An outline of tensorial group field theories and of challenges toward an emergent spacetime

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Quantum Gravity

the quantum theory behind the physical system
we call spacetime

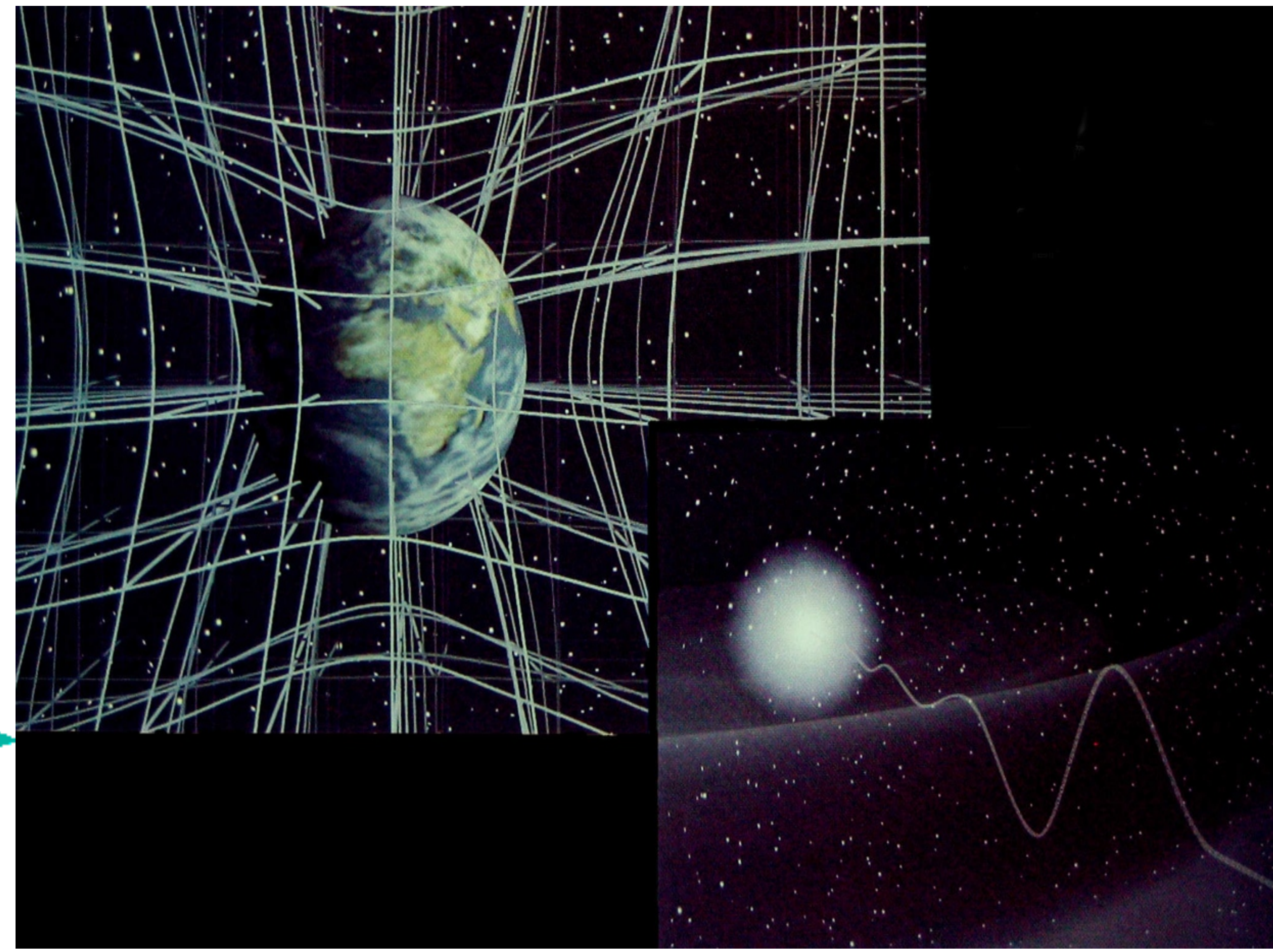
Main lessons from General Relativity (and modern gravitational physics):

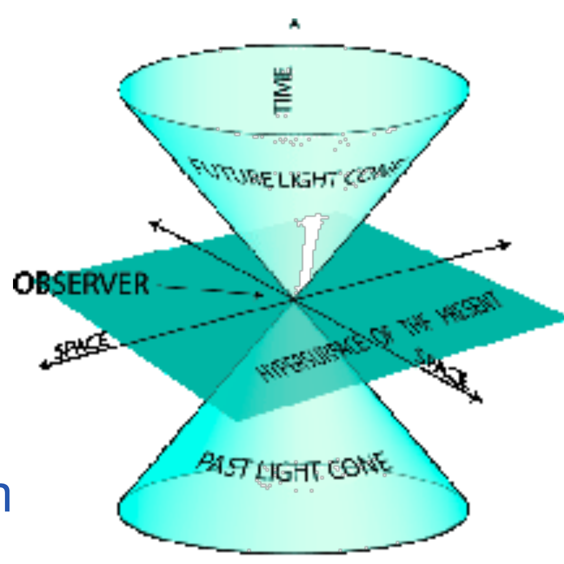
1st Lesson: **spacetime is a physical system**

- gravity = spacetime geometry

$$g_{\mu\nu}(x) \qquad ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$$

(spatial distances, temporal duration, causal structure, curvature, ...)





thus,

spacetime itself is a dynamical, physical system

interacting with other physical systems, via Einstein's eqns:

$$R_{\mu\nu}[g(x)] - \frac{1}{2}R[g(x)] + \Lambda g_{\mu\nu}(x) = 8\pi G_N T_{\mu\nu}[\phi(x), ...]$$

Main lessons from General Relativity (and modern gravitational physics):

1st Lesson: spacetime is a physical system

- ingredients of the world: 4d smooth manifold and Lorentzian metric, plus several other (scalar, vector, ..) matter fields

$$(g\,,\,\phi\,,\ldots,\,\mathcal{M})$$

- gravity = spacetime geometry

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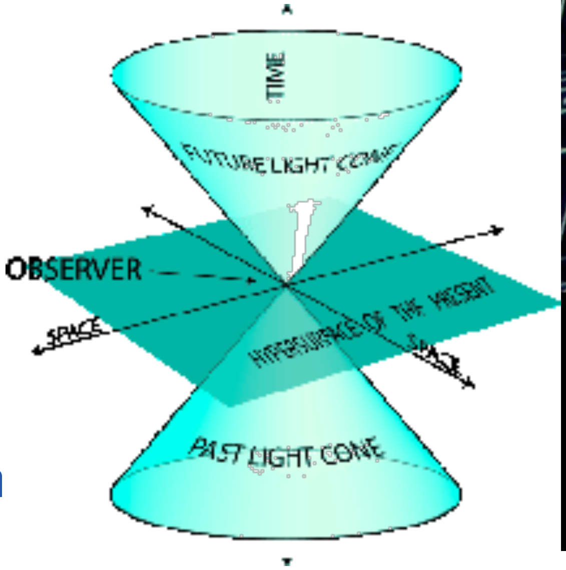

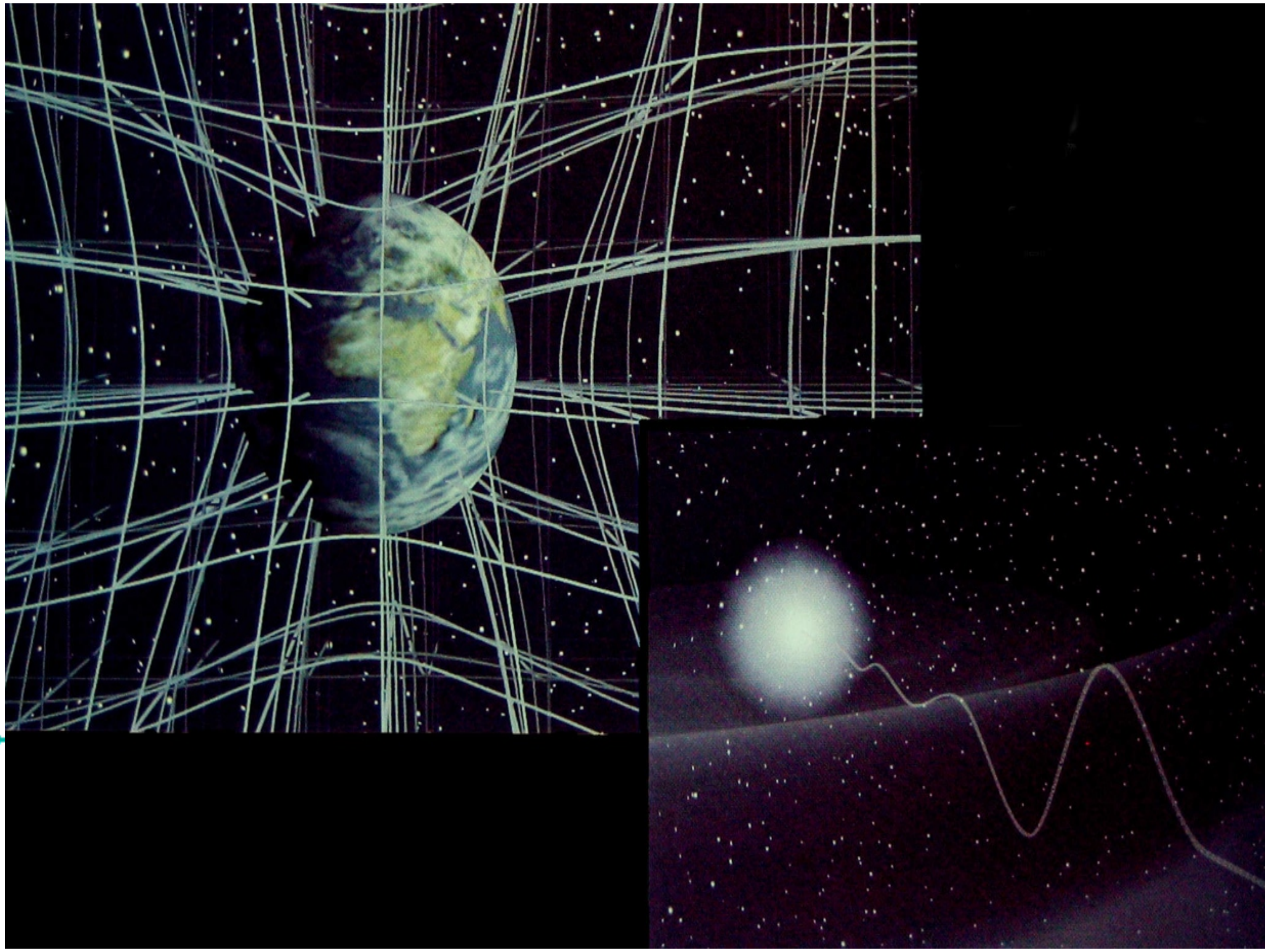

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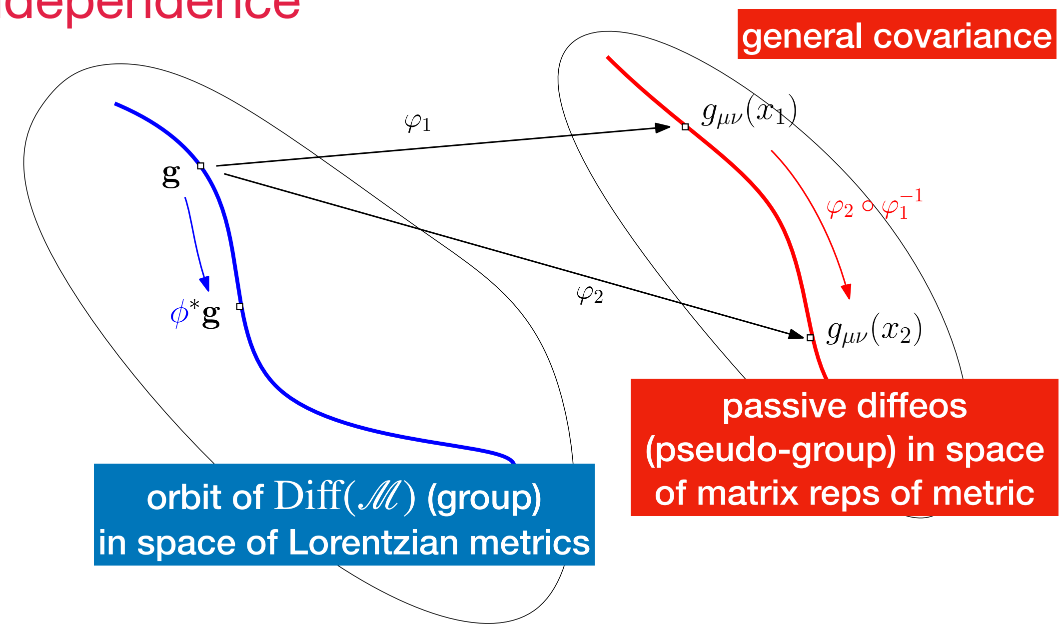
Main lessons from General Relativity (and modern gravitational physics):

