Semiclassical thermodynamics of spacetime

Causal Fermion Systems 2025,
Regensburg, 6-10 Oct 2025

Alessandro Pesci, INFN Bologna

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black hole thermodynamics

$$T = \frac{\kappa}{2\pi} = \frac{\hbar}{c\, k_B} \frac{\kappa}{2\pi}$$

black hole temperature

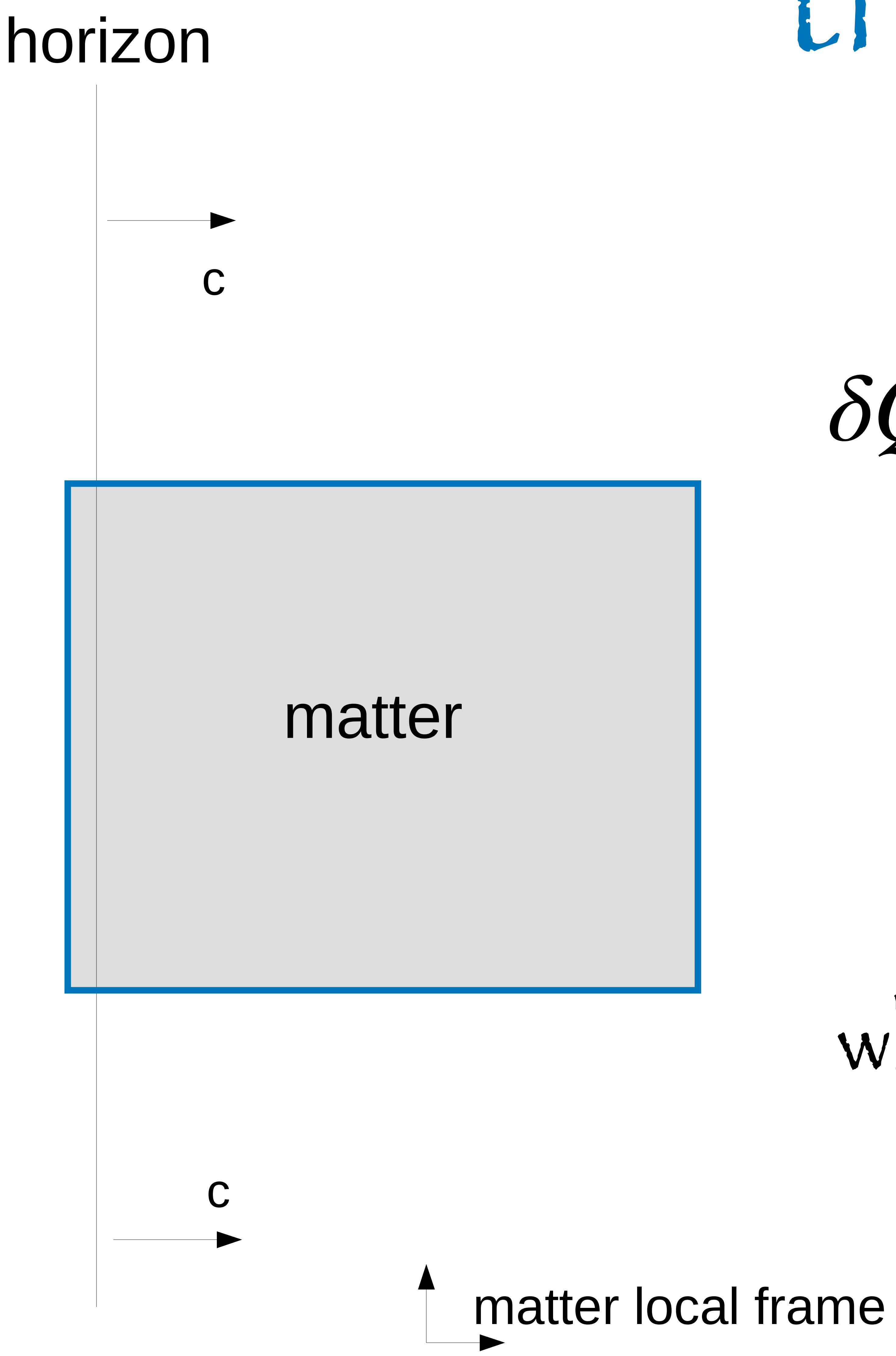
$\kappa =$ surface gravity

Hawking, Nature 248 (1974) 30;
Commun. Math. Phys. 43 (1975) 199

$$S = \frac{A}{4} = \frac{k_B\, c^3}{G\, \hbar} \frac{A}{4} = \frac{1}{4} k_B \frac{A}{l_p^2}$$

black hole entropy

Bekenstein, Nuovo Cim. Lett. 4 (1972) 737;
Phys. Rev. D 7 (1973) 2333;
Phys. Rev. D 9 (1974) 3292
Hawking, 1974



thermodynamics at the centre

Jacobson gr-qc/9504004

$$\delta Q = \int_H T_{ab}\xi^a d\Sigma^b = T\delta S = T\eta\delta A = \int_H \theta \, d\lambda \, dA$$

assuming $S = \eta A$

$$\Updownarrow$$

$$\kappa T_{ab} l^a l^b = \frac{\kappa}{2\pi} \eta R_{ab} l^a l^b$$

Raychaudhuri

which amounts to

$$\frac{2\pi}{\eta} T_{ab} l^a l^b = R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab}$$

with Λ some const.

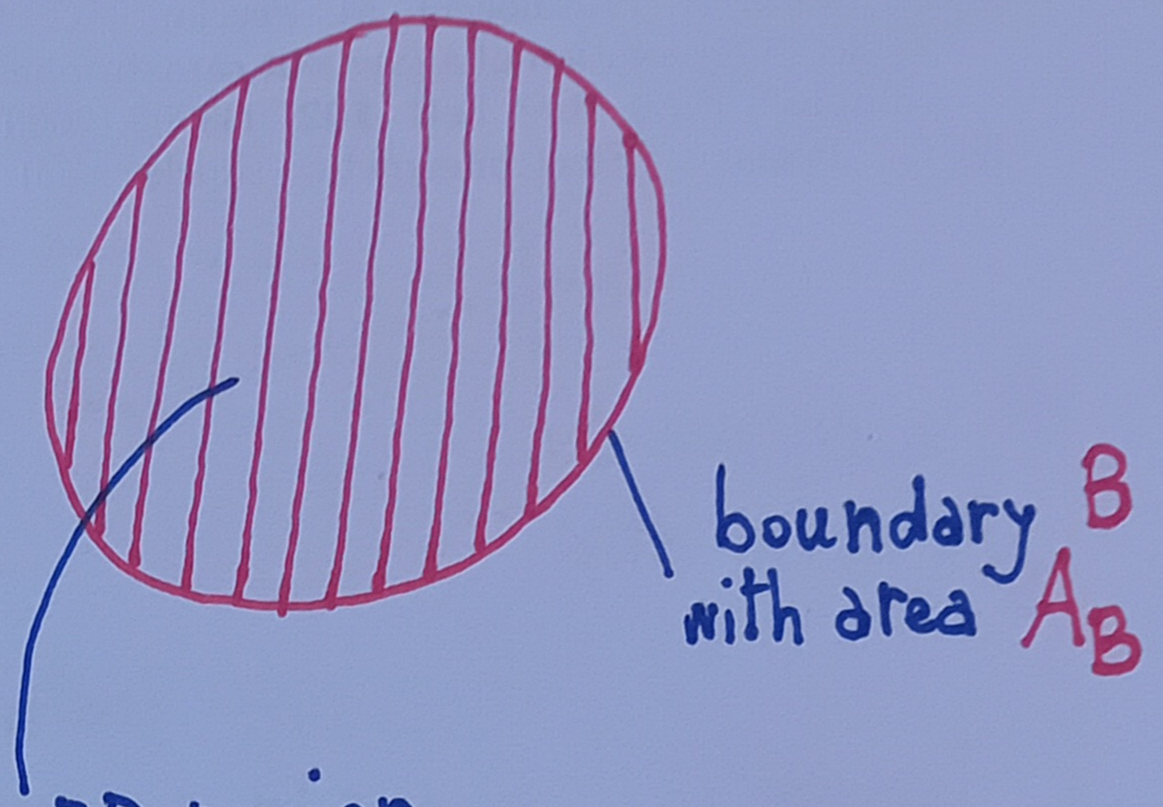
$\Rightarrow$ Einstein eqs. & $\eta = \frac{1}{4}$

