Towards a Completion of Quantum Mechanics[1]

"It seems clear that the present quantum mechanics is not in its final form." (P.A.M. Dirac)

Regensburg, October 10, 2025

[1] Jürg Fröhlich, formerly at ETH Zurich

Contents

1. Dispelling a Curse
2. Diffusion and the Theory of Random Walks
3. The stochastic evolution of individual systems in QM
4. The *ETH*-Approach to Quantum Mechanics (QM)

Concluding Remarks

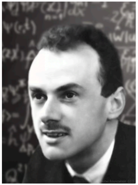
—

The heroes of the year 1925

Heisenberg

Born

Jordan



Dirac

Summary

To cast Quantum Mechanics - incomplete in its text-book version - in a final form, we must understand how to derive the *stochastic dynamics* of *individual isolated open systems* from fundamental physical principles (which are at the core of the so-called *ETH*-Approach to Quantum Mechanics, with *E* standing for "events," *T* for "trees," and *H* for "histories").

I show how, in the non-relativistic limit (with the velocity of light $c \to \infty$), this problem is solved for simple systems of charged matter interacting with the quantized electromagnetic field. An important insight is that the masslessness of photons plays a key role in this endeavor.

1. Dispelling a Curse

"The interpretation of quantum mechanics has been dealt with by many authors, and I do not want to discuss it here. I want to deal with more fundamental things." (P.A.M. Dirac)

Quantum Mechanics (QM) is not a *"Theory of Everything"* – *QM is a theory that has changed everything in our understanding of the microcosm* – for good! – Yet,

QM is afflicted by a curse: Most people who use it in their daily work appear to be unable to coherently describe its meaning, e.g., to say what is meant by **events** and to explain how events arise, such as the successful measurement of a physical quantity, the fluorescence of atoms, decay of unstable nuclei, appearance of particle tracks in a detector, etc.

They view q-m time evolution as *linear and deterministic*, but go on to claim that QM is fundamentally *probabilistic*, as suggested by Einstein (1916) and Born (1926). These seemingly contradictory features of the theory have given rise to a plethora of so-called *"interpretations of QM"* (*Copenhagen, Many-Worlds, Q-bism, ...*) and of *modifications* of QM intended to explain the emergence of events, among which I mention *Bohmian mechanics, GRW, cellular automata*, etc.

Looking for a stochastic Law of Dynamics

"The constant element in physics, since Newton, is not a configuration or a geometrical form, but a Law of Dynamics." (Werner Heisenberg)

I don't know of any compelling "interpretation of QM," and I have not learned of a satisfactory modification/extension of QM – which is why I endeavor to propose my own *attempt towards a completion of QM*.

In text-book QM, the time evolution of *states* is described by a *Schrödinger-von Neumann equation*, or, if dissipative processes are at work, by *Lindblad-type eqs.* – **unless** a *"measurement"* intervenes causing *"wave-function collapse"* governed by *Born's Rule* (Born, von Neumann, Lüders). – However, measurements are *q-m processes,* too, involving interactions between a "small system" and a macro system triggering a measurement process; so why is it not possible to describe the evolution of the **total** system by a *Schrödinger-von Neumann eq.*?

What appears to be correct is to claim that states averaged over a very large ensemble, $\mathcal{E}$, of identical, identically prepared physical systems – *"ensemble states"* – evolve in time according to a *linear, deterministic law*. But states of **indvidual** systems in $\mathcal{E}$ evolve according to a *(non-linear) stochastic Law of Dynamics.* The nature of this law is the subject of ongoing controversies. – It is the **subject of this talk**.