2nd Lesson:

D. Giulini, '06

diffeomorphism invariance (gauge symmetry of GR) + background independence

- no absolute notion of temporal or spatial direction/location/distance
- not physical: local manifold structures (points, paths, coordinates, ...)
- not physical: "field components at a manifold point"
- physical: values of dynamical fields (incl. metric) and their relations



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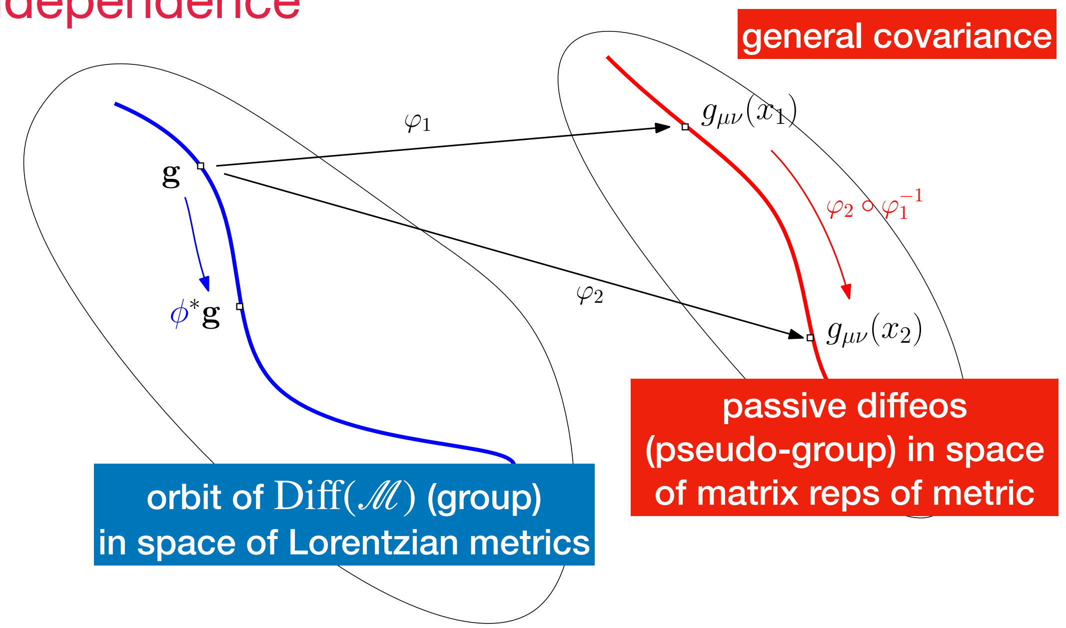


so what are local spacetime observables?

relational strategy:

Rovelli '90s+ [related ideas DeWitt '60s; Bargmann & Komar 90's] Dittrich, Hoehn,.... '00s

- identify internal appropriate d.o.f.s, e.g. matter fields, use them as (approximate) clocks and rods to parametrize evolution and location of other dofs

$R(t) \qquad \Phi(t)$

$\Longrightarrow \qquad R(t) \qquad t(\Phi)$

$\Longrightarrow \qquad R(\Phi)$

spacetime example: $R(g(x)), \varphi_a(x) \rightarrow R(g(\varphi_a))$

spacetime physics should only be expressed in terms of such relational quantities - correlations among field values

points, coordinates, trajectories on manifold are "useful fictions" representing physical frames (clocks and rods) in the limit in which their physical properties (energy, interactions, …) are negligible (so they behave like test fields/coordinates)

do not expect to find manifold etc neither at fundamental QG level, nor in its effective description (before reconstruction)

Two perspectives on quantum gravity

- QG = quantum GR

$$g_{\mu\nu}(t,x) \Longrightarrow \widehat{g_{\mu\nu}(t,x)}$$

quantum metric (connection, tetrad) and
quantum matter fields as fundamental

already radical: quantum geometry, quantum causal structure, ...

- QG = quantum theory of "spacetime constituents"
= "quantum spacetime microphysics"

spacetime fields, including metric,
as effective, collective entities

Emergent spacetime!

quantum spacetime as a (background-independent) quantum many-body system

extraction of spacetime and cosmology similar to typical problem in condensed matter theory
(from atoms to macroscopic effective continuum physics)

- all GR structures and dynamics have to be approximately obtained (in relational language) at effective level

- not just emergent gravity; flat spacetime itself would be emergent, highly excited, collective state of "QG atoms"

The universe as a quantum condensate (in the TGFT formalism)

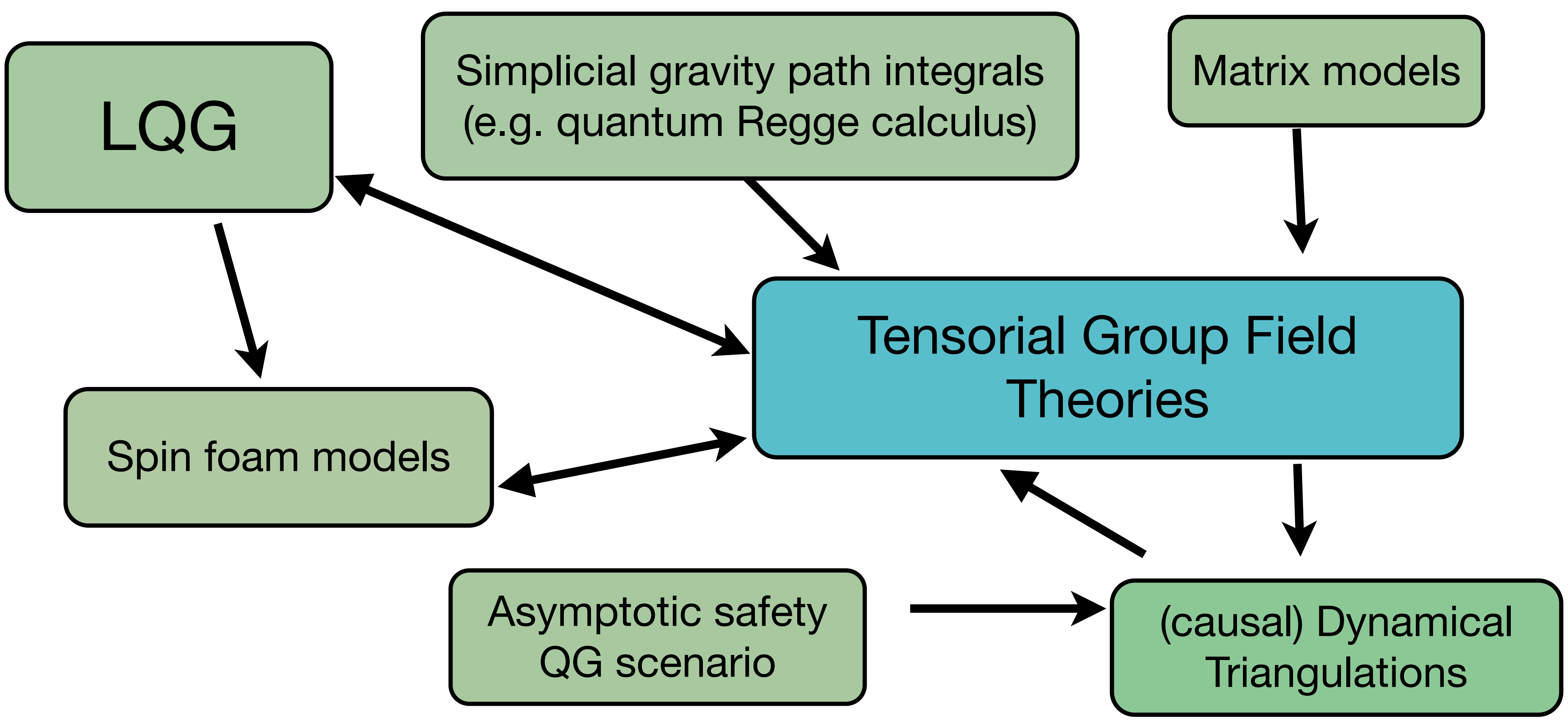
TGFT as an example of spacetime emergence (and convergence of QG formalisms)

goal: give an example of spacetime emergence, in a specific QG formalism

note: TGFT is very general framework, with many different applications:
here quantum geometric models for QG

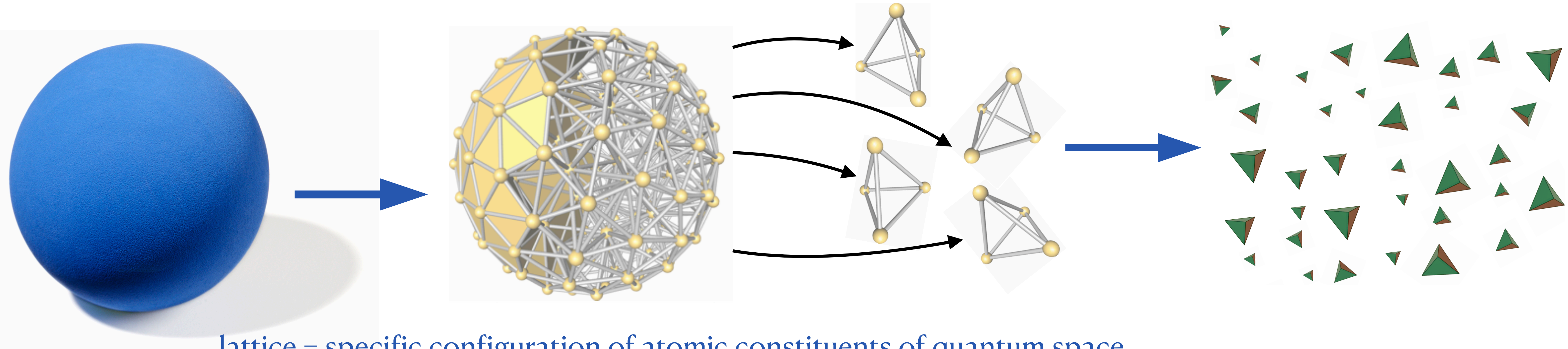
limitation: will not motivate/justify most aspects of formalism, nor give details of specific models or results

important: convergence of QG formalisms, particularly suited for quantum many-body techniques