(semiclassical) holography

Holographic principle ('t Hooft, 1993)

$$N \leq e^{A_B/4}$$

number of orthogonal quantum states
or
number of microscopic configurations



$$N = e^{S_{Boltzmann}} = 2^{S_{Shannon}}$$

AdS/CFT correspondence

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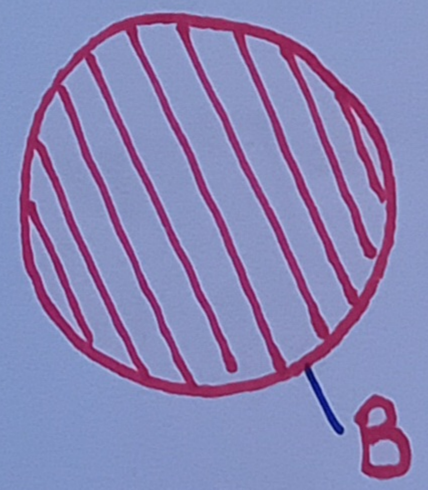
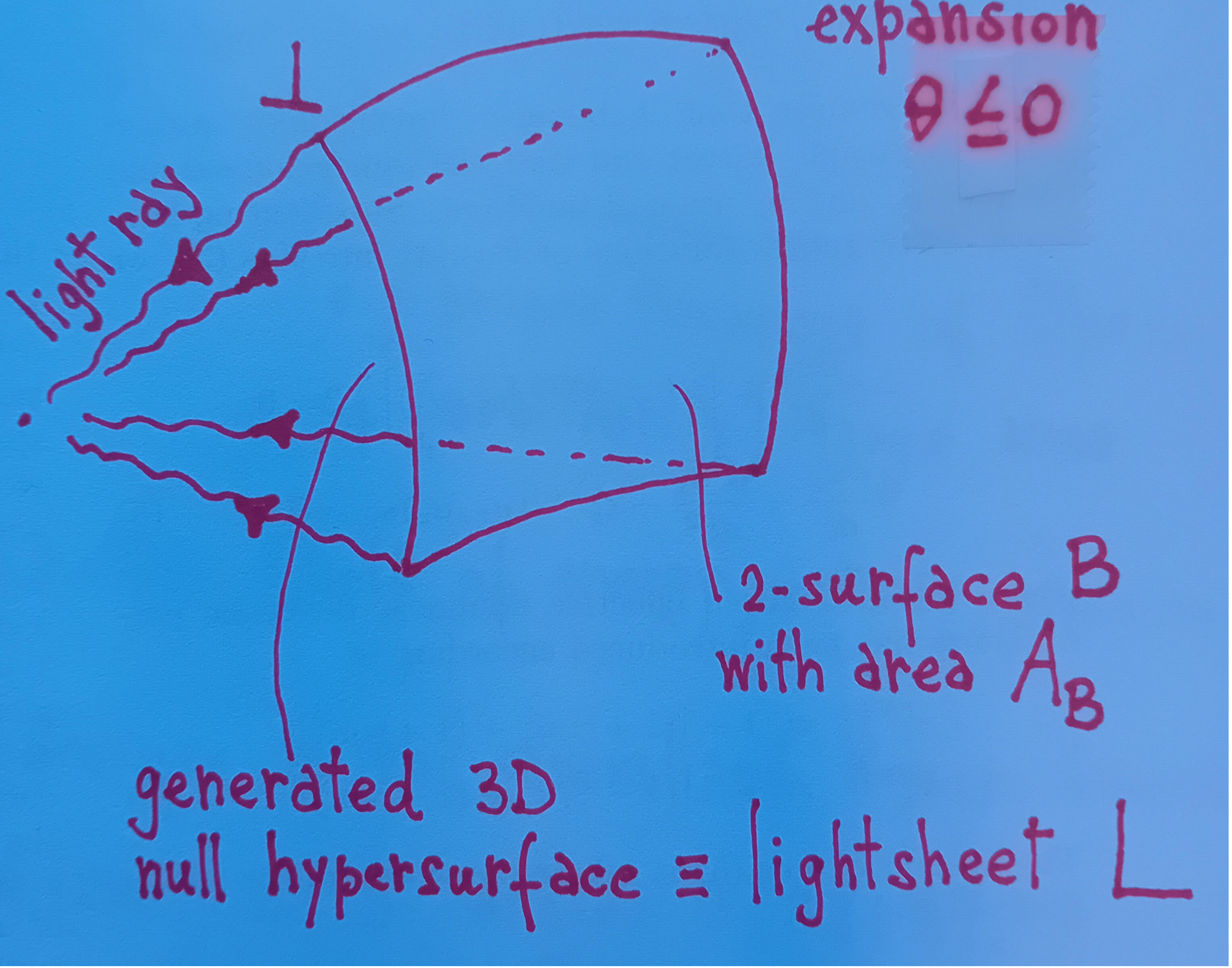
rooted on the
<u>holographic entropy bound</u>:

$$S_{Bo.} \leq \frac{A_B}{4}$$



metric with spherical symmetry;
~ static conditions

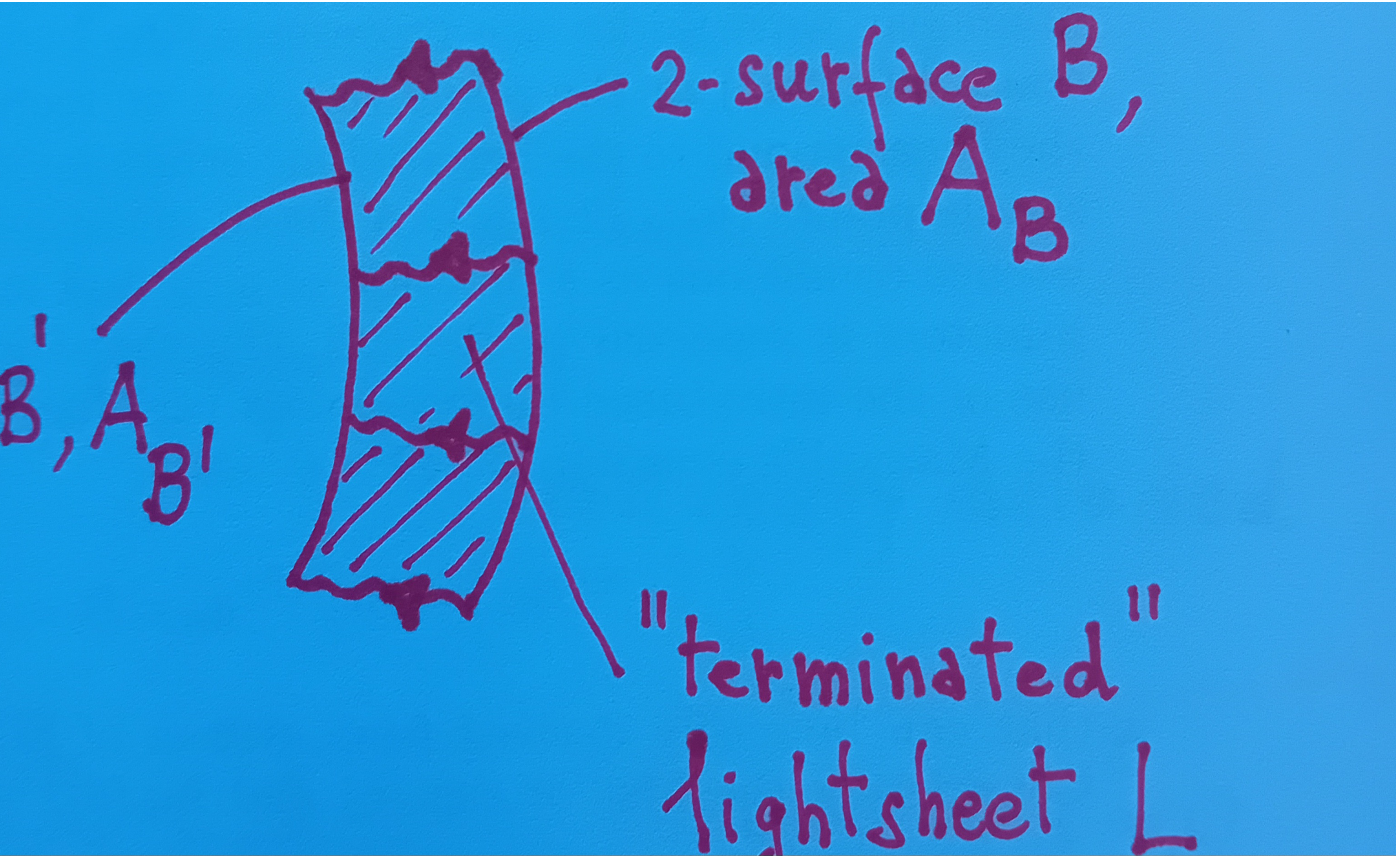


covariant entropy bound

Bousso hep-th/9905177, hep-th/0203101

$$S_L \leq \frac{A_B}{4}$$

generalized bound



Flanagan, Marolf, Wald
hep-th/9908070

Bousso, Flanagan, Marolf
hep-th/0305149

Strominger, Thompson
hep-th/0303067

$$S_L \leq \frac{A_B - A_{B'}}{4}$$

a condition for the bound

$$\ell^* \equiv \frac{1}{\pi}\frac{s}{\rho+p} = \frac{1}{\pi T}\left(1-\frac{\mu n}{\rho+p}\right)$$

$\boxed{\text{when } \mu=0 \quad \ell^* = \frac{1}{\pi T}}$

Gibbs - Duhem : $\rho = Ts - p + \mu n$

rest energy included

$$\ell^* = \frac{1}{\pi}\frac{c\hbar}{k}\frac{s}{\rho+p} = \frac{1}{\pi}\ell_p\frac{T_p}{T}\frac{Ts}{\rho+p}$$

$$1 = \ell_p \equiv \sqrt{\frac{\hbar G}{c^3}} \qquad m_p \equiv \sqrt{\frac{\hbar c}{G}}$$

$$1 = T_p \equiv \frac{m_p c^2}{k} = \sqrt{\frac{\hbar c^5}{G k^2}}$$

- $s=0 \Rightarrow \ell^*=0$
- $\ell^* \gg \ell_p$ in general
- $\ell^* T \approx \hbar$
- $\hbar$ is present, G not!

ℓ^*

if E. eqs. hold

$$S_L \le \frac{1}{4}\left(A_B - A_{B'}\right) \quad \Longleftrightarrow \quad \ell \geq \frac{1}{\pi}\frac{s}{\rho+p} \equiv \ell^*$$

universally true

e.g. proper thickness of layers of (arbitrarily) large area

AP 0708.3729; 0803.2642

consequences for basic thermodynamics

simply from the holographic principle

– with no explicit reference to quantum mech. –

$$\frac{S}{E+pV} \leq \pi\lambda$$

'new law'

in basic thermod.

