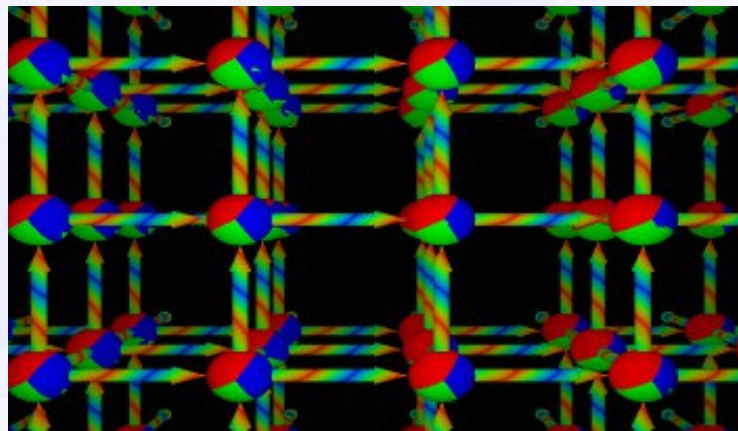
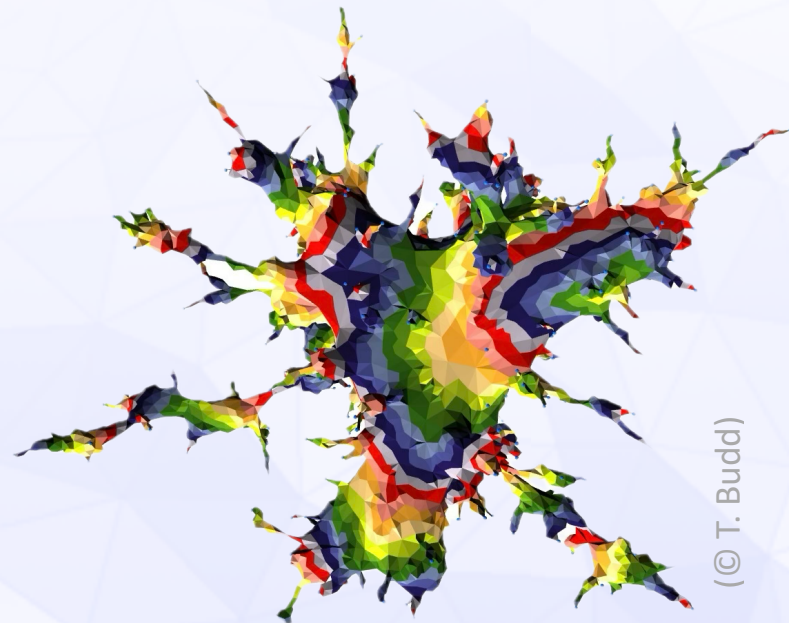


Causal Dynamical Triangulations: Lattice Quantum Gravity Reloaded



(© G. Bergner)



(© T. Budd)

Renate Loll, Radboud University (NL)

Causal Fermion Systems

Regensburg, 7 Oct 2025

What's the problem with quantum gravity?

We are still searching for a fundamental quantum theory of gravity and spacetime (and matter?), which unifies the principles of quantum theory and general relativity, and produces *numbers*.

root problem: perturbative Quantum Gravity (QG) based on the split $g_{\mu\nu}(x) = \underbrace{\eta_{\mu\nu}^{\text{Mink}}}_{\text{flat metric}} + \underbrace{h_{\mu\nu}(x)}_{\text{perturbation}}$ of the spacetime metric is not renormalizable

G. 't Hooft & M. Veltman (1974), M. Goroff & A. Sagnotti (1985), A. v.d. Ven (1992)

⇒ option 1: we need to go beyond perturbation theory (but how?)

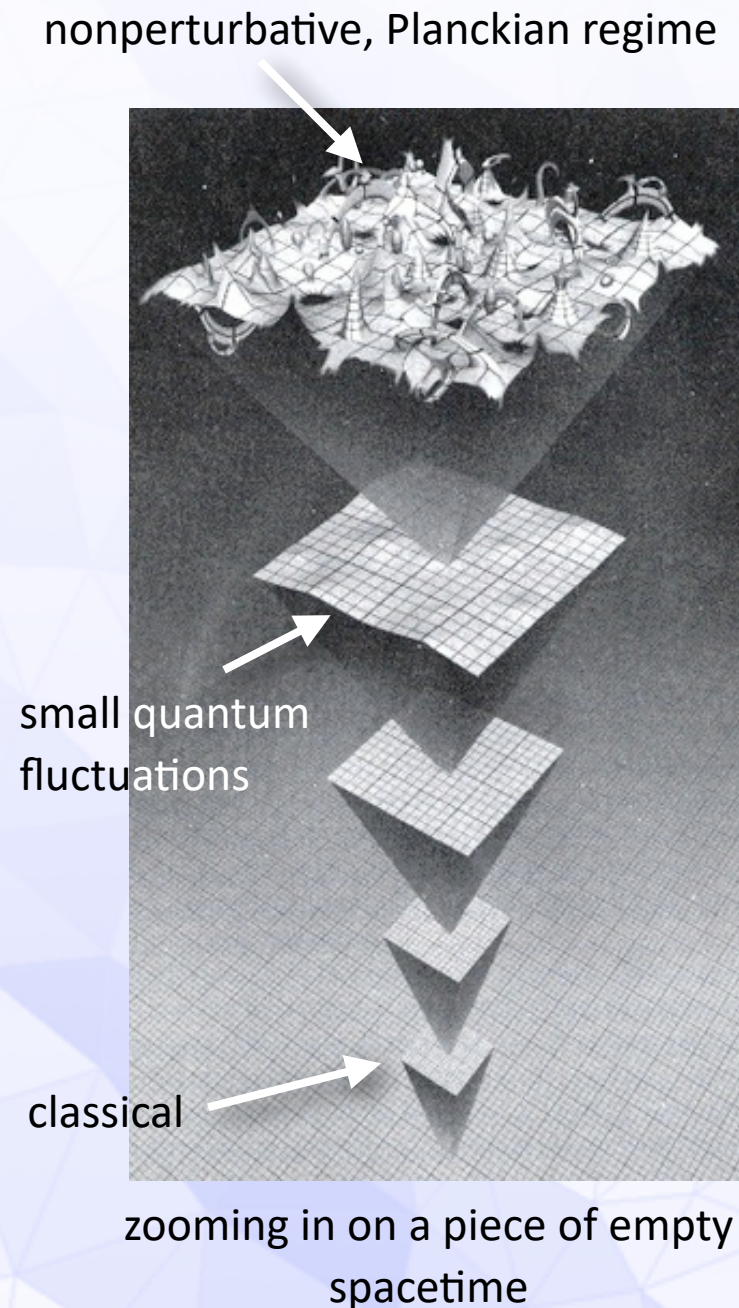
⇒ option 2: a “radically new idea” is needed (but which one?)