Basic intuitions

3d-lattice building blocks = fundamental atomic constituents of quantum 3-space



lattice = specific configuration of atomic constituents of quantum space

microscopic ("atomic") dynamics related to lattice quantum gravity

QG = QFT of atomic constituents of quantum space $\longrightarrow$ Tensorial Group Field Theory

atoms of quantum space = 3-simplices = TGFT field quanta

data associated to single QG atom = geometric data of single 3-simplex

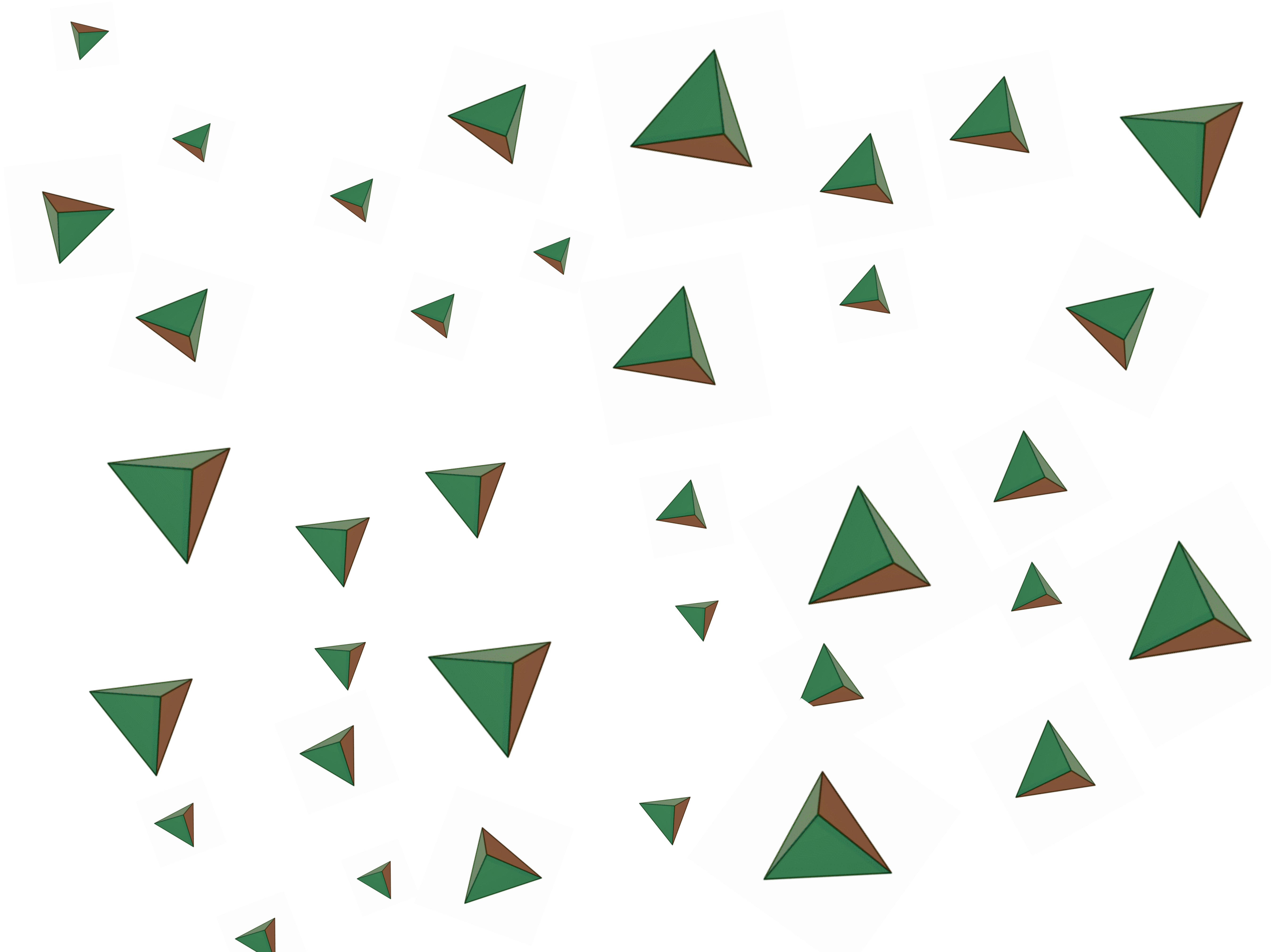
a QFT **of** spacetime and geometry, not **on** spacetime and geometry

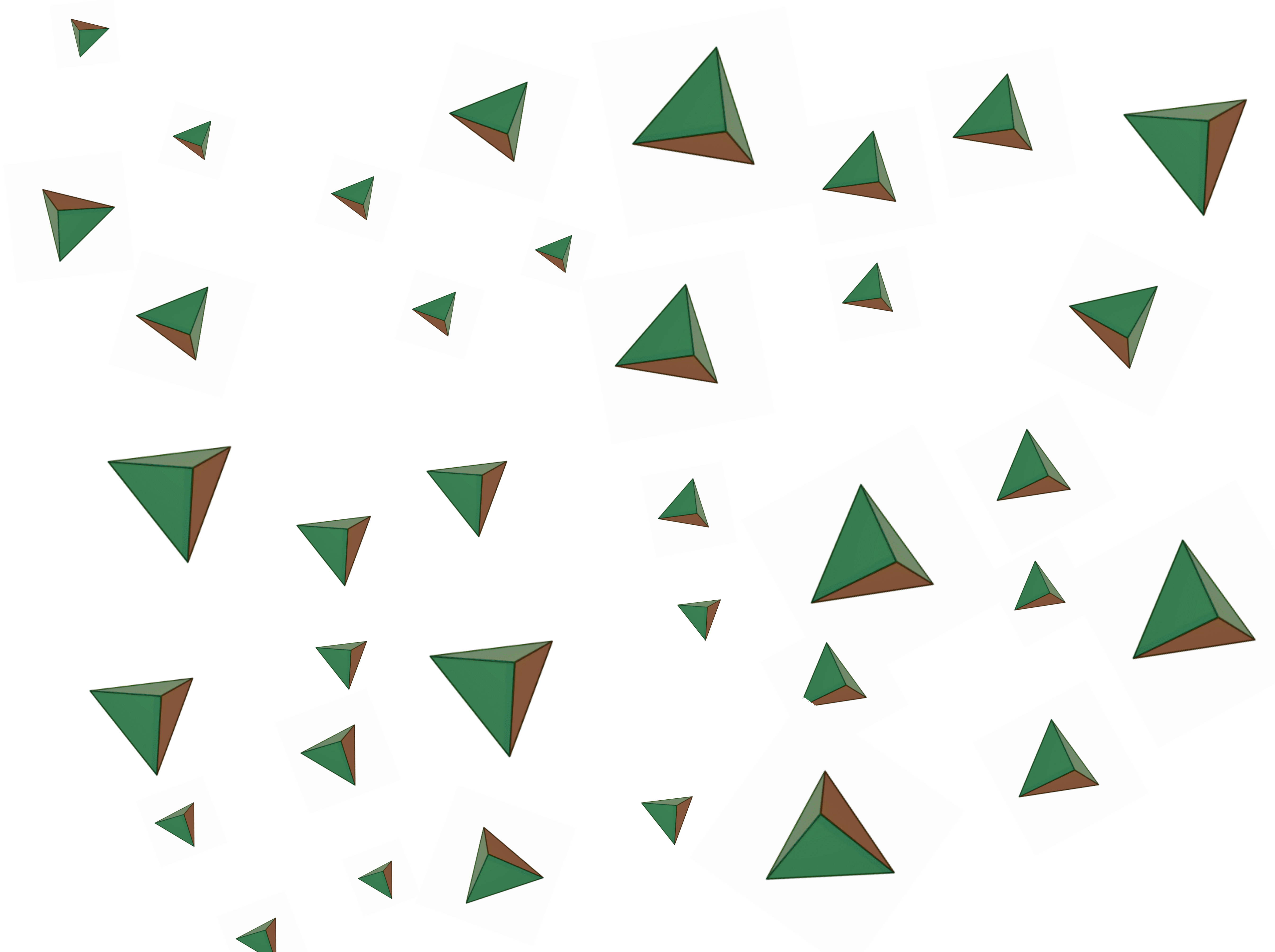
continuum spacetime (and geometry) to be extracted from collective physics

cosmology (most coarse grained dynamics) from QG hydrodynamic regime

The universe as a quantum condensate (in the TGFT formalism)

- a quantum condensate of what?
- Hilbert space and observables

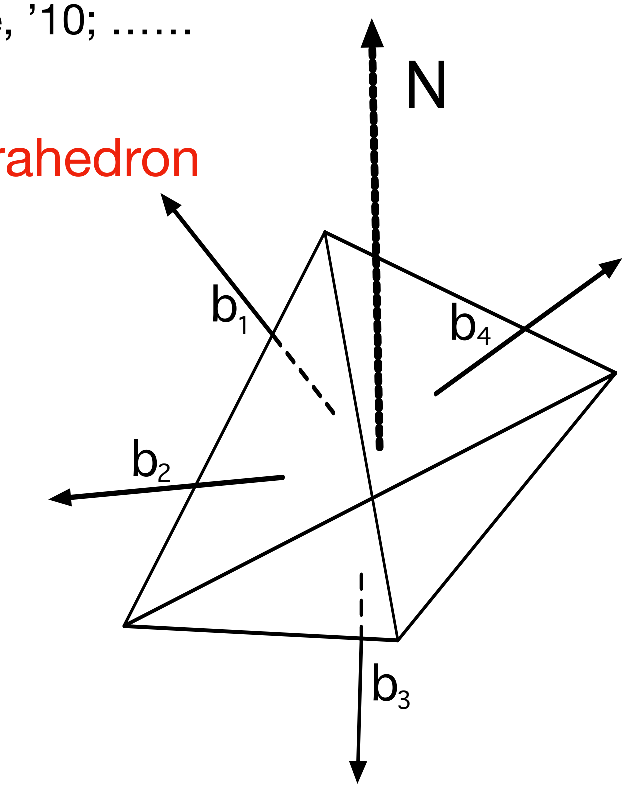


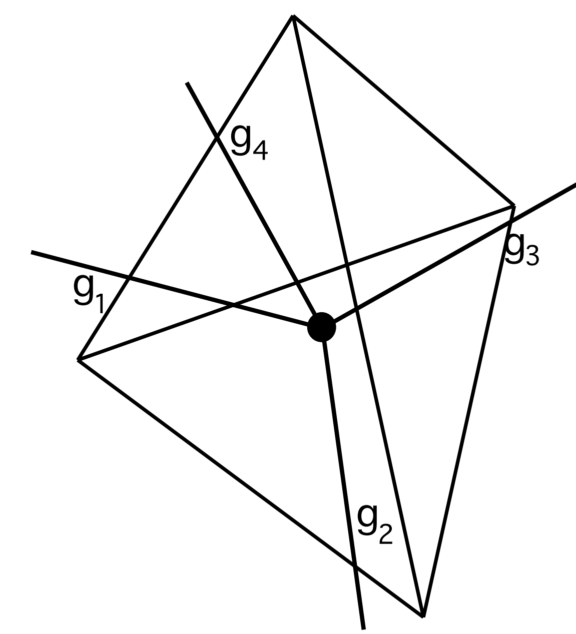


The TGFT "atom of space"

Barbieri '97; Baez, Barrett, '99; Rovelli, Speziale, '06; Bianchi, Dona, Speziale, '10; ……

Elementary building block of 3d space: single polyhedron - simplest example: a tetrahedron





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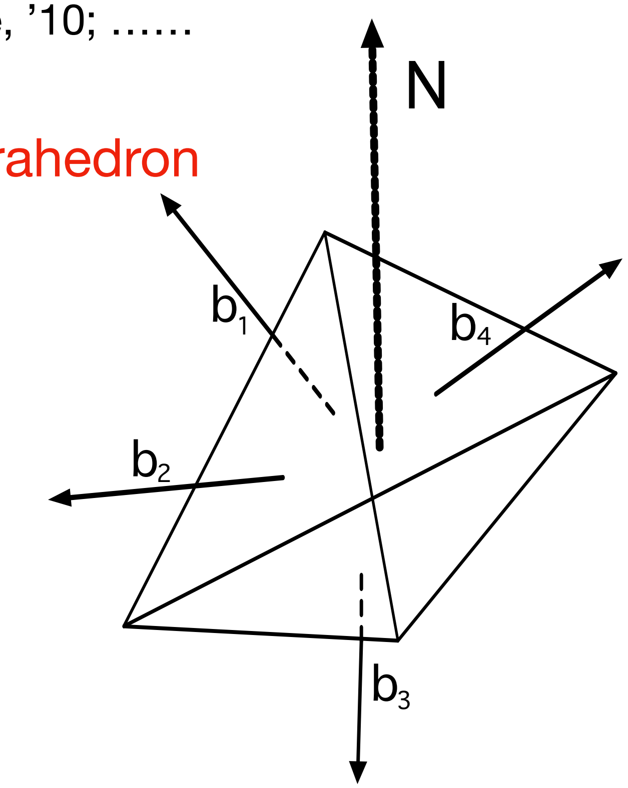