AP 0903.0319

contrast with

$$\frac{S}{E} \leq 2\pi R$$

Bekenstein's bound

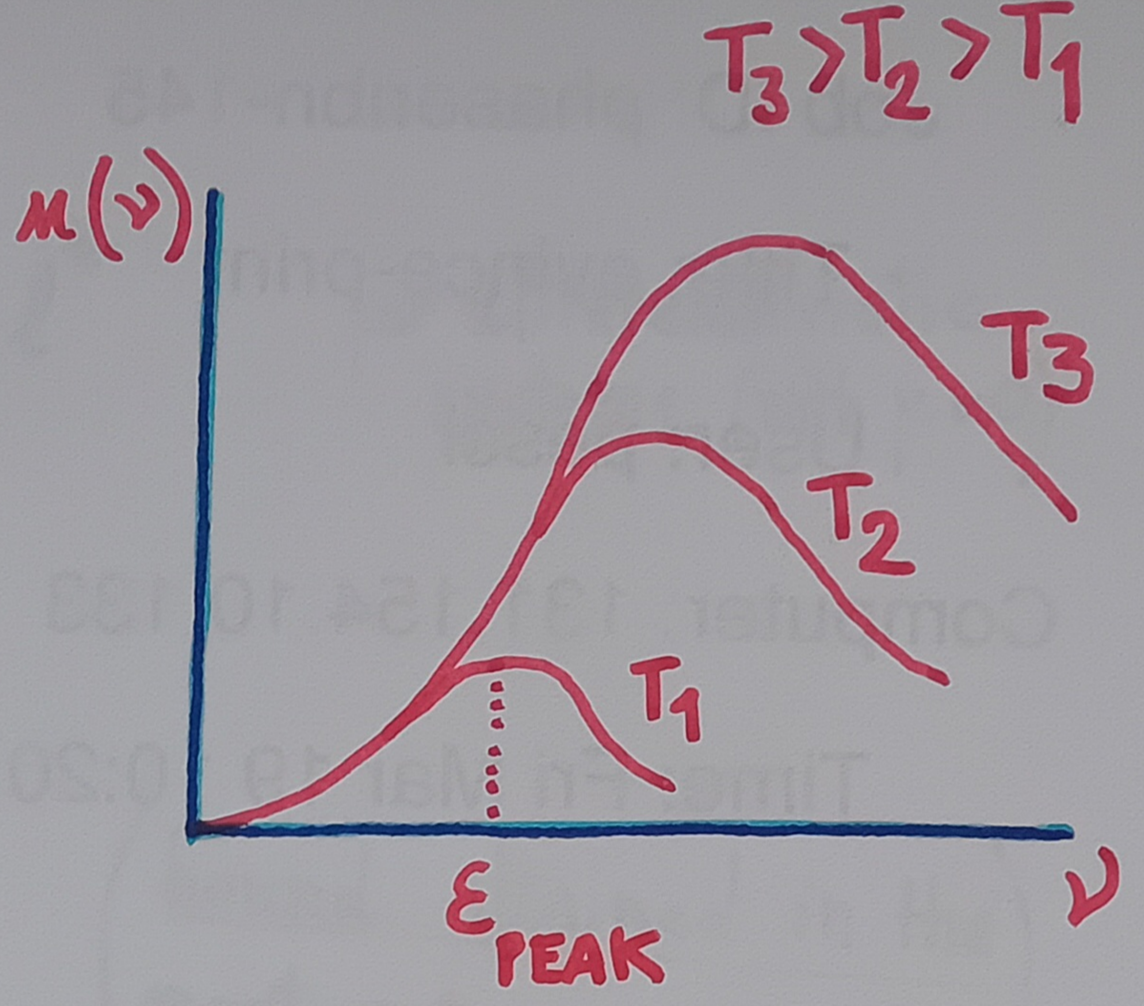
Bekenstein, Phys. Rev. D 23 (1981) 287

attaining the bound

black body radiation

$$\ell^* = \frac{1}{\pi}\,\frac{s}{\rho + p} = \frac{1}{\pi T}$$

$\lambda = ?$



$$\varepsilon_{PEAK} = 2.82\ T$$

assuming $\Delta\varepsilon = \varepsilon_{PEAK}/2$ approximately captures the intrinsic statistical uncertainty in photon energy connected with Planck's law:

$$\lambda\,\Delta\varepsilon = \lambda\,\frac{\varepsilon_{PEAK}}{2} = \frac{1}{2}$$

$$\Rightarrow \lambda = \frac{1}{\varepsilon_{PEAK}} = \frac{1}{2.82\,T} \quad \left(\simeq \frac{1}{\pi T}\right)$$

A slice of a photon gas of thickness $\ell = \lambda = \ell^* = \frac{1}{\pi T}$ just attains the generalized covariant entropy bound !

[A.P., 0803.2642]

relaxation times (1)

Hod gr-qc/0611004

from quantum mechanics, generically:

$$\tau \gg \frac{\hbar}{kT} = \frac{1}{T} \quad (\text{P.u.})$$

$\tau \equiv$ time characterizing the rate of change of a macroscopic quantity when it has some non-equilibrium value

[Landau, Lifshitz, "Statistical physics"]

study of actual systems at 'quantum criticality':

$$\tau = C\,\frac{1}{T} \quad \text{with } C = \text{a few units}$$

[Sachdev, "Quantum phase transitions", Cambridge U.P. 1999]

black holes:

Kerr $(J\neq 0, Q=0)$
Kerr-Newman $(J\neq 0, Q\neq 0)$
Reissner-Nordstrøm $(J=0, Q\neq 0)$

in the limit $T_{bh}\to 0$, $\boxed{\tau = \frac{1}{\pi T}}$

Schwarzschild $(J=0, Q=0)$, $\tau > \frac{1}{\pi T}$ (not in the limit $T_{bh}\to 0$!)

[Hod, 2006]

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from quantum information theory:

$$\frac{I_{max}}{\tau} = \frac{\pi\,\Delta E}{\ln 2} \quad [\text{Bekenstein, 1981}] \Rightarrow \frac{\Delta S}{\tau} \leq \pi\,\Delta E \quad (S = I \ln 2)$$

and from (ordinary) 2nd law:

$$\Delta S \geq \frac{\Delta Q}{T} = \frac{\Delta E}{T} \Rightarrow \frac{\Delta E}{T\,\tau} \leq \pi\,\Delta E \Rightarrow \boxed{\tau \geq \frac{1}{\pi T}}$$

$$\left(\tau \geq \frac{\hbar}{\pi k T}\right)$$

universal bound to relaxation times

[Hod, 2006]

Bremermann, 1967;
Bekenstein, Phys. Rev. Lett 46 (1981) 623

relaxation times (2)

connection with the gen. entropy bound

$$\ell \geq \ell^* = \frac{1}{\pi}\frac{s}{\rho+p} = \frac{1}{\pi T}\left(1 - \frac{\mu n}{\rho + p}\right)$$

$> \ell <$

we assume :

$$\tau = \frac{\ell}{v}$$

where $v =$ speed of propagation of perturbations $=$ velocity of sound / velocity of the particles, if they can move in the layer

we lower $\underline{\ell}$ as we can : $\ell \to \lambda$

$$\begin{cases} \lambda \geq \ell^* \text{ (in general } \lambda \gg \ell^*) \\ v \leq 1 \end{cases}$$

$\Rightarrow$ even when $\ell^* < \frac{1}{\pi T}$ $(\mu > 0)$, this could protect the univ. bound

<u>anyway, for a photon gas</u> :

$\lambda = \ell^* = \frac{1}{\pi T}$ and $v = 1$

$\Rightarrow$ for layers of thickness $\ell = \lambda = \frac{1}{\pi T}$

$$\boxed{\tau = \frac{1}{\pi T}}$$

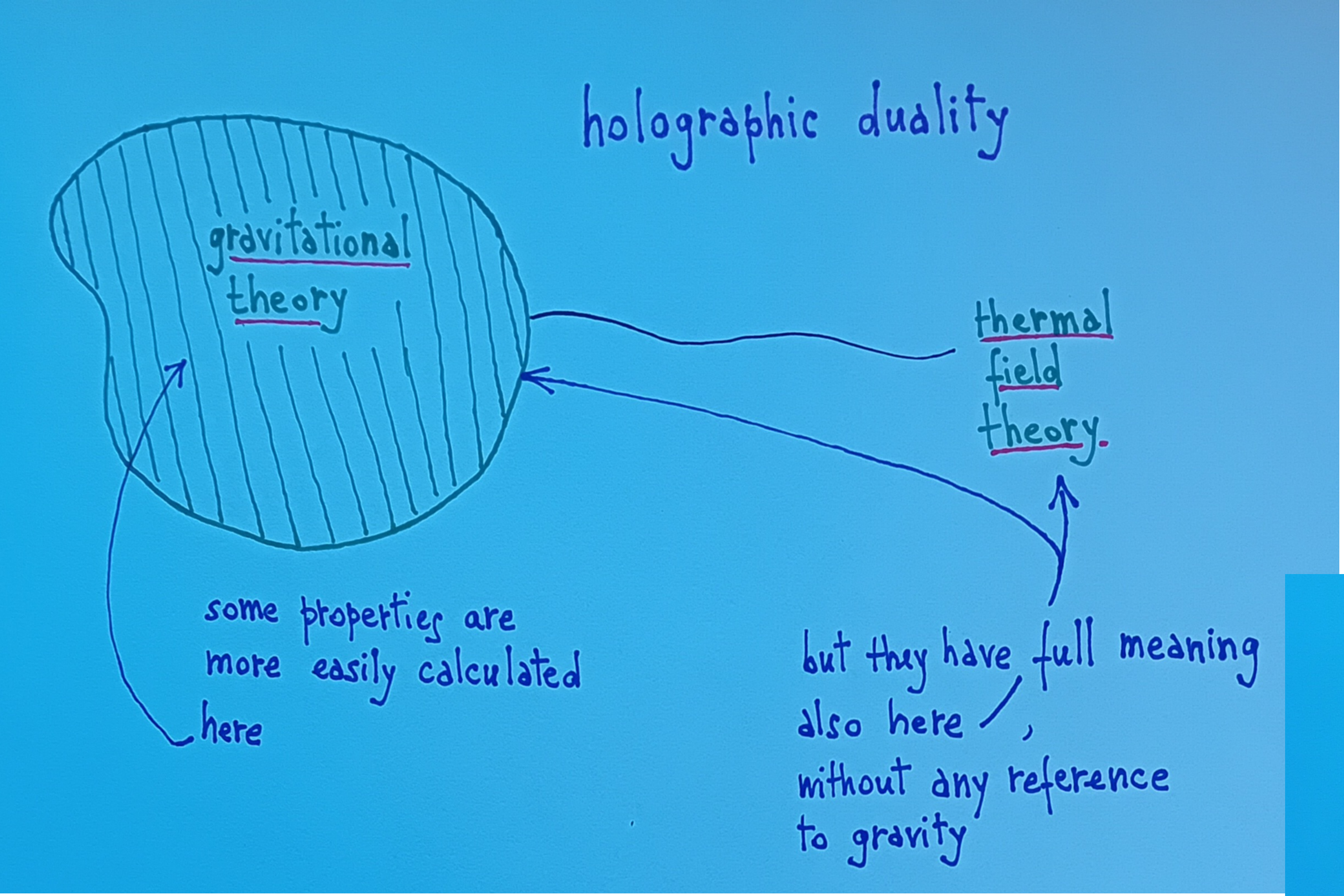
i.e. layers of this thickness of a photon gas do attain the universal bound to relaxation times