birth of QG “approaches”: construct QG using extraneous ingredients, beyond the d.o.f. and symmetries of GR and/or beyond QFT (e.g. extended or discrete fundamental objects, extra dimensions, ...)

⇒ 40 yrs of ever richer approaches, largely detached from physical phenomenology, *and* a growing wishlist of questions that QG should answer

J. Armas, “Conversations on Quantum Gravity”, CUP 2021

Take quantum spacetime foam more seriously!



- beyond perturbation theory: quantum fluctuations near ℓ_{Pl} are those *of spacetime itself*
- in this regime, coordinate systems, the metric $g_{\mu\nu}$ and standard GR tools are *not* applicable!
- by what mechanism should spacetime-as-we-know-it “*emerge*” from such a quantum foam?
- can we compute *anything*, and *how*?
- with available theoretical and computational technology, what can it possibly mean to *solve* a strongly interacting, nonperturbative 4D QFT of dynamical spacetime (or worse)?
- *no experiments or observations* to provide reality checks and keep us honest

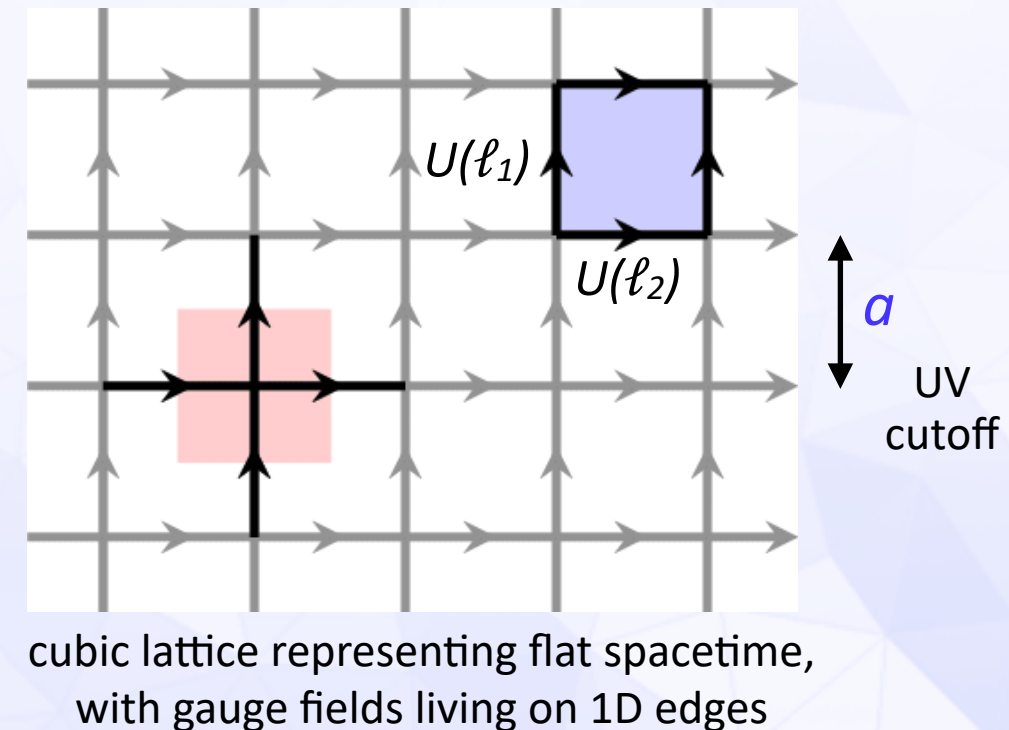
for a larger context, see R. Loll et al.: “Quantum Gravity in 30 Questions”, arXiv:2206.06762

Preview of key results

- we have learned how to put 4D QG (the gravitational path integral) on the lattice and investigate it ***quantitatively beyond perturbation theory***
- ***attractive***: brings QG back into the fold of ***QFT***, w/o exotic ingredients
- it has opened a unique window on ***new and unexpected Planckian physics***, where we can simulate quantum spacetime (QST) reliably
- computing from first principles has produced ***blueprints of QST and emergence***, and a ***new perspective on what QG can realistically deliver***
- ***numerical ‘experiments’*** are an intermediate step to phenomenology, in a regime that *necessarily* has limited analytical control (unlike in 2D)
- road map to phenomenology: ***measuring diffeomorphism-invariant observables*** can inform our understanding of the ***very early universe***
- post-Riemannian: ***nonsmooth, but continuous*** metric spaces; ***no $\hat{g}_{\mu\nu}$*** , but can measure geodesic ***distances and volumes***