2. Diffusion and the Theory of Random Walks

To set the stage for our endeavor I first sketch the solution to a simpler problem: Consider a *gas of non-interacting particles* on a lattice $\mathbb{Z}^\nu$, $\nu = 1, 2, \dots$, subject to thermal noise. The state of the gas at time t is described by the *density*, ρ_t, of particles. This is the analogue of an *ensemble state* in QM.[2] The *linear deterministic evolution equation* of the state ρ_t is the *diffusion equation*, namely

$$\dot{\rho}_t(x) = D\big(\Delta\rho_t\big)(x) = D\Big[\sum_{y:|y-x|=1} \rho_t(y)\Big] - 2\nu D\,\rho_t(x)\,, \quad x \in \mathbb{Z}^\nu\,, \quad (1)$$

where D $(=k_B T/(6\pi\eta\, r))$ is the diffusion constant, Δ is the discrete Laplacian, and the sum on the right side of (1) extends over all sites y that are nearest neighbors of x (indicated by $|y-x| = 1$).

We would like to understand what kind of *stochastic motion* of an **individual** particle, ξ, in the gas implies that the time-dependence of an *"ensemble state"* ρ_t is given by the diffusion equation (1).

The answer can be found by "unraveling" Eq. (1):

[2] given by a density matrix Ω

The stochastic motion of *individual* particles

"Ontology"

The "true" state of an **individual** particle ξ at an arbitrary time t is a **site**, $x_\xi(t) \in \mathbb{Z}^\nu$ – its position at time t. It corresponds to a density $\rho_t(x;\xi) := \delta_{x_\xi(t)}(x)$, $x \in \mathbb{Z}^\nu$, i.e., to a *"pure state"*, where $\delta_y(x) \equiv \delta_{yx}$ is the Kronecker δ.

During a time interval $[t, t+dt)$, a particle ξ may remain at $x_\xi(t)$, or it may jump to a nearest-neighbor site $y = x_\xi(t) + \delta$, where δ is a unit vector (i.e., its state may "jump" from $\delta_{x_\xi(t)}$ to δ_y). The diffusion eq. (1) determines the *probabilities* of these different options:

At time $t + dt$, the state, $x_\xi(t+dt)$, of the particle is given by

$$x_\xi(t+dt) = \begin{cases} x_\xi(t), & \text{with prob. } p_{nj}[t, t+dt] := 1 - 2\nu D\, dt\,, \\ y = x_\xi(t) + \delta, & \text{with prob. } p^\delta[t, t+dt] := D\, dt\,, \end{cases} \tag{2}$$

This *Poisson jump process* is a discrete version of the *Wiener process*.

Law (2) for the stochastic motion of an **individual** particle ξ implies that, when taking an average over the states of **all** particles in the gas, the time-dependence of the ensemble state, ρ_t, is given by the diffusion eq..

Some applications of Law (2)

[A trajectory of ξ, is described by a *simple random walk* $\big(\xi(0), \dots \xi(n)\big)$, $\xi(j) \in \mathbb{Z}^\nu$, $\forall j$; $n \geq 0$ is the number of jumps ξ performs between an initial time $t_0 = 0$ and (final) time $t > 0$. Imagine that the particle ξ makes a nearest-neighbor jump in each time interval $[t_j, t_j + dt_j]$, $j = 1, \dots, n$, tracing out a random trajectory $\underline{\xi}$. According to (2), the probability of $\underline{\xi}$ is given by

$$\mathrm{prob}\big[\underline{\xi}, dt_1 \cdots dt_n\big] = \Big(\prod_{j=1}^{n} e^{-2\nu D(t_j - t_{j-1})} D \, dt_j \Big) \, e^{-2\nu D(t - t_n)}$$

This is the lattice analogue of the *Wiener measure* in the theory of *Brownian motion*. It implies the diffusion equation (1) for the "ensemble state" ρ_t and has many further applications. For example, it yields a formula for the average distance traveled by ξ in time t:

$$\mathbb{E}\big[x_\xi(t) - x_\xi(0)\big]^2 = \sum_n \underbrace{\mathbb{E}\big[\xi(n) - \xi(0)\big]^2}_{=n} \quad \cdot \quad \underbrace{\frac{(2\nu D \cdot t)^n}{n!} e^{-2\nu D \cdot t}}_{\text{prob. to make } n \text{ jumps in time } t - t_0}$$

$$= 2\nu D \cdot t \qquad (\textit{diffusive motion with entropy prod.})]$$

3. The stochastic evolution of individual systems in QM

Inspired by the example of the *diffusion equation* (1) "unraveled" by the *Poisson jump process* describing the stochastic motion of a single particle ξ in a gas (Law (2)), we present a q-m *stochastic Law of Dynamics* of **individual** systems obtained by "unraveling" a q-m analogue of the diffusion equation.