[A.P., 0807.0300]

The gen. entropy bound demands for a discretization both in space and time in the description of statistical systems.

KSS bound (1)

Kovtun, Son, Starinets hep-th/0309213;
hep-th/0405231



η defined through:

$$\frac{F}{A} = \eta \frac{\partial u}{\partial y}$$

KSS bound [Kovtun, Son, Starinets, 2003]

for thermal theories with holographic dual, invariably

$$\frac{\eta}{s} = \frac{1}{4\pi} \qquad (\text{at least for } \mu = 0) \; ;$$

in general

$$\boxed{\frac{\eta}{s} \geq \frac{1}{4\pi}} \qquad \left(\text{with the constants: } \frac{\eta}{s} \geq \frac{\hbar}{4\pi k}\right)$$

The extension to the case $\mu \neq 0$ has been given in Benincasa et al.,
hep-th/0610145

$\boxed{\text{non}}$relativistic systems

$$\frac{\eta}{s} = \frac{1}{3}\frac{L\rho a}{s} \simeq \frac{1}{3} L \frac{\rho+p}{s} a = \frac{1}{3\pi}\frac{L}{\ell^*} a$$

$$\boxed{\eta = \frac{1}{3} \underbrace{L}_{\text{correlation length}} \rho \underbrace{a}_{\text{thermal velocity}}}$$

Now, imagine to increase ρ, s, p while maintaining $\frac{\rho+p}{s}$, λ fixed;

L decreases, not below its q.m. limit λ:

$$\left(\frac{\eta}{s}\right)_{min} = \frac{1}{3\pi}\frac{\lambda}{\ell^*} a$$

$\boxed{\text{problem with the dependence on } a \text{ ?}}$

For a Boltzmann gas (n.r.) [A.P., 0803.2642]: $\frac{\lambda}{\ell^*} \gg 1$.

$$\frac{\lambda}{\ell^*} = (2\pi)^{3/2}\sqrt{\frac{m}{T}}\frac{1}{\underbrace{X}_{\text{a number, } \simeq 0 \text{ at our conditions}} + \underbrace{\frac{g+2}{2}}_{\#\text{ degrees of freedom per particle}}} \propto \sqrt{\frac{m}{T}} \propto \frac{1}{a} \Longrightarrow$$

$\frac{1}{3} m a^2 = T$

in $\left(\frac{\eta}{s}\right)_{min}$ no dependence on a is left

$$(\eta/s)_{min} \approx 0.7 \quad \text{for } g=2.$$

$\boxed{\text{ultra}}$relativistic systems

interacting radiation

model: particles with statistical equilibrium determined by collisions with radiation quanta (γ, ν, g); the particles act as mechanism to transfer momentum through the radiation field.

assuming

$$\eta \approx \frac{1}{3} \tau \rho_\gamma$$

τ: average time between interactions for a quantum

ρ_γ: energy density of radiation

[Misner, 1968] Ap. J. 151, 431

$$\frac{\eta}{s_\gamma} \approx \frac{1}{3}\tau\frac{\rho_\gamma}{s_\gamma} = \frac{1}{4}\tau\frac{\rho_\gamma+p_\gamma}{s_\gamma} = \frac{1}{4\pi}\frac{\tau}{\ell^*_\gamma} = \frac{1}{4\pi}\frac{L}{\ell^*_\gamma}.$$

L: correlation length = average distance between interactions for a quantum

For radiation, $d \simeq \lambda_\gamma$. $\Longrightarrow$ for a field of interacting quanta: 'mean free path of a quantum' $\gtrsim \lambda_\gamma$

d: distance among quanta

If we assume that, for assigned T, L can be decreased down to the limit $L \to \lambda_\gamma$ (which amounts to imply strong coupling as $L_{min} \approx \frac{\lambda}{g^2}$):

see also (still relying on ℓ^*): Hod 0905.4113

$$(\eta/s)_{min} \approx \frac{1}{4\pi}\frac{\lambda_\gamma}{\ell^*_\gamma} = \frac{1}{4\pi}.$$

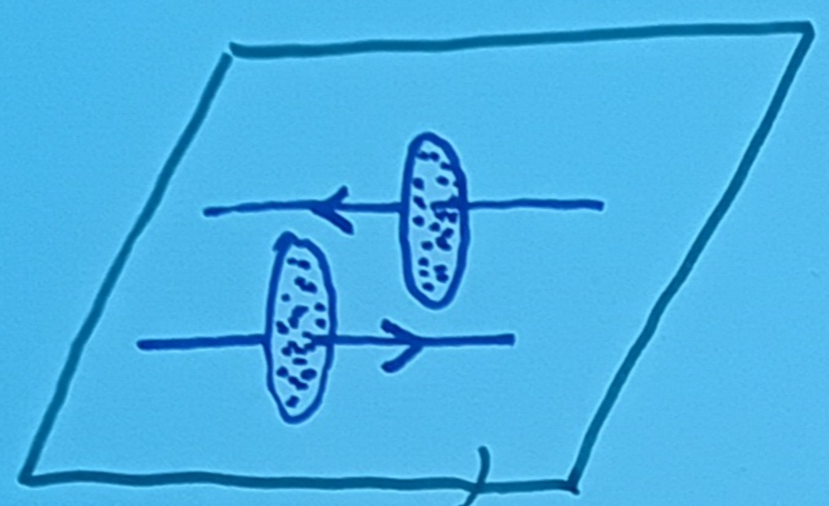
ultrarelativistic systems (with $\mu=0$) have $\lambda = \ell^*$

$\Longrightarrow$ ultrarelativistic systems at $L=\lambda$ conditions do attain the KSS limit

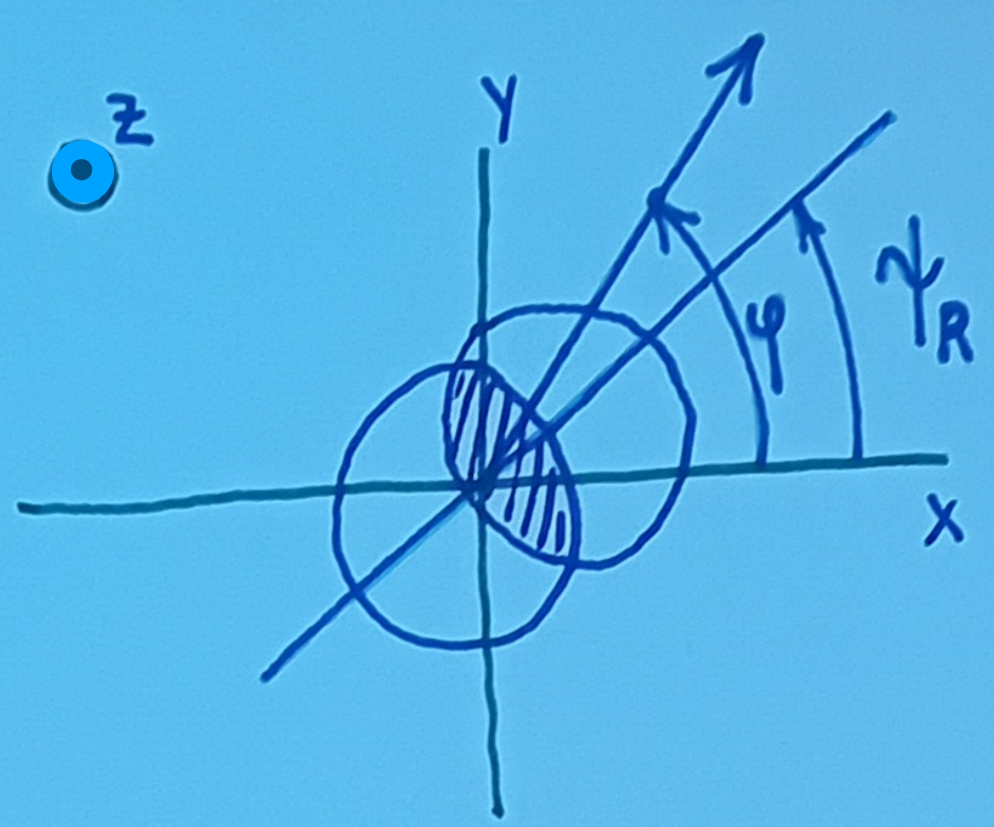
- QGP

KSS b. (2)

an experimental result



reaction plane



KSS bound (3)

