Welcome to Regensburg!

Today: *Geometry*

- *classical* geometry $\leftrightarrow$ *quantum* geometry

Einstein equations in the continuum limit

- Consider linearized gravity $g_{ij} = \eta_{ij} + h_{ij}$
- Construct causal fermion system $(\mathcal{H}, \mathcal{F}, \rho)$.
- Analyze the EL equations of causal action principle. Gives

$$R_{jk} - \frac{1}{2} R\, g_{jk} + \Lambda g_{jk} = \kappa\, T_{jk} + \mathscr{O}\big(h^2\big)$$

  with $\kappa \sim \varepsilon^2$ and *undetermined cosmological constant*.
- Whole setup is *generally covariant*. Scaling of error terms gives higher orders in curvature,

$$\mathscr{O}\big(\varepsilon^4\, \mathrm{Riem}^2\big)$$

Lorentzian quantum geometry

- with A. Grotz (2011)
- $(\mathcal{H}, \mathcal{F}, \rho)$ causal fermion system with spacetime $M := \operatorname{supp} \rho$.
- For $x \in M$ define *spin space* $S_x M := x(\mathcal{H})$.
- There are general notions of

  $$\text{spin connection} \quad D_{x,y} \,:\, S_y M \to S_x M \quad \text{unitary}\,,$$

  and *metric connection* $\nabla_{x,y}$.
- *Curvature* defined via holonomy
- Unfortunately, it seems impossible to reformualte the causal action principle in this language.

Synthetic geometric notions

- two-dimensional *area change* and *matter flux*, connects to T. Jacobson's derivation of the Einstein equations (with E. Curiel and J.M. Isidro)
- *total mass* (with A. Platzer)
- *quasi-local mass, synthetic scalar curvature* (with N. Kamran)
- ...

Extrinsic geometric notions

- Work in progress (C. Krpoun, ...)
- $(\mathcal{H}, \mathcal{F}, \rho)$ causal fermion system with spacetime $M := \operatorname{supp} \rho$.
- $\mathcal{F}$ is *Banach manifold*, Hilbert-Schmidt scalar product gives *Riemannian metric*
- $M \subset \mathcal{F}$ is a subset (or *submanifold*)
- Consider *extrinsic curvature*, *Gauß-Codazzi* equations, ...
- Moreover: $\mathcal{L}(x, y)$ is a *nonlocal kernel*, gives *"smeared derivatives"*, ...
- Hope: Will give
  - geometric reformulation of causal action principle
  - geometric derivation of *generalized Einstein equations* in non-smooth setting
  - ...