We think of a very large ensemble, $\mathcal{E}$, of systems, all identical to a specific system S, consisting of *matter interacting with the quantized radiation field*. In the non-relativistic limit, $c \to \infty$, *ensemble states* of S are described by *density matrices*, Ω, acting on a separable Hilbert space $\mathcal{H}$. As shown in the next section, the time evolution of an ensemble state, Ω_t, is described by a *Lindblad equation,* which is *linear* and *deterministic*:

$$\Omega_{t+dt} = \Omega_t + \mathcal{L}\big[\Omega_t\big]\,dt + \mathcal{O}(dt^2), \qquad \text{with}$$

$$\mathcal{L}\big[\Omega\big] \overset{e.g.}{=} -i\big[H_0, \Omega\big] + \alpha \sum_{k=1}^{K} \Big[V_k \Omega V_k^* - \frac{1}{2}\{\Omega, V_k^*\, V_k\}\Big], \tag{3}$$

“Unraveling” the Lindblad equation

In (3), $\mathcal{L}$ is the Lindblad generator, H_0 is the Hamiltonian of S when matter is decoupled from the radiation field, $V_1, \dots, V_K$ are (“jump”) operators on $\mathcal{H}$, which describe *dissipative effects* arising from interactions of matter with the radiation field, and $\alpha \propto e^2$.

“Ontology”

As in the example of diffusion theory, we have to ask what the state of an **individual** system in $\mathcal{E}$ is. A conventional idea is that it is *pure* at all times t, i.e., given by a rank-1 orth. projection, Π_t, on $\mathcal{H}$.[3] If, at time t, **all** systems in $\mathcal{E}$ were prepared in Π_t the *ensemble state* at time $t + dt$ would be given by the density matrix

$$\Omega_{t+dt} = \Pi_t + \mathcal{L}\big[\Pi_t\big]\, dt + \mathcal{O}(dt^2)\,.$$

The spectral decomposition of Ω_{t+dt} has the form

$$\Omega_{t+dt} = p_{nj}[t, t+dt]\, \Pi^0_{t+dt} + \sum_{\delta>0} p^\delta[t, t+dt]\, \Pi^\delta_{t+dt}\,, \qquad (4)$$

where $\big\{\Pi^\delta_{t+dt} \,\big|\, \delta = 0, 1, 2, \dots\big\}$ are the spectral projections of Ω_{t+dt},

[3] or proportional to a finite-rank orth. projection

State reduction

and "*nj*" stand for "no jump". These spectral projections represent the *possible states* of an **individual** system at time $t + dt$, with the coefficients interpreted as *probabilities* (or frequencies – *Born's Rule*), as explained in Sect. 4. Suppose that Π_t is a rank-1 orthogonal projection (i.e., a *pure* state). Assuming that dt is very **small**, we then find that

$$\begin{gathered}
\Pi^0_{t+dt} \approx \Pi_t\,, \qquad p_{nj}[t, t+dt] \equiv p^0[t, t+dt] = 1 - \mathcal{O}(dt) \approx 1\,, \\
p_{nj} > p^1 > p^2 > \cdots > 0, \quad \text{with} \quad p^\delta = \mathcal{O}(dt),\ \forall\ \delta \geq 1\,, \\
p_{nj}[t, t+dt] + \sum_{\delta>0} p^\delta[t, t+dt] \dim(\Pi^\delta_{t+dt}) = 1\,.
\end{gathered} \tag{5}$$