BRAHMS nucl-ex/0410020; PHENIX nucl-ex/0410003
PHOBOS nucl-ex/0410022; STAR nucl-ex/0501009;
ALICE 1105.3865

transverse momentum

$$E\,\frac{dn}{d^3p} \propto 1 + 2\,v_2(p_T)\cos\left(2(\varphi-\psi_R)\right)$$

"flow"
or
"elliptic flow"

$n \equiv$ particle multiplicity

a direct consequence of the elliptic shape of superposition region and of the hydrodynamic properties of the matter produced

$\boxed{v_2(p_T)}$ is experimentally accessible $\rightarrow$ $\boxed{\eta}$, or more reliably $\boxed{\eta/s}$, can be estimated

result:

$$\eta/s \in (0.08,\ 0.24)$$

[RHIC experiments, 2005]

$\left(\frac{1}{4\pi} \simeq 0.08\right)$

depending on initial conditions and equation of state change of perspective

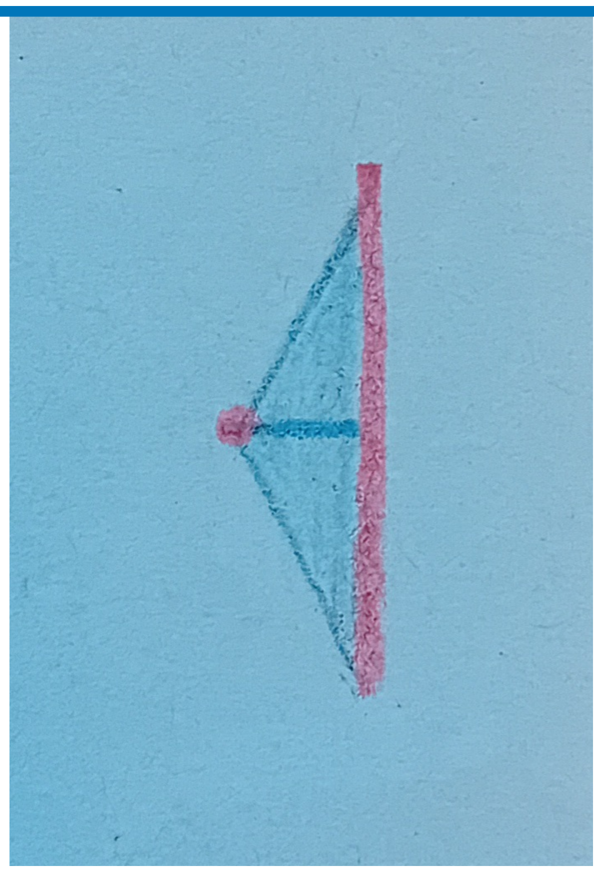
$$S_L \le \frac{1}{4}(A - A')$$

Is there
some entropy content, some piece of information ?

If yes, you $\boxed{\text{must}}$ have focusing.

Is the actual entropy content what can determine focusing ?

focusing



also,
the bound builds upon
existence of a limit length:

$$\frac{1}{3}s_\gamma \le \frac{1}{3}\bar{s}_\gamma = \frac{1}{4\bar{\lambda}_\gamma}$$

$$\Rightarrow \quad \lambda_\gamma \ge \bar{\lambda}_\gamma \approx 0.2\, l_p$$

giving spacetime a limit length

consider spacetime endowed with existence of a minimum length
(i.e., with quadratic intervals -> finite limit at coincidence)
[minimum-length metric or quantum metric or qmetric]
Kothawala 1307.5618; Kothawala, Padmanabhan 1405.4967; JaffinoStargen, Kothawala 1503.03793

allow for this description to include null intervals
AP 1812.01275

apply it e.g. to black hole horizons
Krishnendu N V, S. Chakraborty, A. Perri, AP 2505.22877
M.J. Fahn, AP 2507.16911; 2507.18709

minimum-length metric

Kothawala 1307.5618; Kothawala, Padmanabhan 1405.4967; JaffinoStargen, Kothawala 1503.03793

existence of a minimum length L affects geometry itself in the small scale (i.e., not regarded as L-blurring of sources in an ordinary spacetime)

modification introduced in the quadratic interval $\sigma^2(x, x')$ (before g_{ab}):

$\sigma^2(x, x') \mapsto S(\sigma^2)$ with $S(\sigma^2) \to \epsilon L^2$ finite in the coincidence limit $x \to x'$

(with $S(\sigma^2) \approx \sigma^2$ when $|\sigma^2| \gg L^2$, i.e., when x is far apart from x)

for it, one needs a metric singular everywhere: how to deal with this?

nonlocality

we face the unavoidable nonlocality accompanying gravity in the smallest scales

convenience of nonlocal objects to describe this: use bitensors (just like $\sigma^2(x, x')$, which is a biscalar)

to require

$\sigma^2(x, x') \mapsto S(\sigma^2)$ with $S(\sigma^2) \to \epsilon L^2$ finite in the coincidence limit $x \to x'$
along the connecting geodesic, which such remains (with a same character) also in the new metric

implies

$$g_{ab}(x) \mapsto q_{ab}(x, x') = A\, g_{ab}(x) + \epsilon\, (1/\alpha - A)\, t_a(x)\, t_b(x)$$

$t_a =$ tangent vector

$\alpha = \alpha(\sigma^2), A = A(\sigma^2)$
biscalars

$\epsilon = g^{ab}\, t_a t_b = \pm 1$

causality

q_{ab} turns out to be completely fixed if a condition is additionally posed on the 2-point function $G(x, x')$ of any field (namely, this is about causality):

one requires that, when spacetime is maximally symmetric,

$$G(\sigma^2) \mapsto \widetilde{G}(\sigma^2) = G(S(\sigma^2))$$

where

G and $\widetilde{G}$ are Green functions of $\Box$ and ${}_x\widetilde{\Box}_{x'}$ resp., and ${}_x\widetilde{\Box}_{x'}$ is the d'Alembertian associated to $q_{ab}(x, x')$

the bi-metric

one gets:

Kothawala 1307.5618; Kothawala, Padmanabhan 1405.4967; JaffinoStargen, Kothawala 1503.03793

$$q_{ab}(x,x') = A\, g_{ab} + \epsilon\,(1/\alpha - A)\, t_a t_b$$

t_a unit tangent to connect. geod.

ϵ = -/+ 1 for time/space sep.

with

$$A = \frac{S}{\sigma^2}\left(\frac{\Delta}{\Delta_S}\right)^{\frac{2}{D-1}} \qquad \alpha = \frac{S}{\sigma^2\, S'^2}$$

(D-dim spt.)

$S' \equiv dS/d(\sigma^2)$

$$\Delta(x,x') = -\frac{1}{\sqrt{g(x)g(x')}}\det\left[-\nabla_a^{(x)}\nabla_b^{(x')}\frac{1}{2}\sigma^2(x,x')\right]$$

van Vleck determinant

$\Delta_S = \Delta(\tilde{x},x')$ with $\tilde{x}$ such that $\sigma^2(\tilde{x},x') = S$ on the connecting geodesic

q_{ab} is singular everywhere in the $x \to x'$ limit, and $q_{ab} \approx g_{ab}$ for x, x' far apart

null separations

AP 1812.01275, 2207.12155

what's the meaning of a finite distance limit in this case?

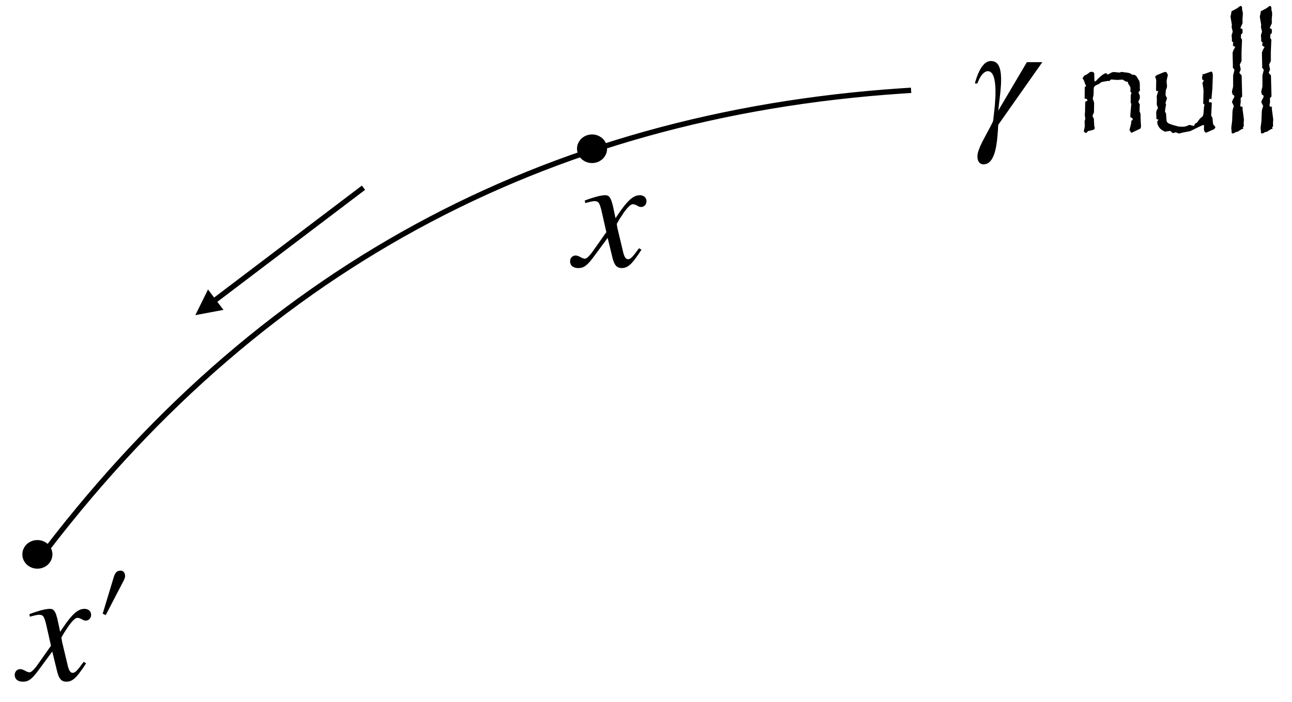
key: affine λ = measure of distance by the canonical observer

qmetric:
this observer at x' will find a finite lower bound L to $\lambda - \lambda_{x'}$

take $\lambda_{x'} = 0$,

$\lambda \mapsto \tilde{\lambda}(\lambda)$, with $\tilde{\lambda} \to L$ when $\lambda \to 0$

