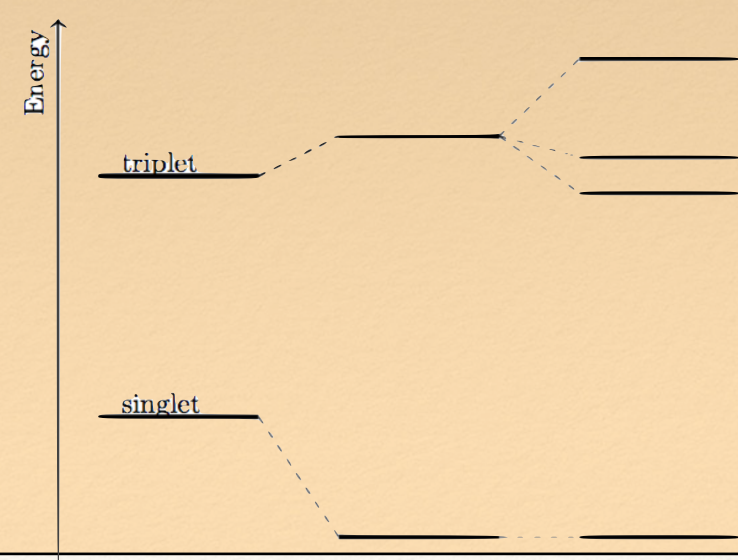

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