Our task is to find explicit expressions for the probability $p_{nj}[t_1, t_2]$ of **not** observing a quantum jump in an interval $[t_1, t_2)$ of times, for the time-dependence of the state Π^0_t, with $t \in [t_1, t_2)$, and for the probability of observing a *"quantum jump"* at time t_2 to a state $\Pi^\delta_{t_2}$, $\delta > 0$, *perpendicular* to $\Pi^0_{t_2}$, for arbitrary times $t_1 < t_2$. This is accomplished with a novel tool of functional analysis called *Infinitesimal Perturbation Theory* (IPT).

Explicit formulae for p_{nj} and Π_t^0, using IPT

Eq. (4) & IPT $\Rightarrow$ (non-linear) *stochastic Law of Dynamics* for states of **individual** systems: [Between two *"quantum jumps"*, at times t_1 and t_2,

$$(i) \qquad p_{nj}[t_1, t_2] = \exp\Big\{ \int_{t_1}^{t_2} \mathrm{Tr}\big(\Pi_t^0\, \mathcal{L}[\Pi_t^0]\big)\, dt \Big\} < 1\,,$$

$$(ii) \qquad \frac{d\Pi_t^0}{dt} \overset{dt \to 0}{=} \big[\Pi_t^0\big]^{\perp} \mathcal{L}[\Pi_t^0]\; \Pi_t^0 + \Pi_t^0\; \mathcal{L}\,\big[\Pi_t^0\big]\, \big[\Pi_t^0\big]^{\perp}, \quad t \in [t_1, t_2).]$$

Eq. (ii) is a *non-linear evolution equation* for state-trajectories between two consecutive *"quantum jumps"*. – If $\alpha = 0$, i.e., if the *dissipative terms* in the Lindblad equation (3) are *absent,* then eq. (ii) describes *unitary Schrödinger-von Neumann evolution*, with $p_{nj} \equiv 1$!

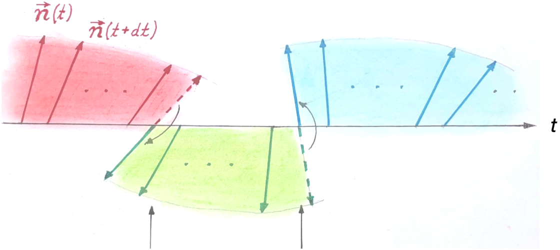
The probability to jump from $\Pi_{t_2}^0$ to a state $\Pi_{t_2+dt_2}^{\delta}$, for some $\delta > 0$, in the time interval $[t_2, t_2 + dt_2]$ is given by the δ^{th} eigenvalue of an explicit non-negative matrix (given by $\big[\Pi_{t_2}^0\big]^{\perp} \mathcal{L}\big[\Pi_{t_2}^0\big]\, \big[\Pi_{t_2}^0\big]^{\perp} dt_2\,.$)

These results uniquely determine a *probability measure* on the space of *state-trajectories* of **individual** systems, S, exhibiting *"quantum jumps"*.

Quantum Poisson Jump Process

The resulting stochastic process on the state-space of S is called a *"quantum Poisson jump process"*. It is the quantum-mechanical cousin of the *random-walk-* or *Wiener process* described in Sect. 2.

[In the example of a *qubit* interacting with the quantized radiation field (with $c \to \infty$), the pure states are parametrized by vectors, $\vec{n} \in S^2$, on the *Bloch sphere* (the corresponding density matrix on $\mathbb{C}^2$ is given by $\rho_{\vec{n}} := \frac{1}{2}\left[\mathbf{1}_2 + \vec{\sigma}\cdot\vec{n}\right]$). The state trajectory of an individual *qubit* is described by a stochastic process on the Bloch sphere with anti-podal *"quantum jumps"* ($\vec{n} \mapsto -\vec{n}$) at discrete times, and, in between two such jumps, $\vec{n}(t)$ evolves according to a *non-linear (cubic) equation of motion*.]



quantum jumps

4. The *ETH*-Approach to Quantum Mechanics

In this last section, I will explain why the q-m evolution of *ensemble states* of *systems of charged matter interacting with the quantized radiation field,* in the limit where $c \to \infty$, is described by a *Lindblad equation* with **dissipative** terms, as considered in the last section; and how the "ontology" – the specific unraveling of the Lindblad evolution – proposed there can be derived from **basic physical principles** at the core the so-called **ETH- Approach to QM** – where E stands for events, T for trees, and H for histories.