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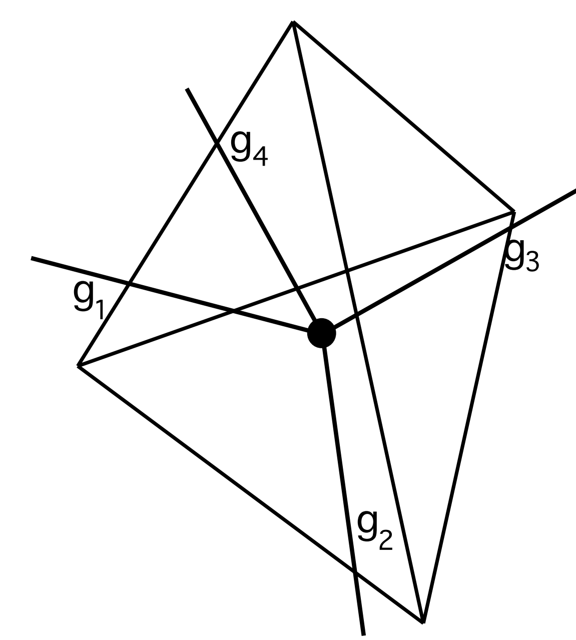
Classical geometry in group-theoretic variables

4 vectors normal to triangles that close (lying in hypersurface with normal N)

$$\boxed{A_i\, n_i^I \;=\; b_i^I \in \mathbb{R}^{3,1} \qquad b_i \cdot N \;=\; 0 \qquad \sum_i b_i \;=\; 0}$$



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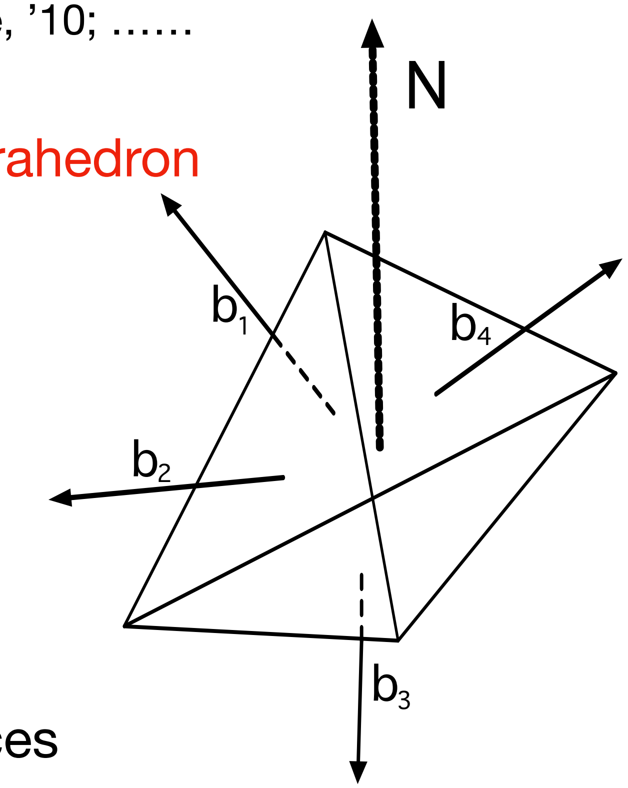

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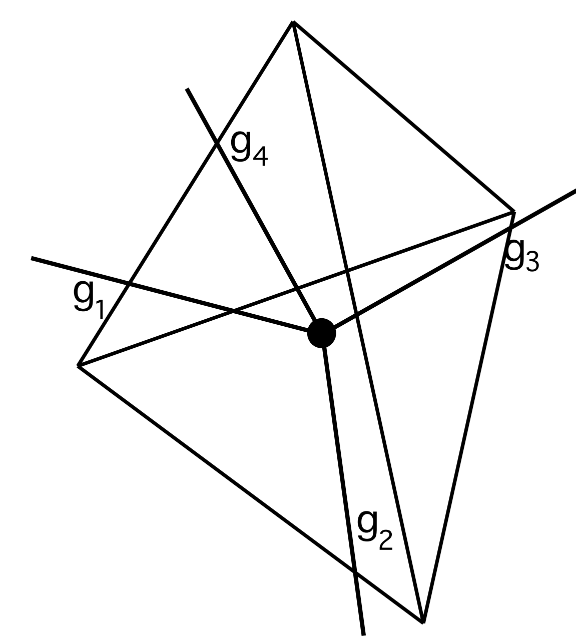
thus vectors are effectively $b_i \in \mathbb{R}^3 \longleftarrow$ note that: $\mathfrak{su}(2) \simeq \mathbb{R}^3$ as vector spaces

- conjugate variables (extrinsic geometric information): group elements $\{g_i\}$ = discrete connection
- thus one can associate to a triangle the phase space: $[\mathcal{T}^* SU(2)]^{\times 4}$



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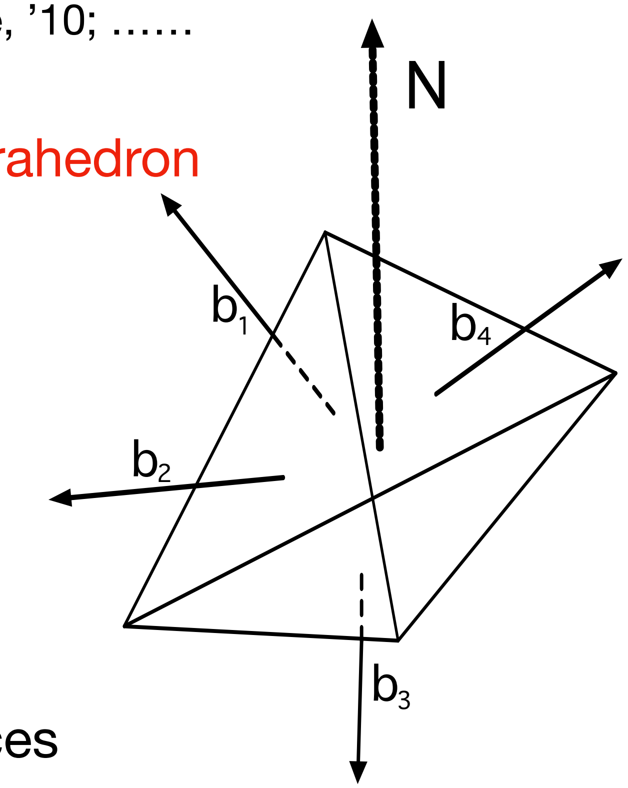

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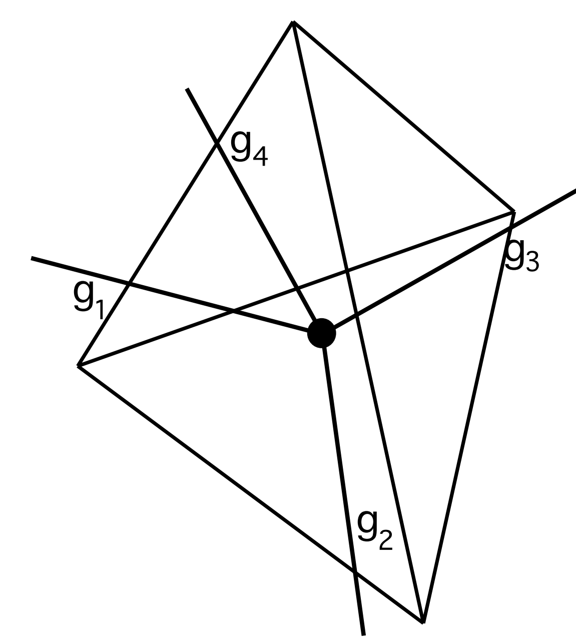

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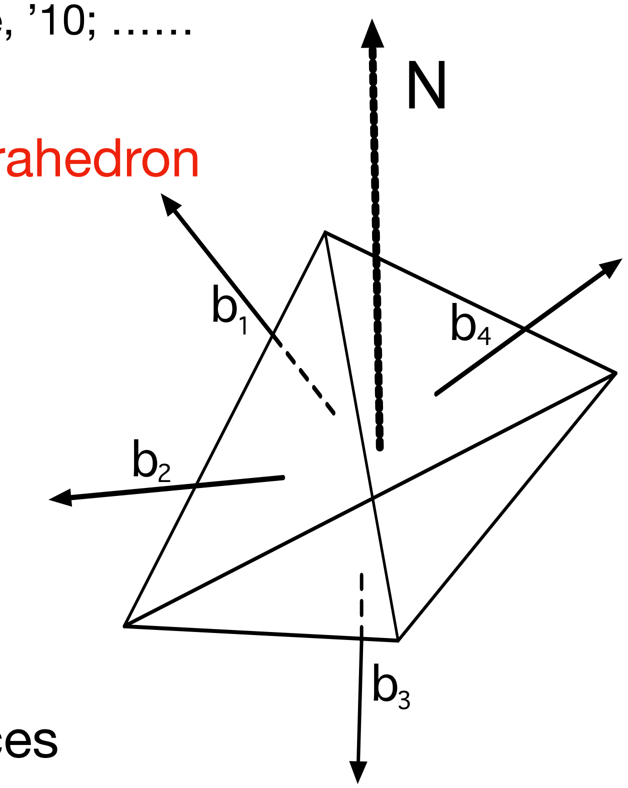

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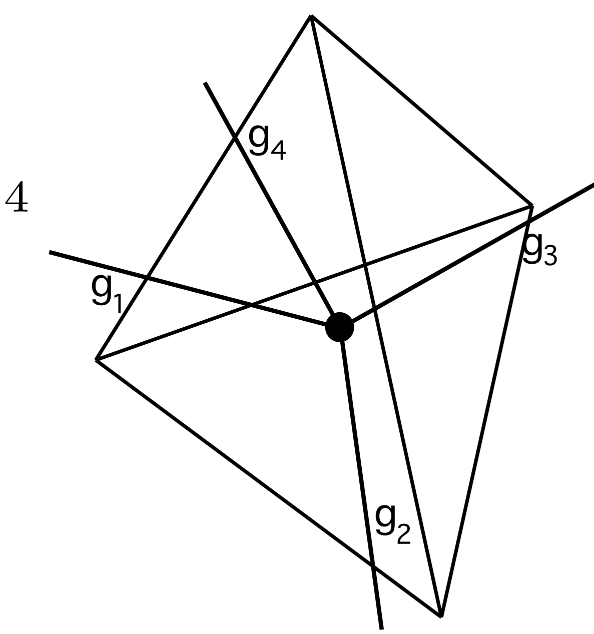
- equivalent formulation in terms of Lorentzian data + constraints

phase space:

$$(\mathcal{T}^* SO(3,1))^4 \simeq (\mathfrak{so}(3,1) \times SO(3,1))^4 \supset (\mathfrak{so}(3) \times SO(3))^4 \simeq (\mathcal{T}^* SO(3))^4$$