Recalling the power of lattice gauge theory

- lattice regularization of the path integral is the go-to methodology for analyzing relativistic QFT *nonperturbatively*, emulating the formidable successes of *lattice QCD*
- **Strategy:** lattice acts as *regulator*, with UV cutoff a ; search for a continuum limit at a 2nd-order phase transition as $a \rightarrow 0$; *no* fundamental discreteness, but *universality* (independence of regularization details)
- breakthrough in Yang-Mills theory: use edge *holonomies* $U(\ell) = P \exp \int_{\ell} A$, which transform under an *exact action* of the gauge group SU(3), despite the discretization! ($A = \text{gauge connection}$)
- attaining just a fraction of the achievements of lattice QCD in quantum gravity would be amazing — can it be done?



K. Wilson, PRD 10 (1974) 2445

Lattice quantum gravity 1.0

Put the *gravitational path integral* (“sum over histories”),

$$Z = \int \mathcal{D}g \, e^{iS_{\text{grav}}[g]}$$

“geometry” $\rightarrow g \in \frac{\text{Lor}(M)}{\text{Diff}(M)}$

on the lattice, following in Wilson’s footsteps.

L. Smolin, “Quantum Gravity on the Lattice”, Nucl. Phys. B 148 (1979) 333:

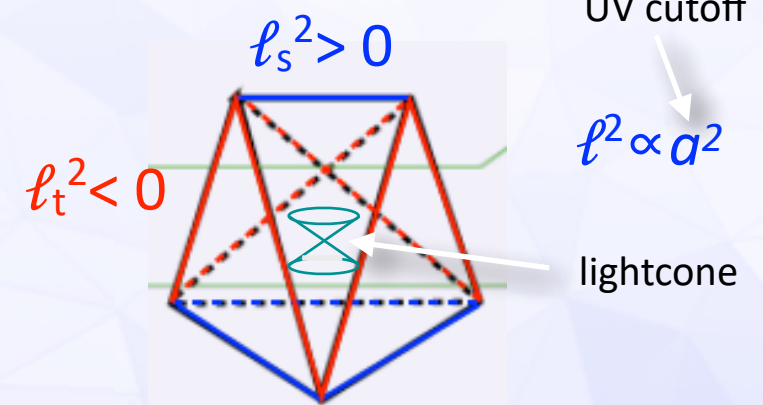
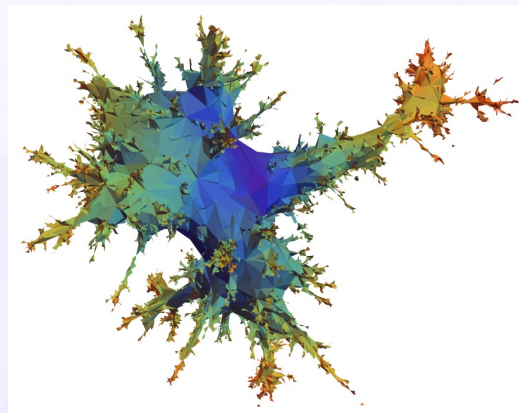
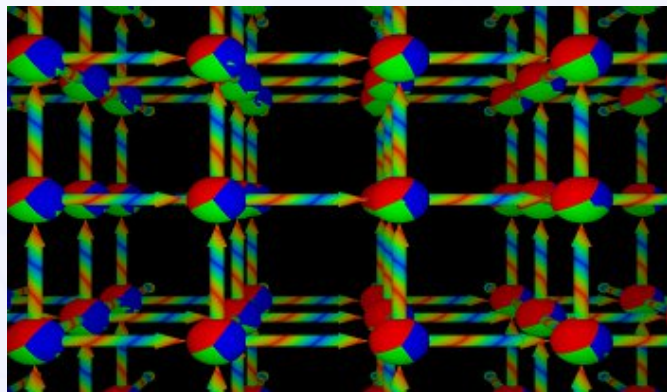
When one begins to think about formulating general relativity on a lattice one runs into an immediate conceptual difficulty. In general relativity the geometry of spacetime itself is dynamical and subject to quantum fluctuations and it is difficult to imagine how to make a lattice theory in which the lattice structure itself is dynamical. This

1980s: first-order, “gauge-theoretic” formulations (vierbein e_μ^I + spin connection ω_μ^{IJ}); **problems:** measure? ‘compactified gravity’? Wick rotation? conformal divergence? where is diffeomorphism symmetry?

Monte Carlo (MC) simulations of lattice QG 1.0 found nothing interesting.

Lattice QG 2.0: triple breakthrough, or, why it took us 40 years

- **Quantum Gravity is not easy:** “incompatibility” of gravity and quantum (field) theory regarding the role of spacetime
⇒ **key 1:** use dynamical, curved instead of rigid, cubic lattices



flat Minkowskian building block in 4D CDT

- **analogue for diffeomorphisms of Wilson’s exact gauge invariance?**
⇒ **key 2:** dynamical triangulations built from equilateral building blocks have an exact labeling invariance; ***no coordinates are needed***
- **need a “Wick rotation” to make the path integral amenable to MC**
⇒ **key 3:** causal dynamical triangulations can be analytically continued