Dissipation in a system S may be caused by interactions of the degrees of freedom in S with an environment E. But one could argue – as did *John Bell* – that one ought to consider $S \vee E$ as the total system, and that the states of the system $S \vee E$ should then be expected to evolve **unitarily** according to a deterministic, linear Schrödinger-von Neumann eq..

It turns out that the **radiation field** – more generally, any field with massless modes – represents an "environment" that causes **fundamental dissipation**, which leads to a stochastic evolution of **individual** systems.

Key ingredients of the *ETH*-Approach

Here are the basic ingredients constituting the *ETH*-Approach.

I. **Potential events**, $\mathfrak{e}$, in an isolated open system S are described by partitions of unity, $\mathfrak{e} = \{\widehat{\pi}_\xi \,|\, \xi \in \mathfrak{X}_\mathfrak{e}\}$, $\sum_{\xi \in \mathfrak{X}} \widehat{\pi}_\xi = \mathbf{1}$, by *mutually disjoint orthogonal projections,* $\widehat{\pi}_\xi$, *of finite rank* ($\mathfrak{X}_\mathfrak{e}$ = "spectrum" of $\mathfrak{e}$ denumerable).

At every time $t \in \mathbb{R}$, $\exists$ a representation of projections of all $\mathfrak{e}$ of S,

$$\mathfrak{e} \ni \widehat{\pi}_\xi \mapsto \pi_\xi(t)\,, \ \forall \xi \in \mathfrak{X}_\mathfrak{e}\,, \ \forall \mathfrak{e} \text{ of } S, \tag{6}$$

by concrete orth. projections, $\pi_\xi(t)$, on a separable Hilbert space $\mathcal{H}$.

Heisenberg time evolution: For all $\widehat{\pi} \in \mathfrak{e}, \forall\, \mathfrak{e}$ of S, and for arbitrary times t and t', $\pi(t)$ and $\pi(t')$ are unitarily conjugated by the propagator of S. $\rightarrow$ Introduce *Algebras of potential events*

$$\mathcal{E}_{\geq t} := \left\langle \pi(t') \,\middle|\, \pi(t') \text{ represents } \widehat{\pi} \in \mathfrak{e}, \forall \mathfrak{e} \text{ of } S, \forall t' \geq t \right\rangle^{-}. \tag{7}$$

Clearly

$$\mathcal{E}_{\geq t'} \subseteq \mathcal{E}_{\geq t}\,, \quad \text{whenever} \quad t' > t\,.$$

The Principle of Declining Potentialities

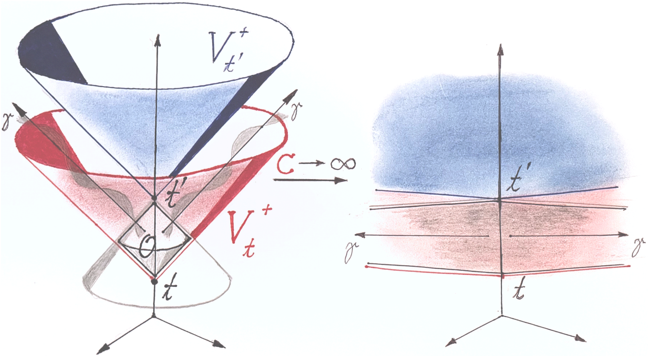
Heisenberg time evolution implies that all the algebras $\mathcal{E}_{\geq t}, t \in \mathbb{R}$, are *unitarily conjugated* to each other; in particular $\mathcal{E}_{\geq t} \simeq \mathcal{E}_{\geq 0} \equiv \mathcal{E}_0 \, , \forall\, t$.