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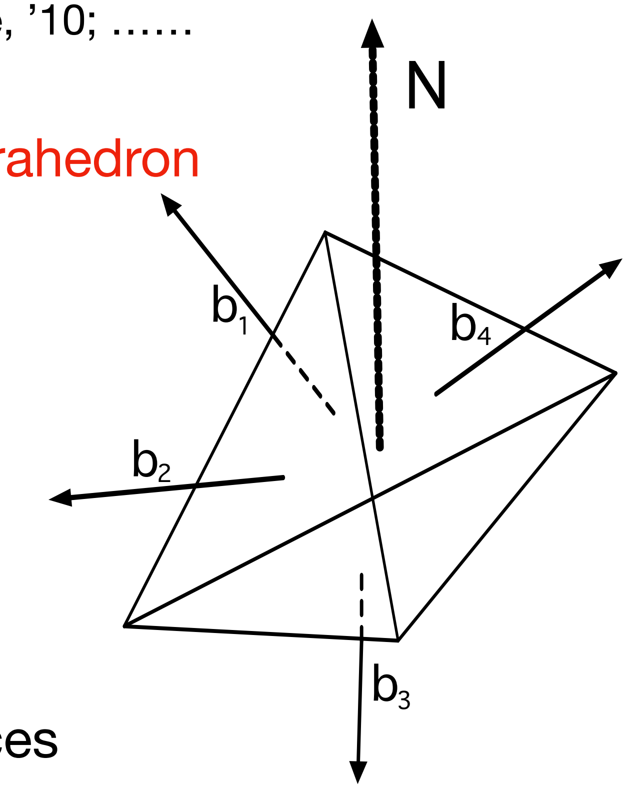

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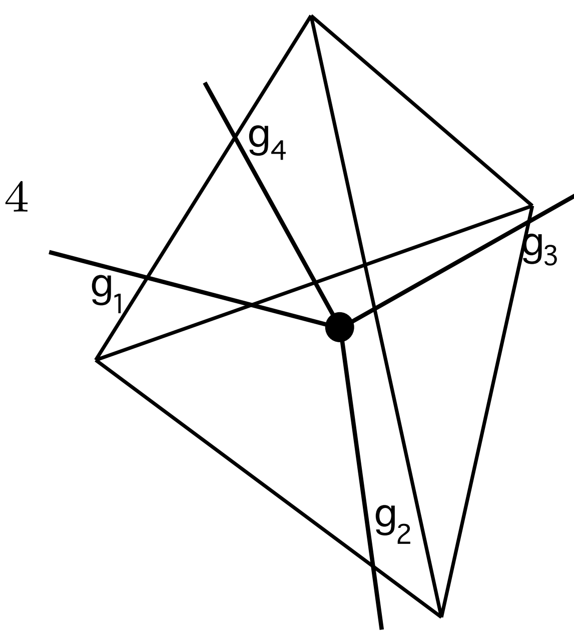

Quantum geometry in group-theoretic variables

Hilbert space

$$\mathcal{H}_v = L^2\left(G^d; d\mu_{Haar}\right)$$

+ constraints on states

wavefunction is thus tensor over product of d group manifolds

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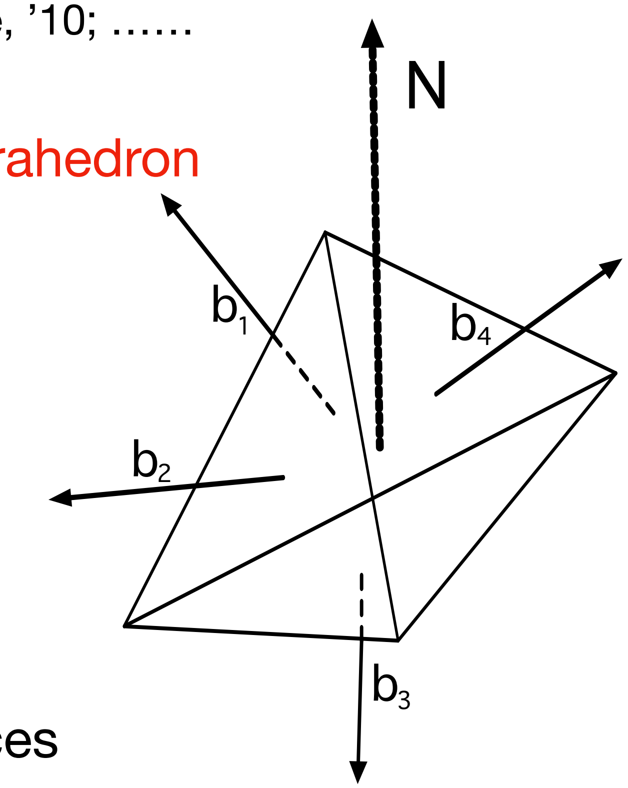

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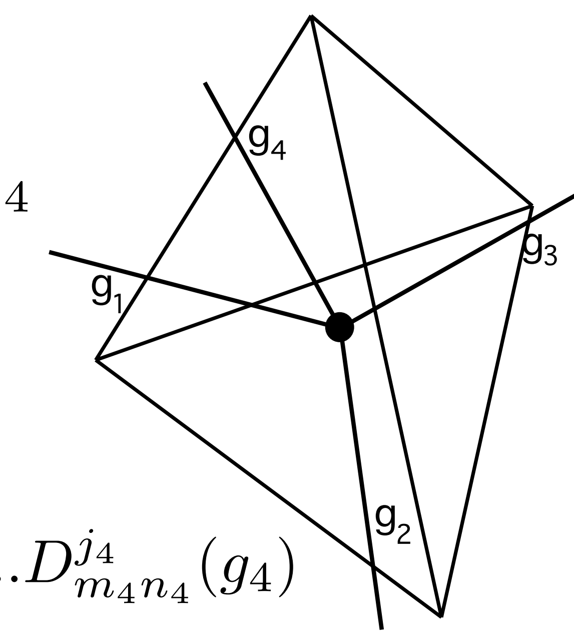

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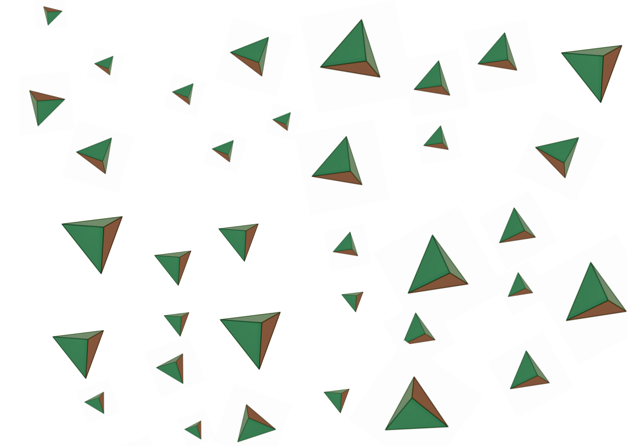
equivalent formulation in terms of irreps of G

$$\psi(g_1, g_2, g_3, g_4) = \sum_{j_i, m_i, n_i} \psi^{j_1 j_2 j_3 j_4}_{m_1 n_1 \ldots m_4 n_4} D^{j_1}_{m_1 n_1}(g_1) \ldots D^{j_4}_{m_4 n_4}(g_4)$$

spin network vertex

wavefunction is thus tensor over product of d group manifolds

Atoms of space



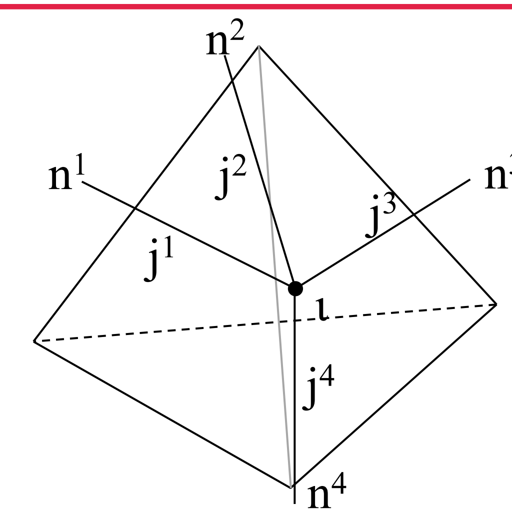
from quantizing the classical geometry of a single tetrahedron in 4d flat spacetime:

Hilbert space of quantum tetrahedron

(in terms of SU(2) irreps)

spin network vertex ~ quantum tetrahedron

$$\mathcal{H}_v = \bigoplus_{\vec{j}_v} \left(\bigotimes_{i=1}^{d} \underbrace{V^{j_v^i}}_{\text{repr. space}} \otimes \underbrace{\mathcal{I}^{\vec{j}_v}}_{\text{intertwiner space}} \right)$$



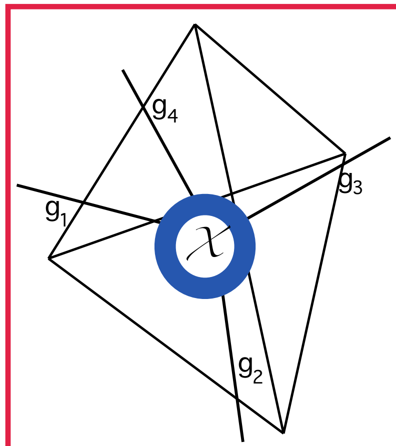
quantum geometric operators (triangle areas, volume,...) act on this Hilbert space

+ additional "geometricity" constraints that can be imposed at dynamical level

Hilbert space of quantum tetrahedron can be given different concrete representations:

$$\Psi(g_1, ..., g_4) = \Psi(g_1 h, ..., g_4 h) = \sum_{\{j_i, m_i; I\}} \Psi^{j_1 ... j_4; I}_{m_1 ... m_4} D^{j_1}_{m_1 n_1}(g_1) ... D^{j_4}_{m_4 n_4}(g_4) C^{j_1 ... j_4 I}_{n_1 ... n_4} \longrightarrow \mathcal{H}_v = L^2\left(\frac{SU(2)^{\times 4}}{SU(2)}\right)$$

Peter-Weyl decomposition



extension to "matter" dofs - example: scalar matter

- domain of wavefunctions extended to include values of scalar fields $\hat{\varphi}(g_I, \chi^a) \equiv \hat{\varphi}(g_I, \chi^1, \ldots, \chi^n)$

  with consequent extension of field operators, quantum states and operators on Fock space

Quantum states of many quantum tetrahedra

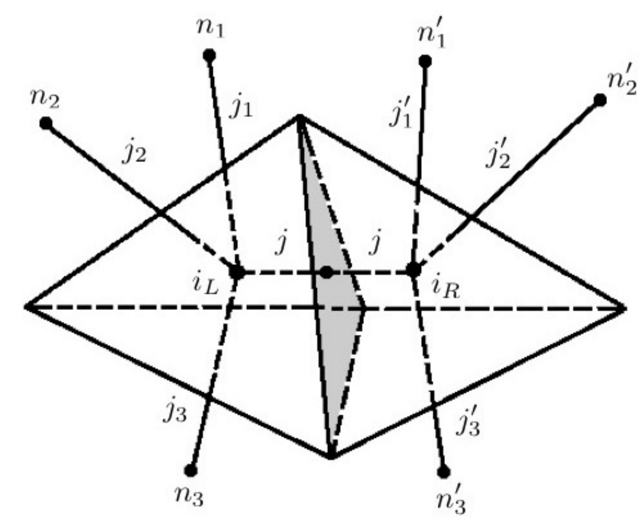
- full Hilbert space (arbitrary number of (connected or disconnected) tetrahedra):

GFT Fock space $$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym\left\{\left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)}\right)\right\} \supset \mathcal{H}_\gamma \quad \forall\gamma$$

  - GFT field operators (creating/annihilating spinnet vertices/tetrahedra):

$$\left[\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = \mathbb{I}_G(\vec{g},\vec{g}') \qquad \left[\hat{\varphi}(\vec{g})\,,\,\hat{\varphi}(\vec{g}')\right] = \left[\hat{\varphi}^\dagger(\vec{g})\,,\,\hat{\varphi}^\dagger(\vec{g}')\right] = 0$$