Lattice quantum gravity à la CDT in a nutshell

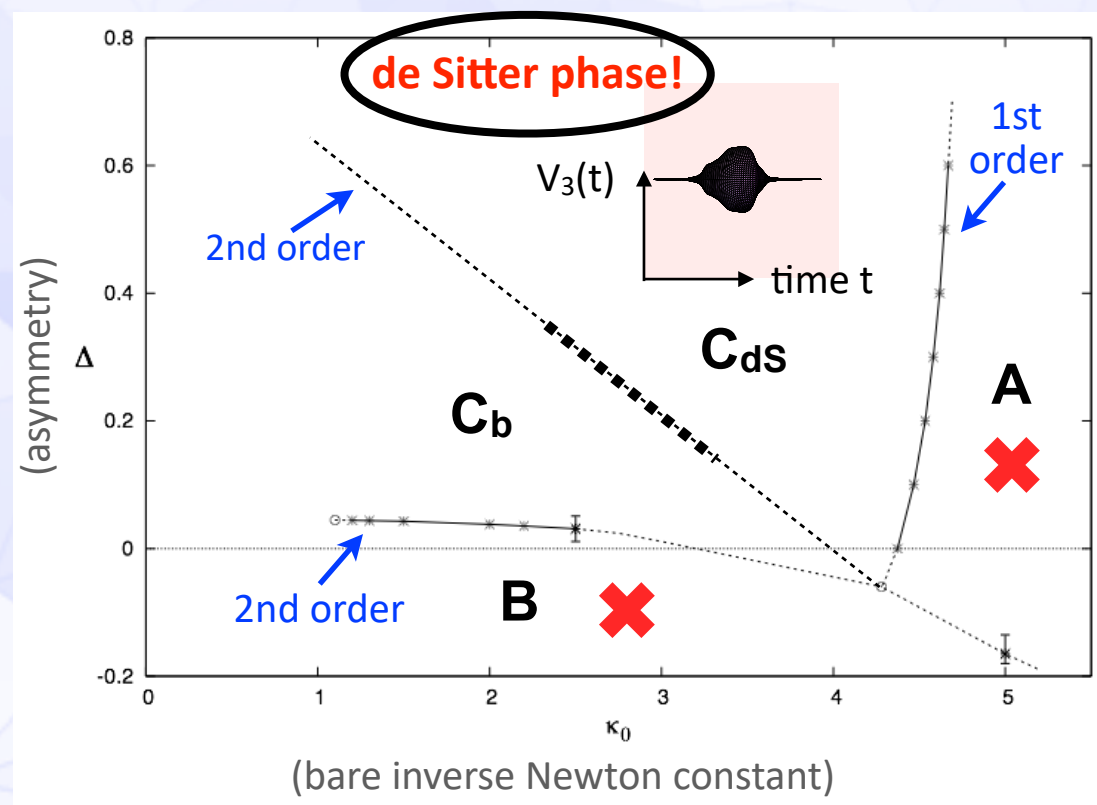
$$Z = \lim_{a \rightarrow 0} \sum_{\substack{\text{causal} \\ \text{triang. } T}} \frac{1}{C(T)} e^{iS^R[T]}, \quad S^R[T] = -(\kappa_0 + 6\Delta)N_0(T) + \kappa_4 N_4(T) + \Delta N_{41}(T)$$

symmetries of T
bare gravitational lattice action
Regge version of the Einstein-Hilbert action

vertices
four-simplices

○ = bare coupling constants

- continuum limit $a \rightarrow 0$, $N_4 \rightarrow \infty$, for finite physical volume $V = N_4 a^4$
- path integral histories obey global hyperbolicity (**causal** structure)
- transfer matrix is reflection positive $\Rightarrow$ **unitary** continuum theory



- Z^{CDT} is computed by Monte Carlo methods after analytic continuation
- phase diagram: 2nd-order transitions!
- can provide independent evidence for UV fixed points (**asymptotic safety**)

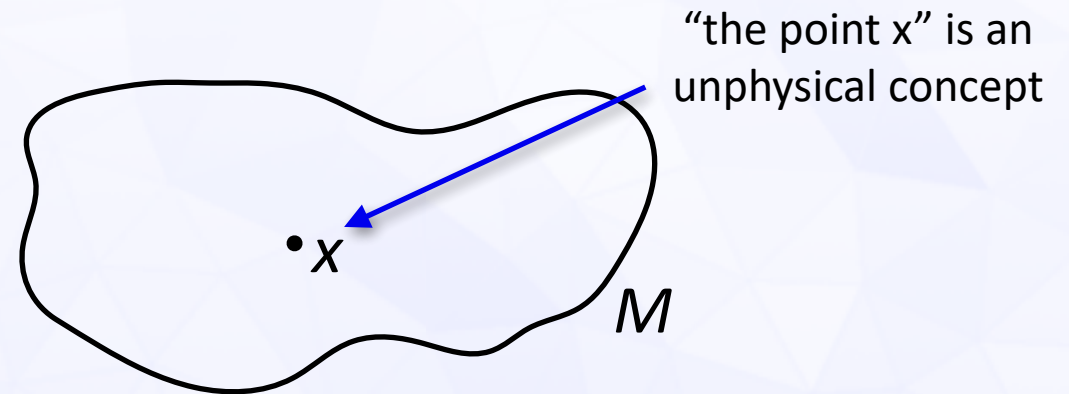
S. Weinberg (1979), M. Reuter & F. Saueressig, PRD 65 (2002) 065016; CDT: J. Ambjørn, A. Görlich, J. Gizbert-Studnicki & D. Nemeth, PRD 110 (2024) 126006, and arXiv: 2411.02330

Lattice quantum gravity à la CDT: Results

- the physics of *quantum spacetime* is captured by the expectation values of diffeomorphism-invariant *quantum observables* $\hat{\mathcal{O}}$:

$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z} \int \mathcal{D}g \, \mathcal{O}[g] e^{-S_{\text{grav}}[g]}$$

- observables $\mathcal{O}$ are typically nonlocal integrals of scalars, like $\int_M d^4x \sqrt{g} R(x)$



- more than that, **there are no meaningful reference frames @ ℓ_{Pl} !**
- “expectation management”: many popular quantum gravity questions do **not** have a Planckian implementation and cannot be defined operationally in a **nonperturbative** background-independent, highly quantum-fluctuating setting; **this is a feature, not a bug!** (*)
- Planckian physics does **not** conform to classical “intuition” and cannot be extrapolated from perturbative/effective quantum gravity

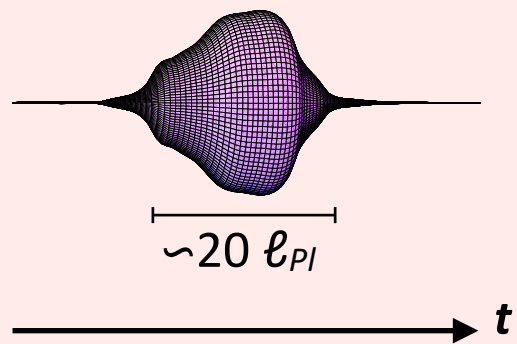
(*) can ask me about black holes later!

Breakthrough result: “emergent classicality”

The measured large-scale Hausdorff & spectral dimension and the shape $\langle V_3(t) \rangle$ (3-volume as function of proper time) of the quantum universe (ground state of the path integral) match those of a classical 4D de Sitter space, **although no background or symmetry assumptions were put in.**

J. Ambjørn, A. Görlich, J. Jurkiewicz, R. Loll, PRL 100 (2008) 091304; N. Klitgaard & R. Loll, Eur. Phys. J. C80 (2020) 990

MC snapshot of the shape $\langle V_3(t) \rangle$ of the universe



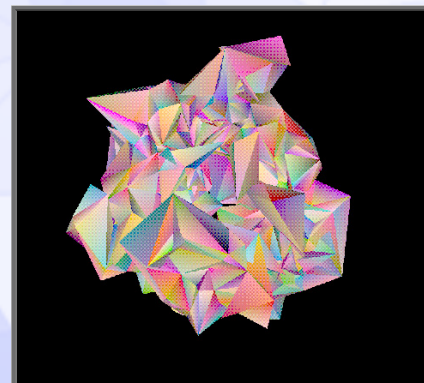
N.B. local fluctuations are **not** depicted here

We find the shape $\langle V_3(t) \rangle \propto \cos^3(ct)$ of a (Euclidean) de Sitter universe. However, **locally** this quantum spacetime is **not at all** approximated by the metric

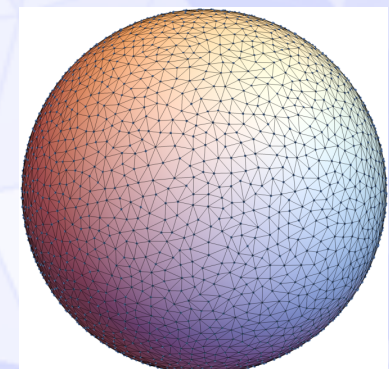
$$ds^2 = dt^2 + c^2 \cos^2(t/c) d\Omega^2$$

of a homogeneous and isotropic de Sitter cosmology.

