Enhanced terms in the Szegő-type asymptotics for the free Dirac operator

When studying scaling laws for the bipartite entanglement entropy of the ground state of a non-interacting free Fermi gas, one finds that the leading-order term is logarithmically enhanced (i.e. it is of order $L^{(d-1)} \log L$ in the scaling parameter $L > 0$, instead of following the usual area law). A proof of this fact relies on the Widom–Sobolev formula, a major breakthrough in the study of Szegő-type asymptotics in the continuum. It is a natural question in which way this situation extends to the relativistic case, where the single-particle Hamiltonian is given by the free Dirac operator. I give a brief introduction to the non-relativistic result and the free Dirac operator, and give an answer to the question above. The proof of the corresponding enhanced area laws relies on a generalisation of the Widom–Sobolev formula to matrix-valued symbols. These results were obtained in collaboration with Peter Müller. Lastly, I discuss a special situation in the massless case, where, instead of an enhanced area law, one obtains a lower-order enhanced term of logarithmic order.