II. **Principle of Declining Potentialities** (PDP): In the *ETH*-Approach to QM, an *isolated open physical system* S capable of producing **events** is characterized by the property that

$$\boxed{\mathcal{E}_{\geq t'} \subsetneq \mathcal{E}_{\geq t}\, , \quad \forall\, t' > t\, .} \qquad \text{(PDP)} \tag{8}$$

Physics: PDP is an expression of *fundamental dissipation*. It holds in theories with massless modes, *photons* (& gravitons); e.g., in relativistic QED (*D. Buchholz*) and in quantum optics in the limit where $c \to \infty$. – Photons and gravitons emitted in a diamond $\mathcal{O} \subset V_t^+$ can escape at the speed of light **without detection** in $V_{\mathbf{t}'}^+$; hence, the energy and information they carry away **cannot** be recovered at times $\geq t'$.

PDP as a consequence of "Huygens' Principle"



Huygens' Principle for $c < \infty$ *Huygens' Principle for* $c \to \infty$

Left: *A subalgebra of operators localized in* $\mathcal{O}$ *commute with all of* $\mathcal{E}_{\geq t'}$

Right: *Em field ops.in different time-slices commute with each other*

As in the genesis of Relativity Theory, the em field (and/or the gravit. field) apparently play a crucial role in the ETH-Approach to QM.

States of physical systems

III.1. **States of physical systems**: *A state, ω, of a system S at time t* is a normalized, positive, linear functional on the algebra $\mathcal{E}_{\geq t}$.

A state at time t enables one to predict the probability of potential events of S actualizing/happening at times $\geq t$, as it ought to.

If PDP holds, states of S at time t depend *non-trivially* (!) on t, *even* in the *Heisenberg picture,* as we will see.

Given an initial state, ω_0, of S at time 0, the *ensemble state,* ω_t, at a later time $t > 0$ corresp. to ω_0 is given by *restricting* ω_0 to the *sub-alg.* $\mathcal{E}_{\geq t} \subsetneqq \mathcal{E}_0$ (PDP). In general, the map from ω_0 to ω_t does *not* preserve *purity* (i.e., the restr. of a pure state ω_0 on $\mathcal{E}_0$ to $\mathcal{E}_{\geq t}, t > 0$, need *not* be pure) – a manifestation of *entanglement* of the system's state with the states of modes that, for *fundamental reasons*, are *unobservable.*

Time evolution of ensemble states

In *non-relativistic QED*, i.e., for $c \to \infty$, the following special property P holds:

- P The algebra $\mathcal{E}_0$ is isomorphic to the algebra, $B(\mathcal{H}_0)$, of bounded operators on a separable Hilbert sub-space, $\mathcal{H}_0 \subseteq \mathcal{H}$.[4]

Hence a state ω_0 on $\mathcal{E}_0$ is given by a *density matrix*, Ω_0, on $\mathcal{H}_0$. The isomorphism $\mathcal{E}_{\geq t} \simeq \mathcal{E}_0$ then implies that the *ensemble state*, ω_t, of S at time $t > 0$, i.e.,

$$\omega_t := \omega_0\Big|_{\mathcal{E}_{\geq t}}$$

can also be represented by a density matrix, Ω_t. If the algebras $\mathcal{E}_{\geq t}, t \geq 0$, were *independent* of t then, in the *Heisenberg picture*, $\Omega_t = \Omega_0, \ \forall\, t > 0$.