(with $\tilde{\lambda}(\lambda) \approx \lambda$ when $\lambda \gg L$)



we seek $q^{(\gamma)}_{ab}$ of the form

$$q^{(\gamma)}_{ab}(x,x') = A_{(\gamma)}\, g_{ab}(x) + (A_{(\gamma)} - 1/\alpha_{(\gamma)})\,(l_a(x)n_b(x) + n_a(x)l_b(x))$$

$$A_{(\gamma)} = A_{(\gamma)}(\lambda)$$
$$\alpha_{(\gamma)} = \alpha_{(\gamma)}(\lambda)$$

n_a null with $l^a n_a = -1$

---

from

$$\tilde{l}^b\, \widetilde{\nabla}_b\, \tilde{l}_a = 0$$

with $\tilde{l}^a = \dfrac{dx^a}{d\tilde{\lambda}} = l^a \dfrac{d\lambda}{d\tilde{\lambda}}$,

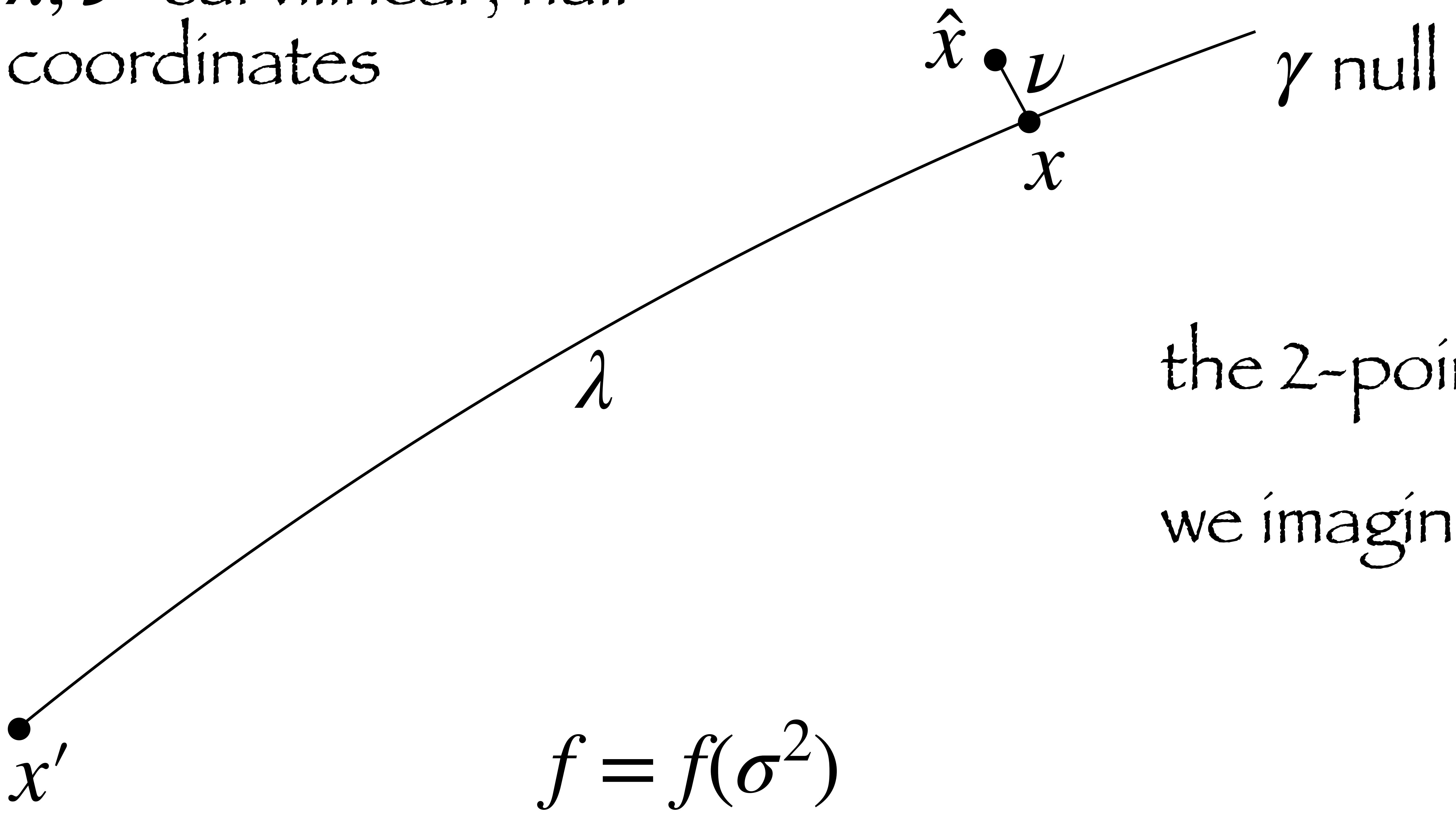
and $\widetilde{\nabla}_b \tilde{v}_a = \nabla_b \tilde{v}_a - \dfrac{1}{2} q^{cd}(-\nabla_d q_{ba} + 2\nabla_{(b} q_{a)d})\,\tilde{v}_c$

we obtain

$$\alpha_{(\gamma)} = \frac{C}{d\tilde{\lambda}/d\lambda},$$

with C real const.

λ, ν curvilinear, null coordinates



the 2-point function $G(x, x')$ diverges on γ

we imagine to be slightly off γ

$$f = f(\sigma^2)$$

$$\Box f = (4 + 2\lambda \nabla_a l^a) \frac{df}{d\sigma^2} \quad \text{at } x \in \gamma \qquad l^a = \frac{dx^a}{d\lambda}$$

we implement then the d'Alembertian condition this way:

$\widetilde{G}(\sigma^2) = \widetilde{G}(S(\sigma^2))$ is solution of

$$(4 + 2\tilde{\lambda}\,\widetilde{\nabla}_a \tilde{l}^a)\frac{d\tilde{G}}{dS}_{|\tilde{\lambda}} = (4 + 2\tilde{\lambda}\,\widetilde{\nabla}_a \tilde{l}^a)\left(\frac{d\tilde{G}}{d\sigma^2}\right)_{|\lambda=\tilde{\lambda}} = 0 \qquad (1)$$

when

$G(\sigma^2)$ is solution of

$$(4 + 2\lambda\,\nabla_a l^a)\frac{dG}{d\sigma^2}_{|\lambda} = 0 \qquad (2)$$

using $\widetilde{\nabla}_b \tilde{l}_a$ and the expression for $\alpha_{(\gamma)}$ we already have, eq. (1) is

$$4 + 2\tilde{\lambda}\frac{d\lambda}{d\tilde{\lambda}}\nabla_a l^a{}_{|\lambda} + \tilde{\lambda}(D-2)\frac{d\lambda}{d\tilde{\lambda}}\frac{d}{d\lambda}\ln A_{(\gamma)} = 0$$

$D =$ spacetime dim.

---

from (2) at $\tilde{\lambda}$, i.e., $4 + 2\tilde{\lambda}\nabla_a l^a{}_{|\tilde{\lambda}} = 0$,

and $\nabla_a l^a{}_{|\lambda} = \dfrac{D-2}{\lambda} - \dfrac{d}{d\lambda}\ln\Delta$, $\quad \nabla_a l^a{}_{|\tilde{\lambda}} = \dfrac{D-2}{\tilde{\lambda}} - \dfrac{d}{d\tilde{\lambda}}\ln\Delta_{\tilde{\lambda}}$,

we obtain

$$\frac{d}{d\lambda}\ln\Big[\frac{\lambda^2}{\tilde{\lambda}^2}\Big(\frac{\Delta_{\tilde{\lambda}}}{\Delta}\Big)^{\frac{2}{D-2}}A_{(\gamma)}\Big] = 0$$

which is

$$A_{(\gamma)} = C' \, \frac{\tilde{\lambda}^2}{\lambda^2} \left(\frac{\Delta}{\Delta_{\tilde{\lambda}}} \right)^{\frac{2}{D-2}}, \qquad C' > 0 \ \text{const.}$$