Local quantum spacetime still looks like a crazy quantum foam



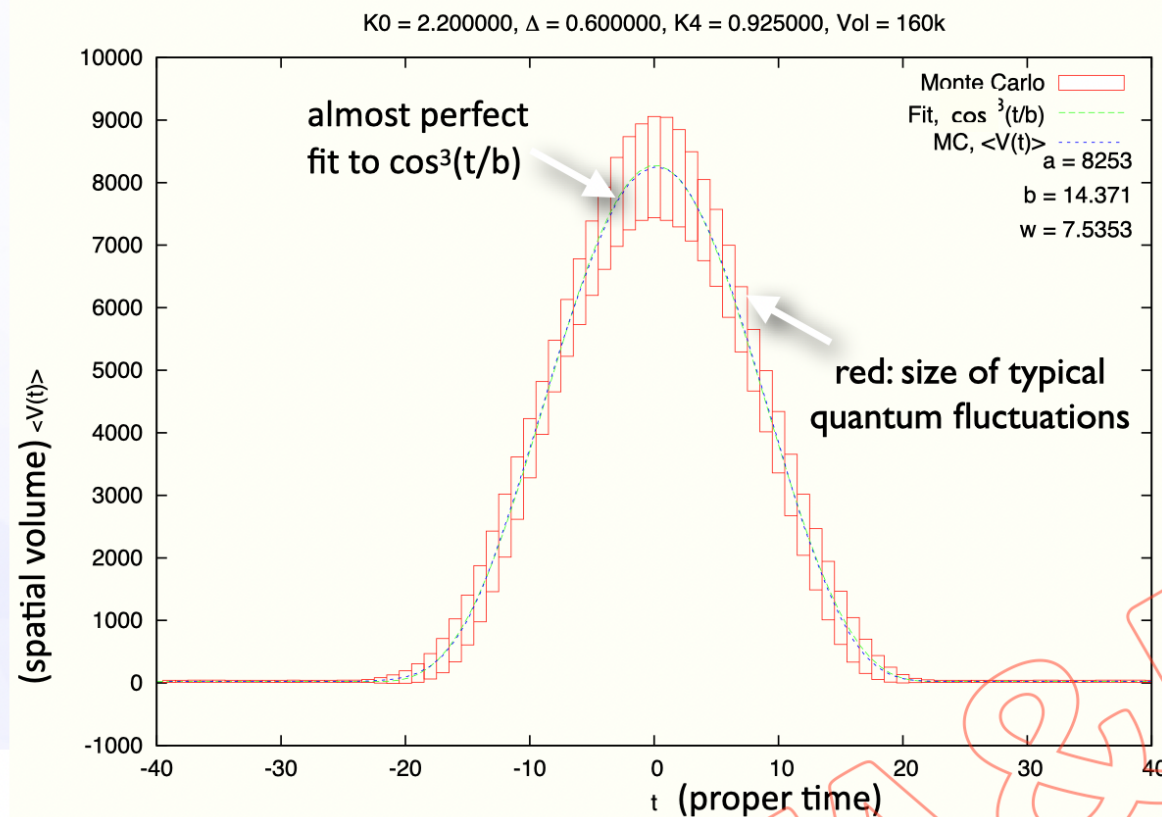
, not like



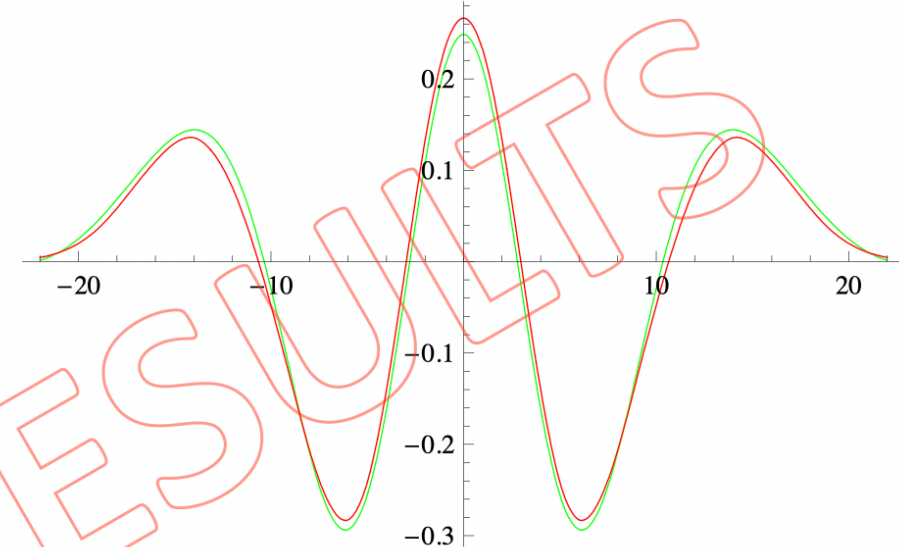
!

