# “How does Quantum Field Theory arise from Causal Fermion Systems?”

Moritz Reintjes
Assistant Professor
City University of Hong Kong

Causal Fermion Systems 2025

The University of Regensburg
October 9, 2025

Collaborators: Claudio Dappiaggi, Felix Finster, Niky Kamran

# We have two ways to connect CFS's with QED:

# We have two ways to connect CFS's with QED:

(I) Construction of an algebraic quantum state, [Finster-Kamran]

$$\omega^t : \mathscr{A} \longrightarrow \mathbb{C},$$

mapping observables to expectation value, at fixed time $t$.

—Works for general CFS.

$\longrightarrow$ Entangled quantum states!

## We have two ways to connect CFS's with QED:

(I) Construction of an algebraic quantum state, [Finster-Kamran]

$$\omega^t : \mathscr{A} \longrightarrow \mathbb{C},$$

mapping observables to expectation value, at fixed time $t$.

— Works for general CFS.

$\longrightarrow$ Entangled quantum states!

(II) Canonical Commutation Relations of QED

are deduced from Euler-Lagrange equations of CFS's.

— Uses stochastic averaging & non-locality.

$\longrightarrow$ Yields dynamics of quantum states & Feynman diagrams.

# (I) Construction of an algebraic quantum state:

- F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\boxed{\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})}$$

• F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

$A \in \mathscr{A}$ (observable)

• F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map

$$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\boxed{\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})}$$

Describes excitation of $\tilde{\rho}$ over $\rho$, as seen (tested) by $A$.

— $(\rho, \mathscr{F}, \mathscr{H})$ is (non-interacting) vacuum CFS

— $(\tilde{\rho}, \tilde{\mathscr{F}}, \tilde{\mathscr{H}})$ some CFS describing interaction

• F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\dots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

$\mathscr{U} \in \mathscr{G}$, unitary transfo. on $\mathscr{H}$;

Degrees of freedom in identifying

Hilbert spaces, $\tilde{\mathscr{H}}$ & $\mathscr{H}$, of $\tilde{\rho}$ & $\rho$.


• F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

Compares $\tilde{\rho}$ with $\rho$, by surface layer integral

$$\gamma^t(\tilde{\rho}, \rho) = \left( \int_{\tilde{\Omega}^t} d\tilde{\rho}(x) \int_{M \setminus \Omega^t} d\rho(y) - \int_{\tilde{M} \setminus \tilde{\Omega}^t} d\tilde{\rho}(x) \int_{\Omega^t} d\rho(y) \right) \mathscr{L}(x, y)$$

$\Omega^t$, $\tilde{\Omega}^t$ ~ past of $t$

$\tilde{M} = F(M)$

Lagrangian of CFS

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

$$Z^t := \int_{\mathscr{G}} e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$ partition function

~ all possible configurations in $\tilde{\rho}$ vs. $\rho$

• F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

"Insertions" $\longleftarrow$ input from CFS & observable $A$

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

"Insertions" $\longleftarrow$ input from CFS & observable $A$

- "bosonic": $(\ldots) \sim D_z \gamma^t(\rho, \mathscr{U}\tilde{\rho})$

differentiation w.r.t. bosonic fields $z$ in $A$

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map
$$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

"Insertions" $\longleftarrow$ input from CFS & observable $A$

- "bosonic": $(\ldots) \sim D_z \gamma^t(\rho, \mathscr{U}\tilde{\rho})$

  differentiation w.r.t. bosonic fields $z$ in $A$

- "fermionic": $(\ldots) \sim$ s.th. similar

  in terms of fermionic fields in $A$

# (I) Construction of an algebraic quantum state:

The quantum state, at time $t$, constructed from CFS's,

is the linear map $$\omega^t : \mathscr{A} \longrightarrow \mathbb{C}$$

on algebra of observables $\mathscr{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathscr{G}} (\ldots) e^{\gamma^t(\tilde{\rho}, \mathscr{U}\rho)} d\mu_{\mathscr{G}}(\mathscr{U})$$

Thm: (i) $\omega^t(A^*A) \geq 0$ for all $A \in \mathscr{A}$ (positivity).