———————

from

$$q^{(\gamma)}_{ab} = A_{(\gamma)} \, g_{ab} + (A_{(\gamma)} - 1/\alpha_{(\gamma)}) \, (l_a n_b + n_a l_b) \ \approx \ g_{ab} \qquad \text{when } \lambda \gg L \, ,$$

we get $C' = 1 = C$

———————

then, final expression is

$$q^{(\gamma)}_{ab} = A_{(\gamma)} \, g_{ab} + (A_{(\gamma)} - 1/\alpha_{(\gamma)}) \, (l_a n_b + n_a l_b)$$

with

$$\alpha_{(\gamma)} = \frac{1}{d\tilde{\lambda}/d\lambda},$$

$$A_{(\gamma)} = \frac{\tilde{\lambda}^2}{\lambda^2} \left(\frac{\Delta}{\Delta_{\tilde{\lambda}}} \right)^{\frac{2}{D-2}}.$$

q_{ab} singular everywhere
when $x \to x'$

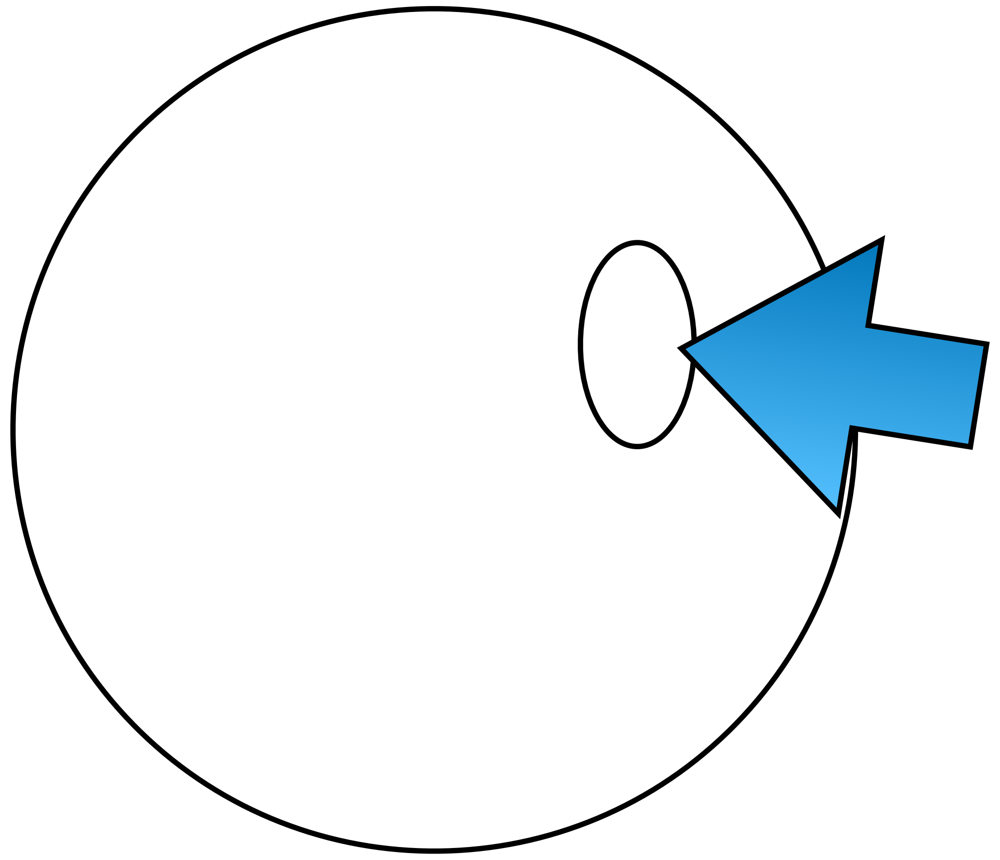
Ricci scalar

$$\lim_{x\to x'} \tilde{R}(x,x') = \epsilon\, D\, R_{ab}t^a t^b \;+\; O(L)$$ time/space sep.

Kothawala, Padmanabhan 1405.4967; JaffinoStargen, Kothawala 1503.03793

$$\lim_{x\to x'} \tilde{R}_{(\gamma)}(x,x') = (D-1)\, R_{ab}l^a l^b \;+\; O(L)$$ null sep. AP 1911.04135

$\delta Q =$ heat flow through horizon



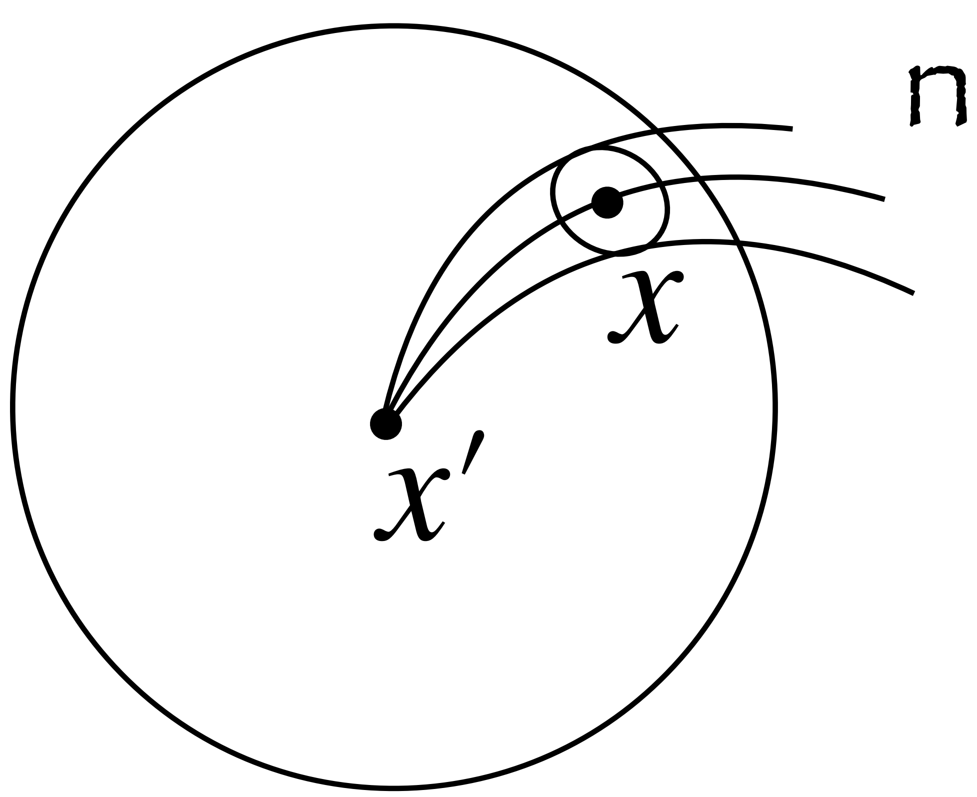
$$\delta Q = \lim_{x\to x'} \tilde{R}$$

qmetric introduces gravitational, local dofs
(geometric)

areas shrink to finite values

transverse metric:

$$\tilde{h}_{ab} = A_{(\gamma)} h_{ab}$$



$$d^{D-2}\tilde{a}(x) =$$

$$= \sqrt{\det \tilde{h}_{ab}(x) / \det h_{ab}(x)}\; d^{D-2}a(x) = \sqrt{\det \tilde{h}_{ab}(x) / \det h_{ab}(x)}\; \lambda^{D-2} d\Omega_{(D-2)} =$$

$$= \tilde{\lambda}^{D-2} \frac{\Delta}{\Delta_{\tilde{\lambda}}} d\Omega_{(D-2)} \;\; \underset{\text{for } x \to x'}{\to} \;\; L^{D-2} \frac{1}{\Delta_{\tilde{\lambda}=L}} d\Omega_{(D-2)} \approx L^{D-2}\, d\Omega_{(D-2)}$$

$$\Delta_{\tilde{\lambda}=L} = 1 + \frac{1}{6} L^2 R_{ab} l^a l^b + \ldots$$

which is finite for a given $d\Omega_{(D-2)}$

Kothawala 1406.2672 ; Padmanabhan 1508.06286; AP 1812.01275

statistical field equations

Padmanabhan 1508.06286; 1702.06136
AP 1511.08665

gravitational dofs (quantum dofs of geometry)

$$S_{\text{flat}} = (\text{number of dofs})_{\text{flat}} \text{ at the point } = \text{ limit area in some units } L^2 = \mathcal{O}(1)$$

$$S_{\text{actual spt}} = \frac{1}{\Delta_L} S_{\text{flat}} = \mathcal{O}(1) \left(1 - \frac{1}{6} L^2 R_{ab} l^a l^b + \dots\right)$$

matter dofs

$$S_{\text{matter}} = \mathcal{O}(1)\ L^4 T_{ab} l^a l^b$$

we want to extremize $(1 - \frac{1}{6} L^2 R_{ab} l^a l^b) + L^4 T_{ab} l^a l^b$ with respect to l^a keeping l^a null;

this gives $-\frac{1}{6} L^2 R_{ab} l^a l^b + L^4 T_{ab} l^a l^b = 0$; we are back then to Jacobson's result ...