Quantum gravity as an “experimental” science

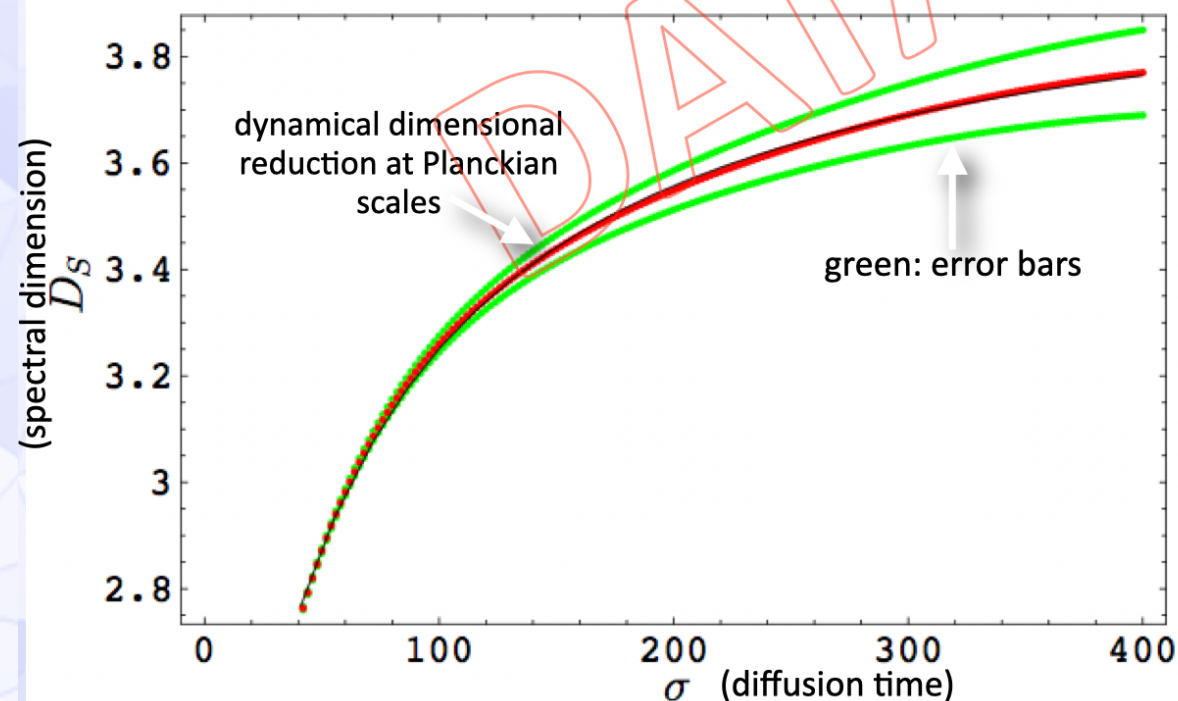
The universe is de Sitter-shaped



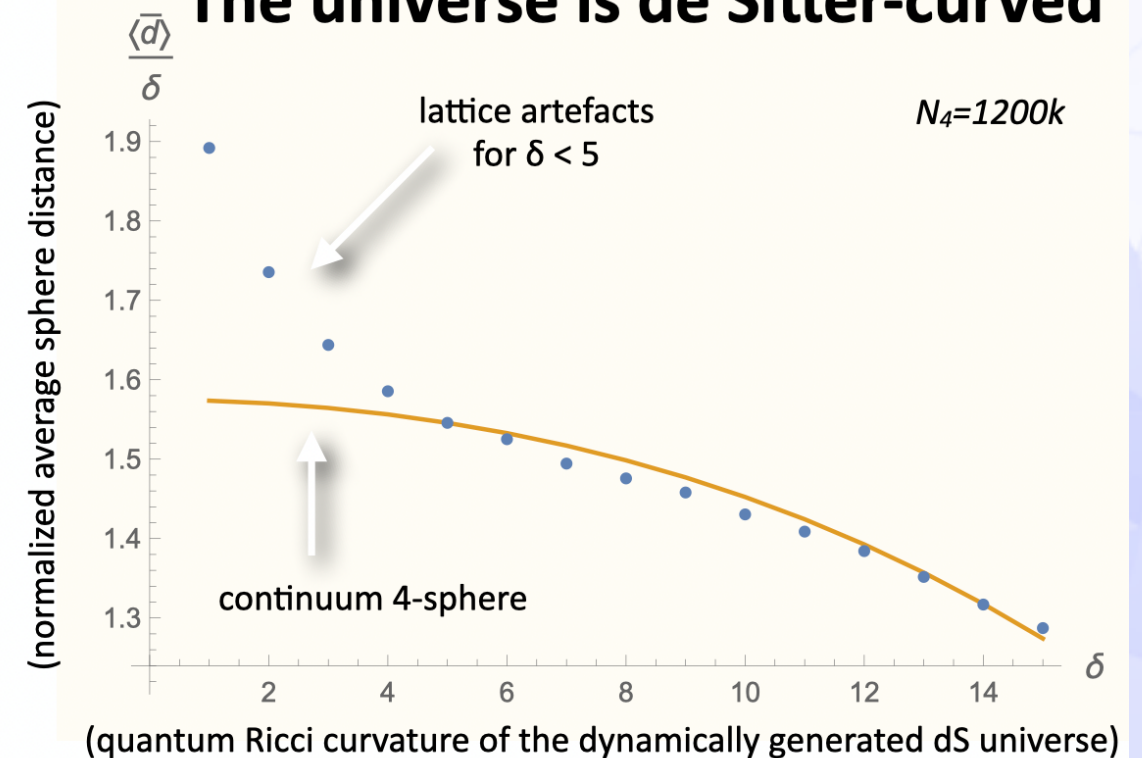
Volume fluctuations around de Sitter



Spectral dimension of the universe



The universe is de Sitter-curved

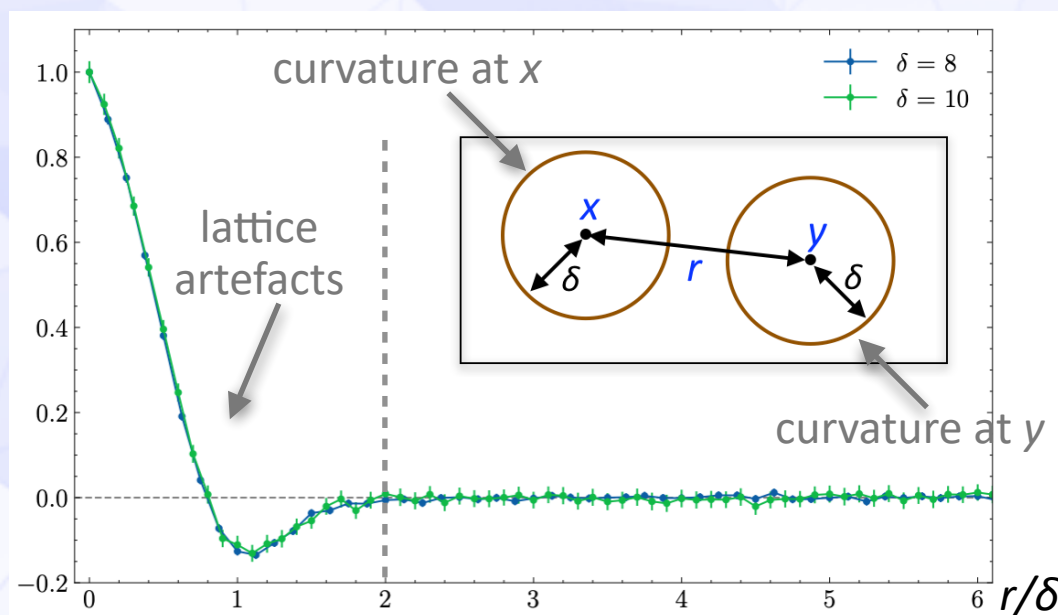


Zoom in on the emergent quantum universe ...