(ii) There exists a density operator $\sigma^t$ on a Fock space $\mathscr{F}$ such that $\omega^t(A) = \mathrm{tr}_{\mathscr{F}}(\sigma^t A)$ (expectation value).

(iii) Refinement of $\omega^t$ describes entanglement.


- F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.
- F. Finster, N. Kamran & M. Reintjes, Adv. Theor. Math. Phys. 27.5, 1463-1589, (2023).

# (II) Derivation of Canonical Commutation Relations (CCR):

- C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$


• C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

- The non-local potential $\mathcal{B}$ is obtained from linearized Euler-Lagrange equations of CFS,

- C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

- The non-local potential $\mathcal{B}$ is obtained from linearized Euler-Lagrange equations of CFS, and is of form

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

with kernel
$$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

- C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

- The non-local potential $\mathcal{B}$ is obtained from linearized Euler-Lagrange equations of CFS, and is of form

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

with kernel
$$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

non-local, linear operator

• C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

- The non-local potential $\mathcal{B}$ is obtained from linearized Euler-Lagrange equations of CFS, and is of form

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

non-local,
linear operator

with kernel $$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

Scalar fields (like el.-mag.)
inducing dephasing effects

• C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

- The non-local potential $\mathcal{B}$ is obtained from linearized Euler-Lagrange equations of CFS, and is of form

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

non-local,
linear operator

with kernel $$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

Solves linearized EL-eqn's of CFS

Scalar fields (like el.-mag.) inducing dephasing effects

• C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# (II) Derivation of Canonical Commutation Relations (CCR):

- Starting point is Dirac equation

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

- The non-local potential $\mathcal{B}$ is obtained from linearized Euler-Lagrange equations of CFS, and is of form

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

with kernel $$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

A multitude of fields:

$$N \sim \frac{\text{length scale of non-locality}}{\text{Planck scale}}$$

# (II) Derivation of Canonical Commutation Relations (CCR):

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

$$\mathcal{B}$$

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

$$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\left(\frac{x+y}{2}\right) L_a(y-x)$$

- Averaging, $\langle\langle\,\cdot\,\rangle\rangle$, over suitable stochastic background, for suitable choices of potentials, yields CCR, up to errors $\mathscr{O}\left(\frac{1}{N}\right)$.

# (II) Derivation of Canonical Commutation Relations (CCR):

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

$$\mathcal{B}$$

$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x,y)\,\psi(y)\,d^4y$$

$$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j\, A_a^j\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

- Averaging, $\langle\langle \cdot \rangle\rangle$, over suitable stochastic background, for suitable choices of potentials, yields CCR, up to errors $\mathcal{O}\Big(\frac{1}{N}\Big)$:

$$\boxed{\ll\big[\hat{\mathcal{B}}^j_q, \hat{\mathcal{B}}^k_{q'}\big]\otimes\hat{\mathcal{B}}^l_r\otimes\hat{\mathcal{B}}^{l'}_{r'}\gg \;\approx\; (2\pi)^4\delta^4(q+q')\hat{K}^{jk}(q)\;\mathbb{1}\otimes\ll\hat{\mathcal{B}}^l_r\otimes\hat{\mathcal{B}}^{l'}_{r'}\gg}$$

- Averaging, $\langle\langle \cdot \rangle\rangle$, over suitable stochastic background, for suitable choices of potentials, yields CCR, up to errors $\mathscr{O}\left(\frac{1}{N}\right)$.

One gets standard perturbative QED, up to error terms $\mathscr{O}\left(\frac{1}{N}\right)$.

- C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

- Averaging, $\langle\langle \cdot \rangle\rangle$, over suitable stochastic background, for suitable choices of potentials, yields CCR, up to errors $\mathscr{O}\left(\frac{1}{N}\right)$.

$\Downarrow$

One gets standard perturbative QED,
up to error terms $\mathscr{O}\left(\frac{1}{N}\right)$.

$\longrightarrow$ Quantifying error terms will yield corrections of CFS to standard QED…


- C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).

# Thanks for listening!


- F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.
- F. Finster, N. Kamran & M. Reintjes, Adv. Theor. Math. Phys. 27.5, 1463-1589, (2023).
- C. Dappiaggi, F. Finster, N. Kamran & M. Reintjes, Ann. Henri Poincaré 23 (2025).