In the *Schrödinger picture*, the density matrix Ω_t repr. ω_t at time $t > 0$ is defined by

$$\mathrm{Tr}\big(\Omega_t \cdot \pi(0)\big) := \mathrm{Tr}\big(\Omega_0 \cdot \underbrace{\pi(t)}_{\in\, \mathcal{E}_{\geq t}\ (\subsetneq \mathcal{E}_0)}\big), \quad \forall\, \widehat{\pi} \in \mathfrak{e},\ \forall\ \mathfrak{e}. \tag{9}$$

[4] *In relativistic QED, $\mathcal{E}_0$ would be a von Neumann algebra of type III_1 or II_1.*

Lindblad equation, states of individual systems

The map $\Omega_0 \mapsto \Omega_t$ is *linear, positivity-preserving* and *trace-preserving*, for all $t > 0$; in fact, it is *completely positive*. Hence it is given by a *Lindblad semi-group*, as assumed in Sect. 3.

III.2. **States of individual systems**: The true state of an **individual** system S characterized by ingredients I, II, III.1 and Property P is given by a density matrix proportional to an *orthogonal projection*, Π_t, in $\mathcal{E}_{\geq t}$ of *finite rank* (generically of rank 1), $\forall$ times t.

Let Ω_{t+dt} ($dt > 0$: time step) be the density matrix obtained by restricting the state at time t, i.e., $\dim(\Pi_t)^{-1}\,\Pi_t$, to $\mathcal{E}_{\geq(t+dt)} \subsetneqq \mathcal{E}_{\geq t}$, see (9). According to the spectral theorem, Ω_{t+dt} is given by a convex combination of its spectral projections,

$$\Omega_{t+dt} = p^0[t, t+dt]\,\Pi^0_{t+dt} + \sum_{\delta>0} p^\delta[t, t+dt]\,\Pi^\delta_{t+dt}\,,$$

$$\text{with} \quad p^0 \gg p^1 > \cdots \geq 0.$$

If the time step dt is tiny then $\Pi^0_{t+dt} \approx \Pi_t$.

The State-Selection Postulate

The *potential event* $\mathfrak{e}$ repr. by the projections $\{\Pi^{\delta}_{t+dt} \,|\, \delta = 0, 1, \dots\}$ **actualizes** at time $t + dt$ according to the following

IV. **State-Selection Postulate** (*"Ontology"* of QM): The true state at time $t + dt$ of an *individual* isolated system S, whose state at time t is $\propto$ to the projection Π_t, is a density matrix proportional to

- Π^0_{t+dt}, with *probability* $p^0[t, t+dt]\,\dim(\Pi^0_{t+dt}) = 1 - \mathcal{O}(dt)$,
  or to one of the projections
- Π^{δ}_{t+dt}, with *probability* $p^{\delta}[t, t+dt]\,\dim(\Pi^{\delta}_{t+dt}) = \mathcal{O}(dt)$,
  for some $\delta > 0$.

The State-Selection Postulate says that, at each time, a state is selected (i.e., an *event actualizes*) in accordance with *Born's Rule*. – The limit, as $dt \to 0$, is under control!

Conclusions: (i) In this talk I have sketched a *completion of non-relativistic QM* derived from the *ETH-Approach*. Property P and IV completely justify the analysis presented in Sect. 3! The *ETH*-Approach solves the infamous *"measurement problem"* and the *"unitarity paradox"*.

Concluding Remarks

(ii) The *ETH*-Approach can be generalized to encompass *relativistic quantum theory* (coupled to gravity). But the fine print of this generalization remains to be worked out.
(iii) . . .

Achknowledgement: I thank *C. Albert, Ph. Blanchard, Z. Gang, A. Pizzo, B. Schubnel* and *S. Zivi* for many useful discussions and collaboration on problems related to this talk. – I am indebted to *Felix Finster* for providing me with an opportunity to present these thoughts.

—

To conclude let me express my hope that **Western European countries** *will welcome back – among other peoples – the* **Russian people** *in the European family, that they will solve conflicts by negotiations, rather than by war, and that they will work for* **Trust** *and* **Peace** *and the* **future of humankind**, *rather than waste absurd amounts of resources for rearmament!*