... by measuring more fine-grained observables:

- classical de Sitter space is maximally symmetric; can homogeneity and isotropy of quantum spacetime “emerge”? we have constructed ***nonperturbative homogeneity measures*** A. Silva, R.L., PRD 107 (2023) 086013
- fluctuations/inhomogeneities are captured by ***diffeomorphism-invariant two-point functions*** of local scalars $\mathcal{O}$: operator dependence!

$$\langle \mathcal{O}\mathcal{O} \rangle(r) = \frac{1}{Z} \int \mathcal{D}g \, e^{-S[g]} \underbrace{\int d^4x \sqrt{g(x)}}_{\text{x-integral}} \underbrace{\int d^4y \sqrt{g(y)}}_{\text{y-integral}} \underbrace{\mathcal{O}(x)\mathcal{O}(y)}_{\text{operators at x \& y}} \overbrace{\delta(d_g(x,y)-r)}^{\text{operator dependence!}}$$



$\langle (K_q(x) - \bar{K}_q)(K_q(y) - \bar{K}_q) \rangle$ in 2D CDT on T^2 , $N=100k$

curvature-curvature correlator in 2D CDT: no correlations for $r > 2\delta$

J. vd Duin & R. Loll, Eur. Phys. J. C84 (2024) 7

new: use tools from TDA to characterize local quantum geometry

J. vd Duin, R. Loll, M. Schiffer & A. Silva, appears tomorrow

Summary and outlook

- ***computer-assisted theory construction***: new, data-driven insights; lattice QG is ‘just’ nonperturbative QFT, not an ‘approach’; may be the ***primary gateway*** to the strongly coupled quantum regime @ ℓ_{Pl}
- ***what fundamental quantum gravity can deliver***: observables tailored to the Planck regime — universal behaviour, scaling relations, new quantum signatures, what does (or doesn’t) ‘emerge’
- ***quantum gravity is not “GR with hats”*** — rich NP dynamics, related to new mathematics of beyond-Riemannian and random geometry
- ***path to early-universe phenomenology***: look for emergence of symmetry (homogeneity, isotropy) and structure (correlators)! test standard assumptions of cosmology! derive new predictions!

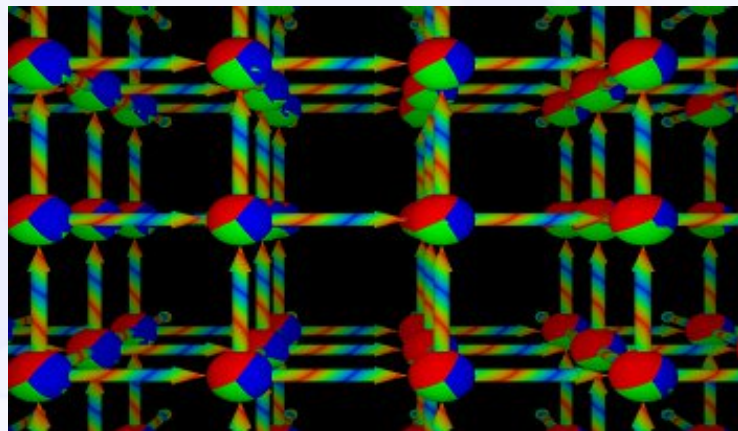
Reviews of CDT lattice quantum gravity:

J. Ambjørn, A. Görlich, J. Jurkiewicz, R.L., Phys. Rep. 519 (2012) 127, arXiv:1203.3591

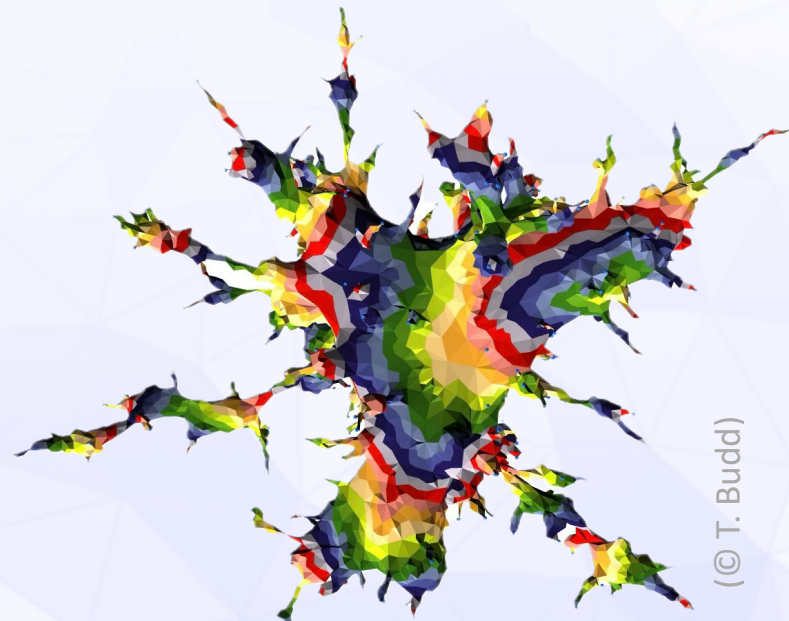
R.L., Class. Quant. Grav. 37 (2020) 013002, arXiv:1905.08669

J. Ambjørn, R.L., Encyclopedia of Mathematical Physics, arXiv:2401.09399

Causal Dynamical Triangulations: Lattice Quantum Gravity Reloaded



(© G. Bergner)



(© T. Budd)

Thank you!

Causal Fermion Systems

Regensburg, 7 Oct 2025

So what about black holes?

Aren't black holes (and BH thermodynamics) supposed to be the **key to quantum gravity**? From a nonperturbative perspective, apparently **not**.

Entropy and area in the Bekenstein-Hawking formula $S_{BH} = A/4$ are semi-classical and **not operationally defined @ ℓ_{Pl}** .

The nonperturbative ground state of lattice QG resembles a de Sitter space, not a black hole. — Anyway, what **is** a black hole @ ℓ_{Pl} ?

Shouldn't nonperturbative QG reproduce semi-classical gravity? **No**.

There is **no empirical evidence for SCG**, although its results are widely believed.

(cf. talks by E. Curiel)

In the bifurcation phase of CDT QG, there is a string-like structure, which **may** be a “seed” of a primordial BH.

(G. Clemente & R. Loll, w.i.p.)

