

“How does Quantum Field Theory arise from Causal Fermion Systems?”

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Causal Fermion Systems 2025

The University of Regensburg
October 9, 2025

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(I) Construction of an algebraic quantum state, [Finster-Kamran]

$$\omega^t : \mathcal{A} \longrightarrow \mathbb{C},$$

mapping observables to expectation value, at fixed time t .

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(II) Canonical Commutation Relations of QED

are deduced from Euler-Lagrange equations of CFS's.

— Uses stochastic averaging & non-locality.

—→ Yields dynamics of quantum states & Feynman diagrams.

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The quantum state, at time t , constructed from CFS's,

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on algebra of observables $\mathcal{A}$ (build from field operators), defined by

path integral-like structure

$$\omega^t(A) := \frac{1}{Z^t} \int_{\mathcal{G}} (\dots) e^{\gamma^t(\tilde{\rho}, \mathcal{U} \rho)} d\mu_{\mathcal{G}}(\mathcal{U})$$

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$A \in \mathcal{A}$ (observable)

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Describes excitation of $\tilde{\rho}$ over ρ , as seen (tested) by A .

- $(\rho, \mathcal{F}, \mathcal{H})$ is (non-interacting) vacuum CFS
- $(\tilde{\rho}, \tilde{\mathcal{F}}, \tilde{\mathcal{H}})$ some CFS describing interaction

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$\mathcal{U} \in \mathcal{G}$, unitary transfo. on $\mathcal{H}$;

Degrees of freedom in identifying
Hilbert spaces, $\tilde{\mathcal{H}}$ & $\mathcal{H}$, of $\tilde{\rho}$ & ρ .

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Compares $\tilde{\rho}$ with ρ , by surface layer integral

$$\gamma^t(\tilde{\rho}, \rho) = \left(\int_{\tilde{\Omega}^t} d\tilde{\rho}(x) \int_{M \setminus \Omega^t} d\rho(y) - \int_{\tilde{M} \setminus \tilde{\Omega}^t} d\tilde{\rho}(x) \int_{\Omega^t} d\rho(y) \right) \mathcal{L}(x, y)$$

$\Omega^t, \tilde{\Omega}^t \sim \text{past of } t$

$\tilde{M} = F(M)$

Lagrangian of CFS

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$$Z^t := \int_{\mathcal{G}} e^{\gamma^t(\tilde{\rho}, \mathcal{U} \rho)} d\mu_{\mathcal{G}}(\mathcal{U}) \quad \text{partition function}$$

~ all possible configurations in $\tilde{\rho}$ vs. ρ

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• “bosonic”: $(\dots) \sim D_z \gamma^t(\rho, \mathcal{U}\tilde{\rho})$

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• “fermionic”: $(\dots) \sim$ s.th. similar

in terms of fermionic fields in A

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Thm: (i) $\omega^t(A^*A) \geq 0$ for all $A \in \mathcal{A}$ (positivity).

(ii) There exists a density operator σ^t on a Fock space $\mathcal{F}$ such that $\omega^t(A) = \text{tr}_{\mathcal{F}}(\sigma^t A)$ (expectation value).

(iii) Refinement of ω^t describes entanglement.

- F. Finster & N. Kamran, Ann. Henri Poincaré 23 (2022), 1359–1398.
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$$(\mathcal{B}\psi)(x) = \int_M \mathcal{B}(x, y) \psi(y) d^4y$$

with kernel

$$\mathcal{B}(x, y) = \sum_{a=1}^N \gamma_j A_a^j \left(\frac{x+y}{2} \right) L_a(y-x)$$

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Solves linearized
EL-eqn's of CFS

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A multitude of fields:

$$N \sim \frac{\text{length scale of non-locality}}{\text{Planck scale}}$$

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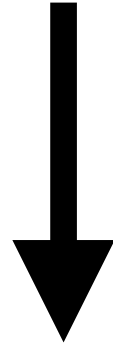
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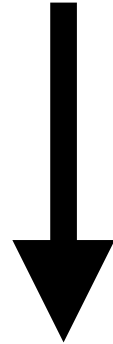
$$\langle\langle [\hat{\mathcal{B}}_q^j, \hat{\mathcal{B}}_{q'}^k] \otimes \hat{\mathcal{B}}_r^l \otimes \hat{\mathcal{B}}_{r'}^{l'} \rangle\rangle \approx (2\pi)^4 \delta^4(q+q') \hat{K}^{jk}(q) \mathbf{1} \otimes \langle\langle \hat{\mathcal{B}}_r^l \otimes \hat{\mathcal{B}}_{r'}^{l'} \rangle\rangle$$

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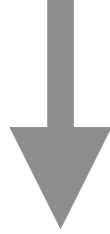
One gets standard perturbative QED,
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→ Quantifying error terms will yield
corrections of CFS to standard QED...



Thanks for listening!

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