- gluing quantum tetrahedra with entanglement

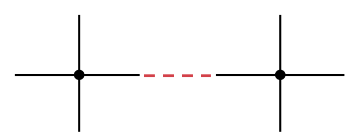


link state

quantum states for extended simplicial 3-complexes (spin network graphs) =
entangled many-body states of many quantum tetrahedra (spin network vertices)

gluing
(entangling edge dof)

$$|\psi\rangle \longrightarrow |\psi_\gamma\rangle = \left(\bigotimes_{e\in L}\langle e|\right)|\psi\rangle$$



many- body wave-function
for V open vertices

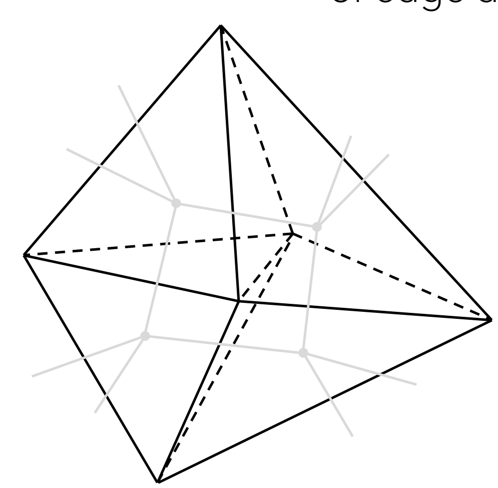
maximal entanglement

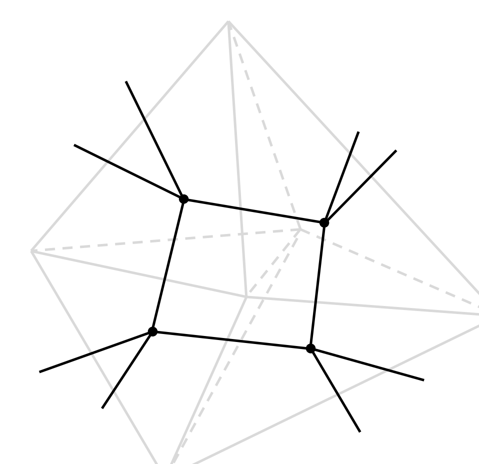
internal links
of combinatorial pattern γ

$$|e\rangle = \bigoplus_j \frac{1}{\sqrt{d_j}} \sum_n |jn\rangle \otimes |jn\rangle$$

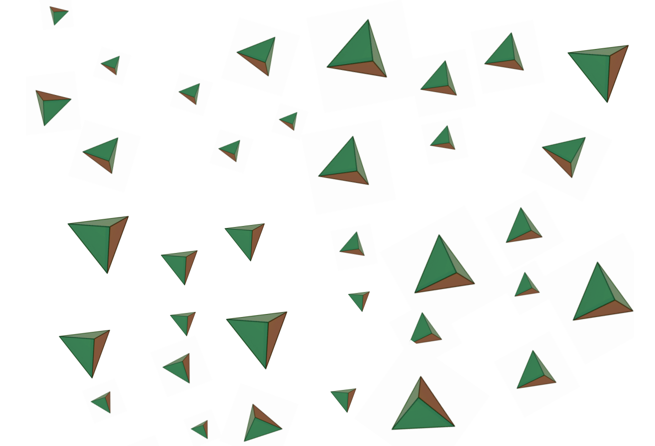
maximally entangled state
of edge degrees of freedom

- spin network states for arbitrary graphs
  ~ arbitrary quantum simplicial lattices





Observables

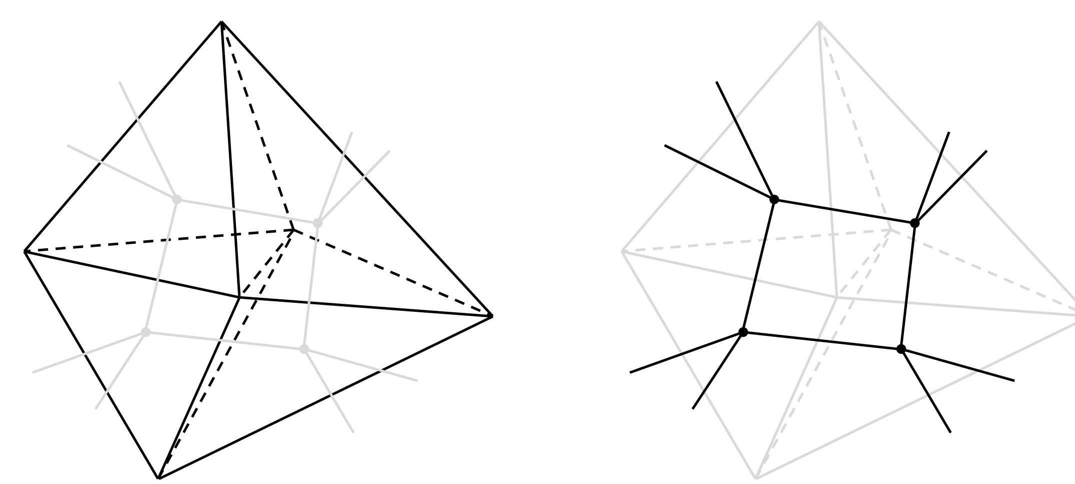


geometric quantities (areas, volume, curvature, ...) and observables for (scalar) matter become operators acting on the single-tetrahedron Hilbert space and then on the many-tetrahedra Fock space

examples:

- number operator $$\hat{N} = \int \mathrm{d}^n\chi \int \mathrm{d}g_I\, \hat{\varphi}^\dagger(g_I,\chi^a)\hat{\varphi}(g_I,\chi^a)$$
- universe volume $$\hat{V} = \int \mathrm{d}^n\chi \int \mathrm{d}g_I\, \mathrm{d}g'_I\, \hat{\varphi}^\dagger(g_I,\chi^a) V(g_I,g'_I)\hat{\varphi}(g'_I,\chi^a)$$
- value of scalar fields $$\hat{X}^b \equiv \int \mathrm{d}^n\chi \int \mathrm{d}g_I\, \chi^b \hat{\varphi}^\dagger(g_I,\chi^a)\hat{\varphi}(g_I,\chi^a)$$
- momentum of scalar fields $$\hat{\Pi}_b = \frac{1}{i}\int \mathrm{d}^n\chi \int \mathrm{d}g_I\, \left[\hat{\varphi}^\dagger(g_I,\chi^a)\left(\frac{\partial}{\partial\chi^b}\hat{\varphi}(g_I,\chi^a)\right)\right]$$

many-body operators are more complicated, like curvature
(eg holonomies along paths across several tetrahedra)



The universe as a quantum condensate
(in the TGFT formalism)

- which fundamental quantum dynamics?

TGFT dynamics

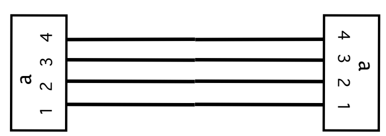
Boulatov, Ooguri, Barrett, Crane, De Pietri, Freidel, Krasnov, Rovelli, Reisenberger, Perez, DO, Livine, Baratin, Chirco, Girelli, Ryan, Gurau, Girelli, Rivasseau, ...

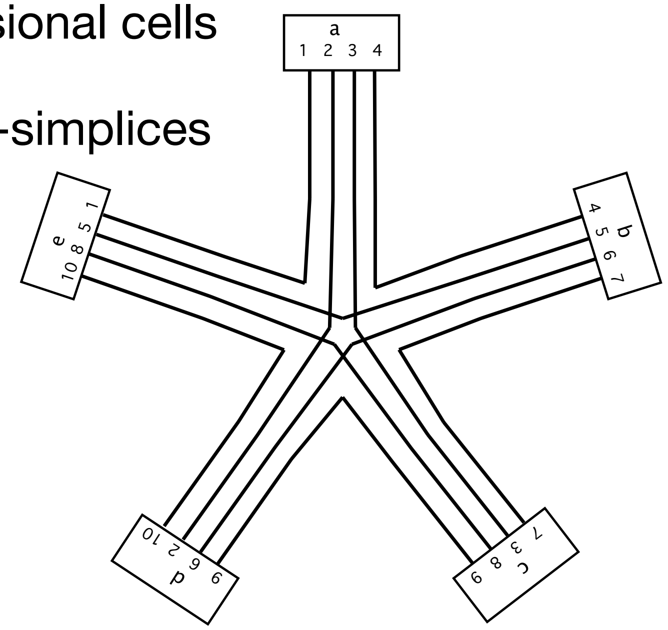
- main guideline for model building

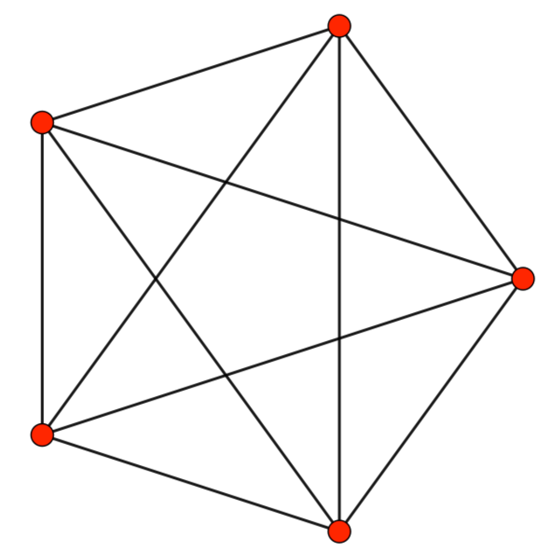
define models that produce, in perturbative expansion (i.e. where lattice structures are relevant),

- interaction processes (Feynman diagrams) of quantum simplices corresponding to 4d lattices,
- quantum (Feynman) amplitudes given by lattice gravity path integrals (coupled to scalar fields)

- general structure of action
  - interaction: gluing of 3-simplices to form 4-dimensional cells
  - kinetic term: gluing conditions for 4-cells across 3-simplices







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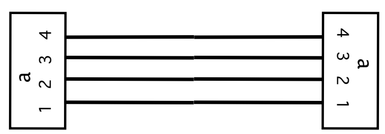

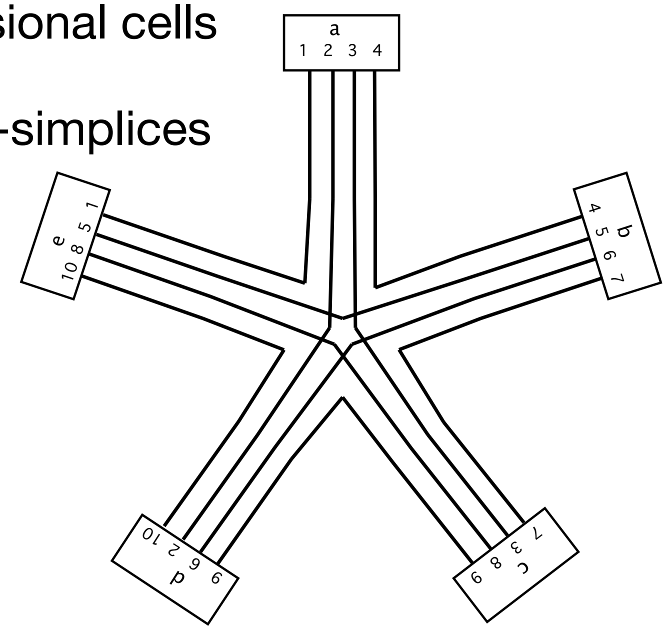

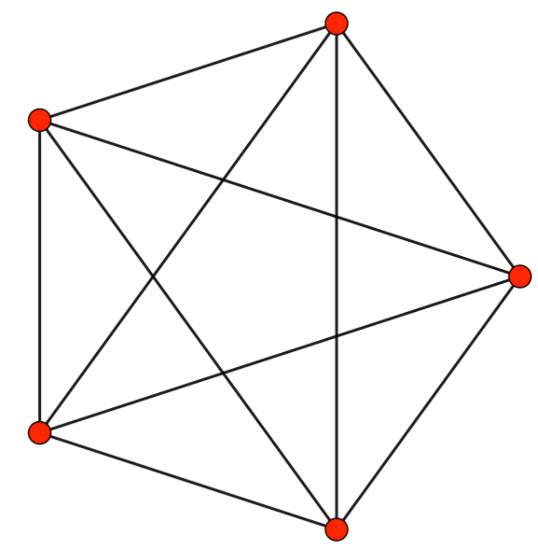

- example:

EPRL model

$$S = \sum_{j_{v_i}\ldots} \bar{\varphi}^{j_{v_1}\iota_1}_{m_{v_1}} \varphi^{j_{v_2}\iota_2}_{m_2} \left(\mathcal{K}_2\right)^{j_{v_1}j_{v_2}\iota_1\iota_2}_{m_{v_1},m_{v_2}} + V$$

$$V = \sum_{j_i,m_i,\iota_i} \left[\varphi^{j_1j_2j_3j_4\iota_1}_{m_1m_2m_3m_4} \varphi^{j_4j_5j_6j_7\iota_2}_{m_4m_5m_6m_7} \varphi^{j_7j_3j_8j_9\iota_3}_{m_7m_3m_8m_9} \varphi^{j_9j_6j_2j_{10}\iota_4}_{m_9m_6m_2m_{10}} \varphi^{j_{10}j_8j_5j_1\iota_5}_{m_{10}m_8m_5m_1} \times \tilde{\mathcal{V}}_5(j_1,\ldots,j_{10};\iota_1,\ldots,\iota_5) \right]$$

$$\tilde{\mathcal{V}}_5(j_{ab},i_a) = \sum_{n_a} \int d\rho_a (n_a^2+\rho_a^2) \left(\bigotimes_a f^{i_a}_{n_a\rho_a}(j_{ab}) \right) 15j_{SL(2,\mathbb{C})}\left((2j_{ab},2j_{ab}\gamma);(n_a,\rho_a)\right)$$

$$f^i_{n\rho} := i^{m_1\ldots m_4}\,\bar{C}^{n\rho}_{(j_1,m_1)\ldots(j_4,m_4)} \qquad \rho = \gamma n \qquad n = 2j$$

SL(2,C) data (and further geometricity conditions) in interaction term (encode covariant info)

TGFT dynamics: dynamics of quantum atomic geometry

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma$$

Feynman diagrams = stranded diagrams dual to cellular complexes of arbitrary topology

De Pietri, Petronio, '00; R. Gurau, '10; ...

random tensor models literature

labelled by group-theoretic data (group elements, group irreps, ...)

Feynman amplitudes (model-dependent) = sum over group-theoretic data associated to complex dual to Feynman diagram

Reisenberger,Rovelli, '00

A. Baratin, DO, '11

M. Finocchiaro, DO, '18

- GFT Feynman amplitudes = lattice gravity path integrals (in group/algebra variables) on lattice dual to GFT Feynman diagram = spin foam models (in irreps variables)

connection to lattice gravity in GFT perturbative expansion; non-perturbative GFT = continuum limit of lattice gravity

$$\mathcal{Z} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{i\,S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\mathcal{A}_\Gamma = \sum_\Delta w(\Delta)\int \mathcal{D}g_\Delta\; e^{i\,S_\Delta(g_\Delta)} \equiv \int \mathcal{D}g\; e^{i\,S(g)}$$

TGFTs: example

Boulatov model - topological 3d euclidean QG (no matter)

$\varphi : SU(2)^{\times 3} \to \mathbb{C}$ ~ quantum triangles

$$S(\varphi) = \frac{1}{2}\int [dg]\varphi^2(g_1, g_2, g_3) + \frac{\lambda}{4!}\int [dg]\varphi(g_1, g_2, g_3)\,\varphi(g_3, g_4, g_5)\,\varphi(g_5, g_2, g_6)\,\varphi(g_6, g_4, g_1) + \text{cc}$$

for fields satisfying: $\varphi(g_1, g_2, g_3) = \varphi(hg_1, hg_2, hg_3) \quad \forall h \in SU(2)$

partition function & perturbative expansion

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i.e. quantum covariant dynamics
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discretization of Palatini gravity: $S(e,\omega) = \int Tr(e \wedge F(\omega))$

The universe as a quantum condensate (in the TGFT formalism)

- continuum approximation and collective physics

Continuum approximation and collective physics

TGFTs: field theories of quantized simplices with non-local interactions (describing how simplices form higher-cells)

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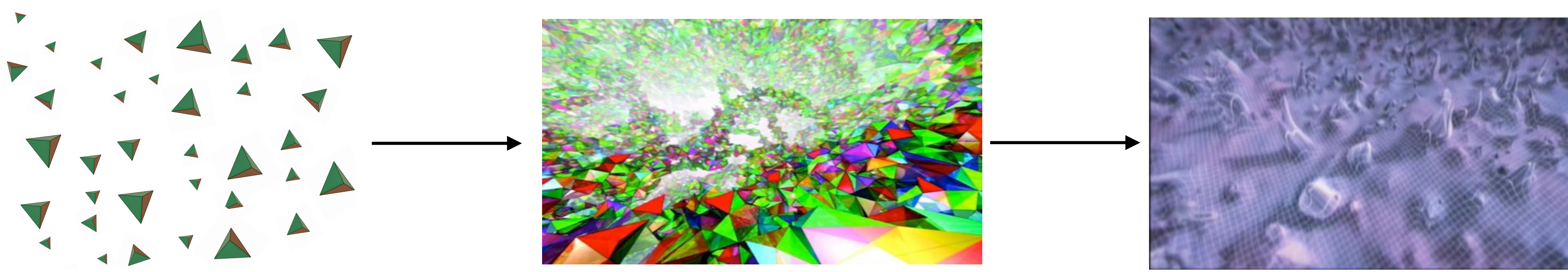
give definition of non-perturbative lattice gravity path integrals in their perturbative expansion

TGFTs can be well defined, controllable QFTs (renormalization, continuum limit, ...)

Ben Geloun, Carrozza, Tanasa, Toriumi, Krajewski, Martini, DO, Rivasseau,
Gurau, Lahoche, Ousmane-Samary, Benedetti, Marchetti, Pithis, Thürigen, ...

see S. Carrozza, 2404.07834 [math-ph]

key task is to control collective physics (in continuum approximation, upon coarse-graining)



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key task is to control collective physics (in continuum approximation, upon coarse-graining)

hypothesis: relevant regime is QG hydrodynamics

TGFT condensate hydrodynamics

hypothesis: geometric phase is QG condensate phase

(universe as QG fluid)

TGFT mean field hydrodynamics

$$F_\lambda(J) \;=\; \ln Z_\lambda[J] \qquad \Gamma[\phi] \;=\; sup_J\left(J\cdot\phi - F(J)\right) \qquad \langle\varphi\rangle = \phi$$

✳ simplest approximation: mean field hydrodynamics - infinite resummation of lattice gravity path integrals

$$\Gamma[\phi] \;\approx\; S_\lambda(\phi)$$

mean field ~ condensate wavefunction

- corresponding quantum states:

(simplest): GFT condensate, GFT field coherent state
- infinite superposition of discrete dofs

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle$$

$$\hat{\sigma} := \int d^4g\;\sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) \;=\; \sigma(g_I)$$

Universe as BEC (TGFT condensate)

The universe as a quantum condensate (in the TGFT formalism)

- TGFT condensate cosmology
- reconstruction of effective gravitational dynamics for (diffeo-invariant, relational) observables

TGFT condensate cosmology - general features

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- In quantum geometric TGFT models, cosmological interpretation of hydrodynamics is immediate:

isomorphism between domain of TGFT condensate wavefunction and minisuperpsace

$$\begin{aligned} \sigma\left(\mathcal{D}\right) \qquad\qquad \mathcal{D} &\simeq \{\text{geometries of tetrahedron}\} \simeq \\ &\simeq \{\text{continuum spatial geometries at a point}\} \simeq \\ &\simeq \text{minisuperspace of homogeneous geometries} \end{aligned}$$

S. Gielen, DO, L. Sindoni, '13

S. Gielen, '15

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- general form of resulting (Gross-Pitaevskii-like) equations of motion for condensate wavefunction (mean field):

$$\int [dg'] d\chi' \mathcal{K}(g,\chi;g',\chi')\sigma(g',\chi') + \lambda \frac{\delta}{\delta\varphi}\mathcal{V}(\varphi)|_{\varphi\equiv\sigma} = 0$$

Gielen, DO, Sindoni, '13; DO, Sindoni, Wilson-Ewing, '16

polynomial functional of condensate wavefunction

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Gielen, DO, Sindoni, '13; DO, Sindoni, Wilson-Ewing, '16

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cosmology as QG hydrodynamics ~ non-linear extension of (loop) quantum cosmology

that is, in isotropic restriction and with just one matter field, using standard variables on minisuperspace, i.e. scale factor and matter scalar field:

$\sigma(a,\phi)$ "wavefunction" on minisuperspace

$\mathcal{K}(a,\partial_a,\phi,\partial_\phi)\sigma(a,\phi) + \mathcal{V}\left[\sigma(a,\phi)\right] = 0$ hydrodynamic (non-linear, possibly non-local) eqn on minisuperspace

TGFT condensate cosmology - cosmological observables

need to turn general TGFT mean field equations into equations for cosmological observables

we have (many) operators acting on TGFT Fock space - need to turn them into "local" spacetime quantities

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apply relational strategy! can be done in different ways S. Gielen, A. Calcinari, '22, '23

operators defined in full QG theory

used to define collective relational (spacetime localized) observables

as expectation values in "good clock+rods" condensate states

(i.e. TGFT coherent states peaking on specific values of clock&rods scalars)

$$\boxed{\begin{aligned} N(x^0,x^i) &\equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{N} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle & V(x^0,x^i) &\equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \\ X^\mu(x^0,x^i) &\equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \simeq x^\mu & \Pi(x^0,x^i) &\equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi_\nu} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \\ \phi(x^0,x^i) &\equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{\Phi} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle & \Pi_\phi(x^0,x^i) &\equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi}_\phi | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \end{aligned}}$$

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$$N(x^0, x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{N} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \qquad V(x^0, x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

$$X^\mu(x^0, x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{V} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \simeq x^\mu \qquad \Pi(x^0, x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi_\nu} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

$$\phi(x^0, x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \hat{\Phi} | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle \qquad \Pi_\phi(x^0, x^i) \equiv \langle \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} | \widehat{\Pi}_\phi | \sigma_{\epsilon,\delta,\pi_0,\pi_x,x^\mu} \rangle$$

- eqn for condensate wavefunction ------> eqn for geometric/cosmological observables

(differential equations wrt to "clock-time"and "rod-space")

spatiotemporal dynamics for cosmological observables can be then compared with GR
(and models from theoretical cosmology)

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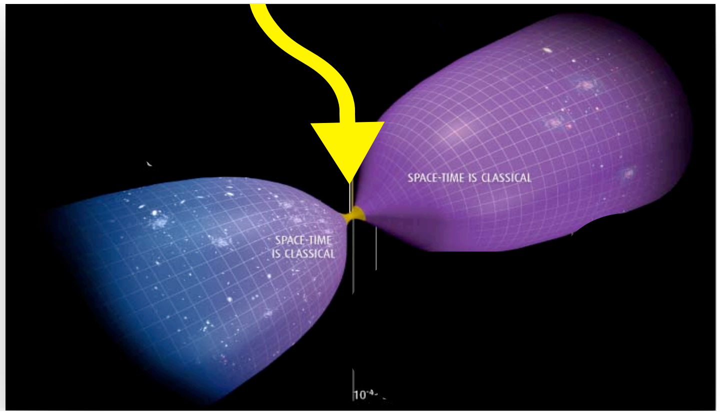
quantum bounce
(no big bang singularity)!