... but with the addition of some statistical mechanical flavor!

since we start from micro dofs, and get the result from an extremization of entropy

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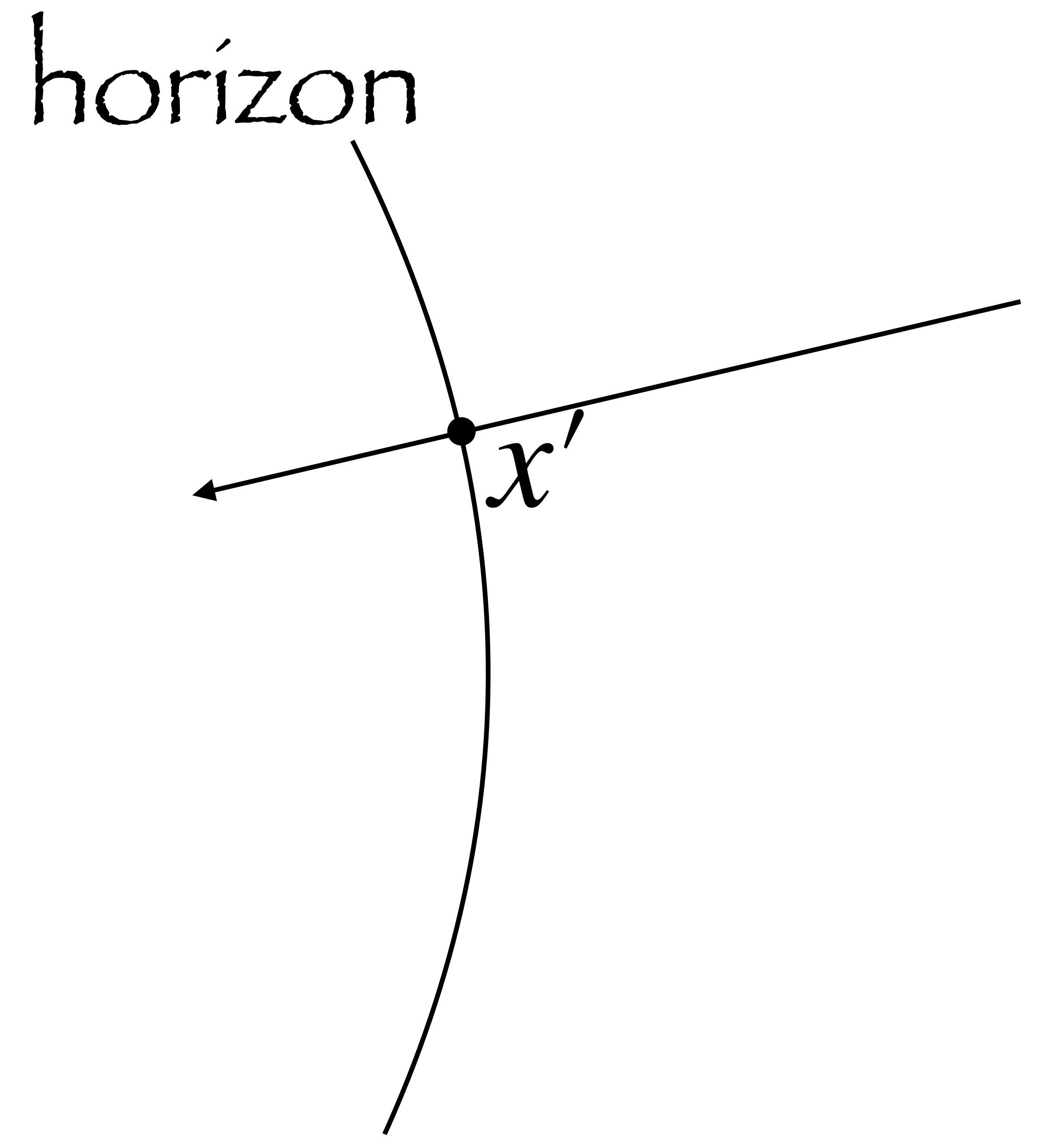
an extremization based on a different hypothesis about the underlying quantum states of the geometry has been investigated in Isidro, Paganini, P. 2407.13317 .

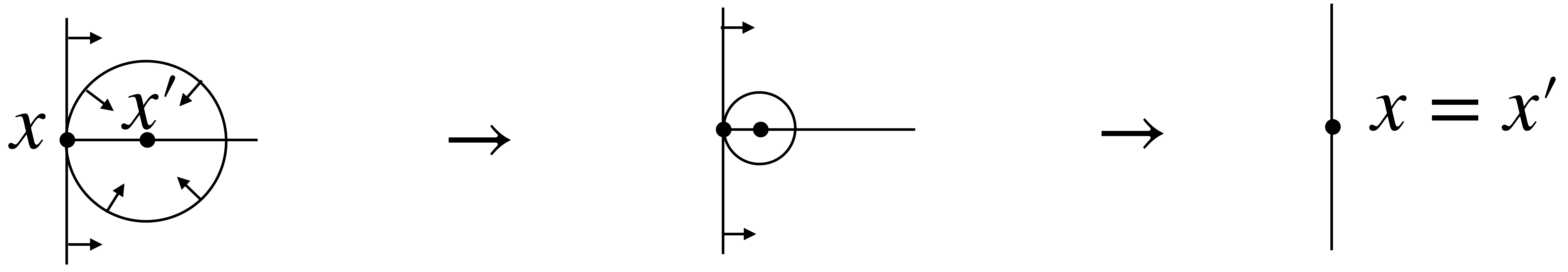
use on horizons

(collabs. with Krishnendu N.V. (ICTS, Bengaluru), S. Chakraborty (IACS, Kolkata), A. Perri (Bologna), and with M.J. Fahn (Bologna))

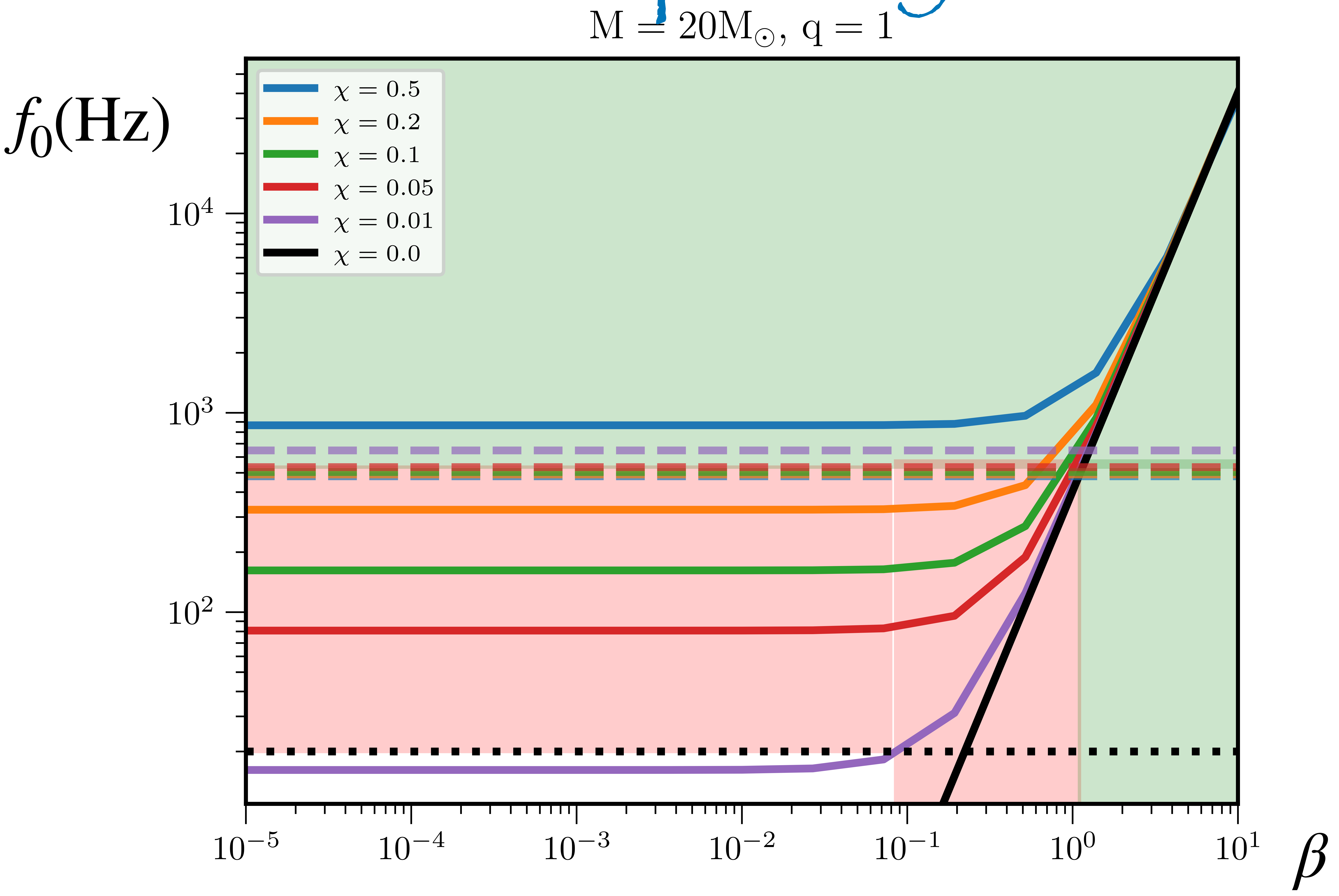
x' = event of crossing of the horizon by some chunk of energy



we describe the coincidence event according to the local observer at x'



thresh. frequency vs $\beta\ (= L/l_p)$



$$\chi = \frac{J}{M^2}$$

(from Krishnendu N.V.)

Krishnendu, Chakraborty, Perri, AP 2505.22877

quantum systems near the horizon

the presence of a horizon can affect the coherence of a quantum system

Danielson, Satishchandran, Wald 220506279; 2301.00026; ...

the existence of a limit length can have significant impact on the results

M.J. Fahn, AP 2507.16911; 2507.18709

➡ see the poster by M.J. Fahn

conclusions

we have seen that

the long history (53y) of gravity-thermo.-quantum_info connection apparently is strictly intertwined with the notion of a limit length

and that

the latter provides hints/means to endow spacetime with quantum features

some open problems:

– why $\lambda \geq \frac{1}{\pi} \frac{s}{\rho + p}$?

– why is the gen. cov. entropy bound attained (and not just satisfied by far) ?

– should any quantum theory of gravity foresee (effectively at least) a limit length ?

– gravity is definitely intertwined with thermod./information: can we recast its geometric description fully in the language of the latter ?