L. Sindoni, DO, E. Wilson-Ewing, '16; ....



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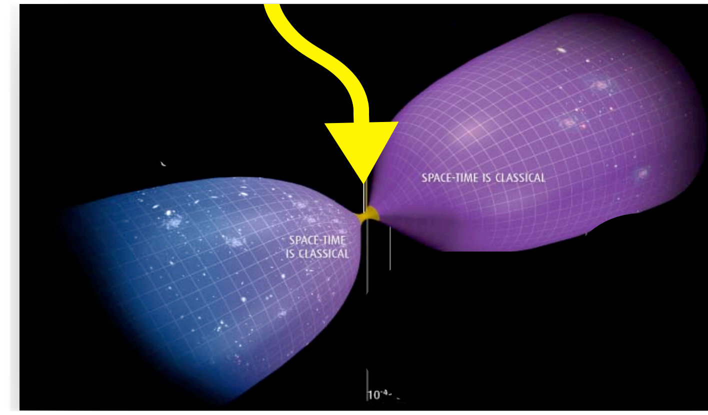

- intermediate times: large volume - QG interactions still subdominant

(here written neglecting matter contribution)

$$\boxed{\left(\frac{V'}{V}\right)^2 = \frac{V''}{V} = 12\pi\tilde{G}}$$

classical Friedmann dynamics in GR
(wrt relational clock, with effective Newton constant) - flat FRW

- special case: fixed-spin condensate - reproduces LQC dynamics

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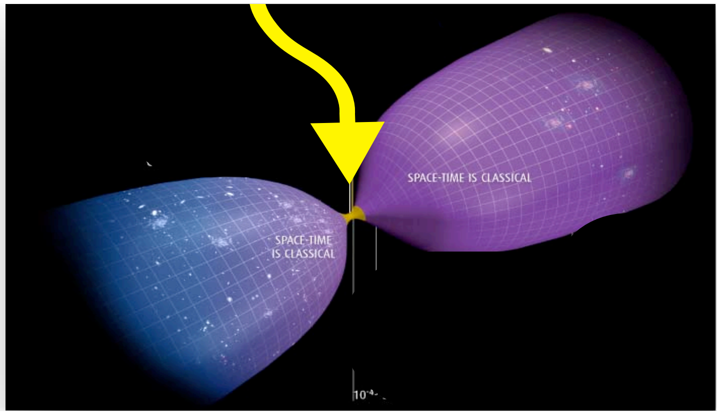

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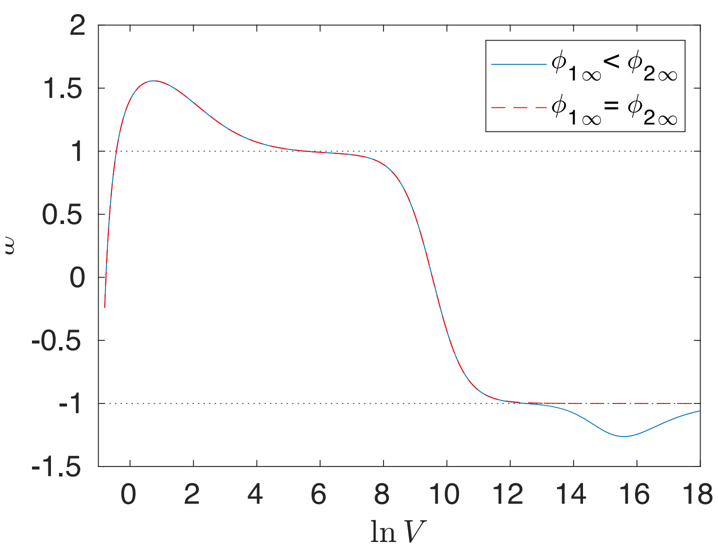

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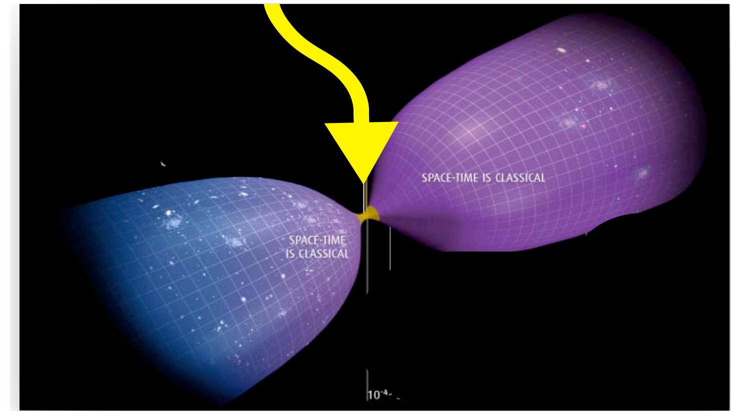

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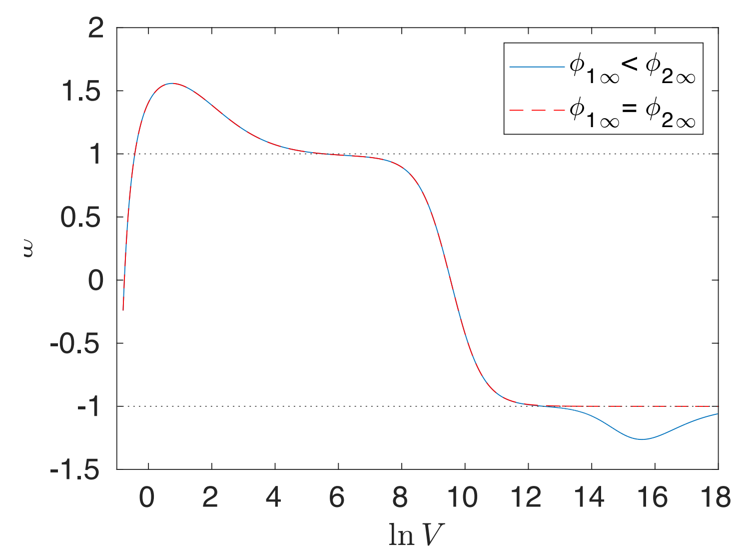

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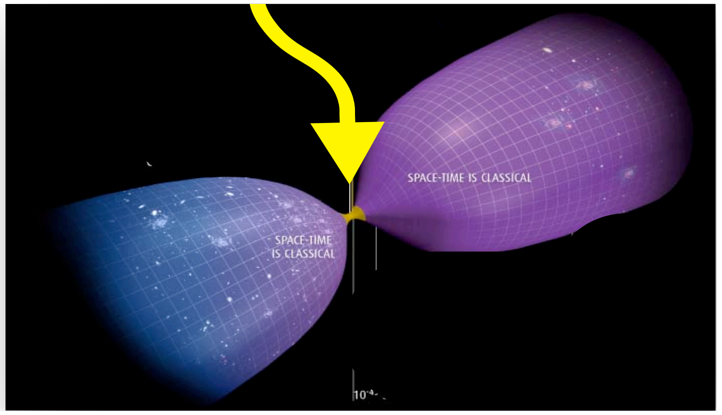

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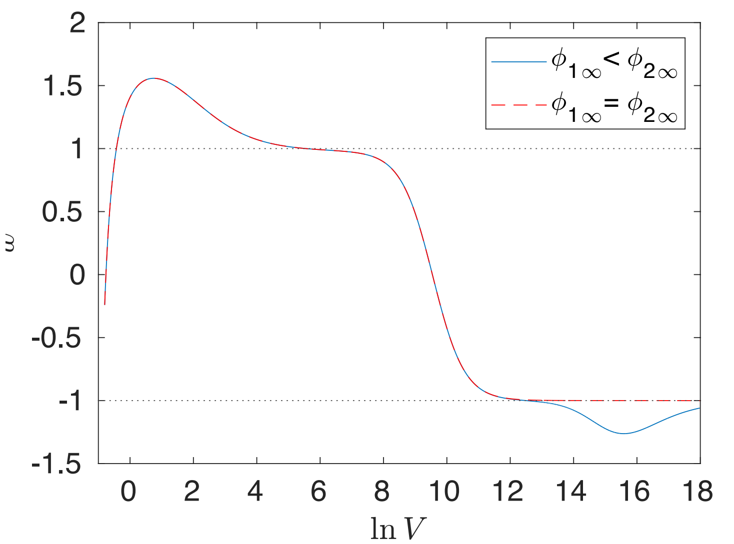

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- cosmological perturbations (inhomogeneities)

R. Dekhil, F. Gerhardt, S. Gielen, A. Jercher, S. Liberati, L. Marchetti, DO, A. Pithis, E. Wilson-Ewing, ...

- ....

Some lessons

1.Cosmology as hydrodynamic approximation of QG system: collective, coarse grained QG dynamics

- focus on cosmological dynamics = focus on few global observables = (from point of view of QG theory as well as local effective theory (GR)) focus on collective variables and collective states = result of coarse graining

- symmetry reduction = drastic coarse graining (sharp removal of (infinite) dofs) + restriction to few observables

- cosmological wavefunction on minisuperspace = order parameter labelling collective state, not quantum state
  no corresponding Hilbert space of "quantum cosmology" within larger Hilbert space of QG states

$$\langle\Omega|\widehat{\varphi}|\Omega\rangle = \Psi(a,\phi) \qquad \Psi(a,\phi) = \rho(a,\phi)e^{i\theta(a,\phi)}$$

- relevant observables are matched with continuum gravitational physics as averages, not eigenvalues

$$a_U = \int a\,\rho^2(a,\phi)$$

- important role, in observables and dynamics, of "number of QG dofs": modulus of cosmological wavefunction = average number operator (literally, in GFT), new observable wrt standard quantum cosmology

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2. Cosmology is emergent dynamics, taking form of non-linear extension of (loop) quantum cosmology

- dynamics of cosmological wavefunction = non-linear extension of LQC, i.e. QG hydrodynamics on minisuperspace

$$\mathcal{K}(a,\phi)\,\Psi(a,\phi) + \lambda_i \int \mathcal{V}(a_1,\phi_1;...;a_i,\phi_i)\Psi(a_1,\phi_1)\cdots\Psi^*(a_i,\phi_i) = 0$$

- gravitational couplings (in emergent cosmological dynamics) are functions of underlying QG ones (not directly gravitational)

- "standard" quantum cosmology setting obtained when neglecting non-linear corrections, i.e. underlying QG interactions, or linearizing the full dynamics - here, in GFT, FRW is reproduced at large volumes

- non-linear contributions important; example: in GFT can produce cosmic acceleration (phantom-like dark energy, inflation, asymptotic deSitter universe) without additional dofs

- large-scale effects of direct QG origin and linked to small-scale effects (failure of EFT intuition and principles)

3. Cosmological evolution is relational evolution with respect to physical clock

- relational evolution requires conditions of (good) clock, implemented as conditions on relevant quantum states
- which clock? possible mismatch between "fundamental" and "effective" clock dofs
  eg: "massless free scalar field" at fundamental level =/= massless free scalar field at effective (hydrodynamic) level
- how does the cosmological physics (e.g. bounce, cosmological perturbations, etc) depend on the choice of clock?
- what is invariant under change of physical clock? what is general covariance with respect to transformations between (realistic) physical frames? what is invariant?

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4. Also spatial localization and thus local physics, including cosmological perturbations, is relational

- cosmological perturbations: spatial localization to be defined also via relational strategy, i.e. by additional rod fields
  - dynamics obtained for each choice of relational frame should be compared with GR dynamics in particular gauge

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$\Longrightarrow$ Most principles of QFT and GR are emergent & approximate (locality, unitarity, local causality, ...)

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6. Effective cosmological dynamics is affected by RG flow (and critical behaviour of underlying QG dynamics

- effective cosmological dynamics takes place within one particular phase of underlying QG system, thus need to check values of couplings, their running and what happens close to criticality
- bounce (when reproduced) happens in hydrodynamic approximation of full QG
  - assuming universe volume is an extensive quantity, this happens at small values of (average) number operator, thus in the "danger zone" for validity of hydrodynamic approximation
  - bounce scenario may not be viable and one may need to resort to more fundamental QG dynamics with no fully "geometric/spatiotemporal description
- cosmological interpretation of underlying QG phase transitions? if early universe is close to QG phase transition, what is impact on cosmological observables, e.g. CMB spectrum?

Thank you for your attention!