The breakdown of microcausality in quantum gravity via purely virtual particles

D. Anselmi

In this talk I review the concept of purely virtual particles (particles that are always off the mass shell) and their applications to quantum field theory, cosmology, and phenomenology

Then I focus on the theory of quantum gravity developed using this concept and derive its main prediction regarding primordial inflation ($4/10000 < r < 3/1000$)

In the rest of the talk I discuss the prices to pay in order to reconcile unitarity and renormalizability within quantum field theory

In particular, I describe the causality violations expected at small scales and explain why they remain consistent with current experimental and observational data

Finally, I argue that imposing absolute causality in the universe is both unnecessary and incompatible with a proper approach to investigating nature

Three equivalent formulations of purely virtual particles have been worked out so far

First, a nonanalytic Wick rotation was introduced as a way to get rid of ghosts with complex masses, and reformulate the Lee-Wick models into models of new type (see later in this talk). Its key ingredient is the average continuation around the branch cuts of amplitudes

- D. A. and M. Piva, A new formulation of Lee-Wick quantum field theory, JHEP 06 (2017) 066 and arXiv:1703.04584
- D. A. and M. Piva, Perturbative unitarity of Lee-Wick quantum field theory, Phys. Rev. D 96 (2017) 045009 and arXiv: 1703.05563 [hep-th]

The method can be extended to remove ghosts with real masses (as well as physical particles), and, among the other things, give sense of quantum gravity as a power counting renormalizable theory, like the standard model

- D. A., On the quantum field theory of the gravitational interactions, JHEP 06 (2017) 086 and arXiv:1704.07728

The **second**, equivalent formulation of purely virtual particles was introduced by means of the diagrammatic threshold decomposition, and the spectral optical identities derived from it.

- D. A., Diagrammar of physical and fake particles and spectral optical theorem, JHEP 11 (2021) 030 and arXiv:2109.06889

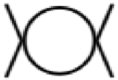
The **third** formulation, equivalent to the other two, is based on a minimally non time-ordered product

– D. A., A new quantization principle from a minimally non time ordered product, JHEP 12 (2022) 088, arXiv:2210.14240 [hep-th]

A possible **fourth** formulation is currently under study

Diagrammatic formulation via (minimally) non T-ordered correlation functions

– D. A., A new quantization principle from a minimally non time-ordered product,
JHEP 12 (2022) 088, arXiv:2210.14240 [hep-th]

We commonly use Feynman diagrams 

We meet the Cutkosky-Veltman diagrams  in the unitarity equation

$$S^\dagger S = 1$$

We know we can build diagrams for in-in correlation functions

Can we study more general diagrams?

Yes! Actually, we can study all possibilities at once by including non-T ordered diagrams

Ingredients

Propagators

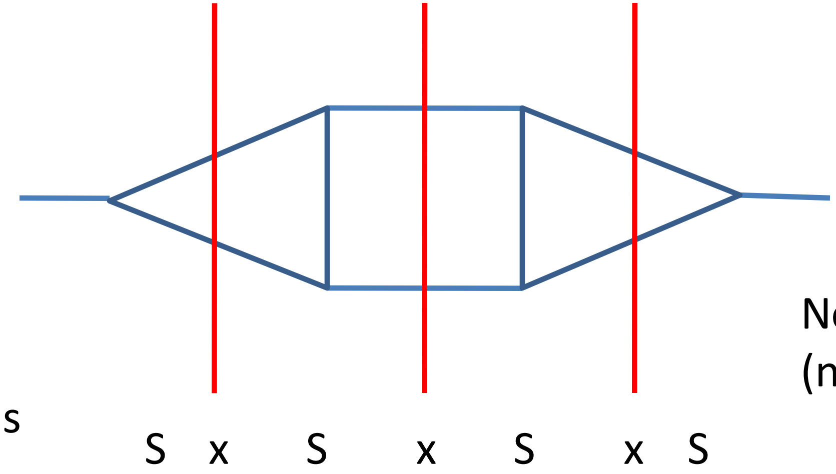
Cut propagators $\theta(p^0)(2\pi)\delta(p^2-m^2)$ in an **arbitrary** number of cuts

~~Conjugate propagators~~

Vertices

~~Conjugate vertices~~

New diagrams are like



which are generated by products (not T-ordered products) of ordinary diagrams

No shadows here (no conjugates)

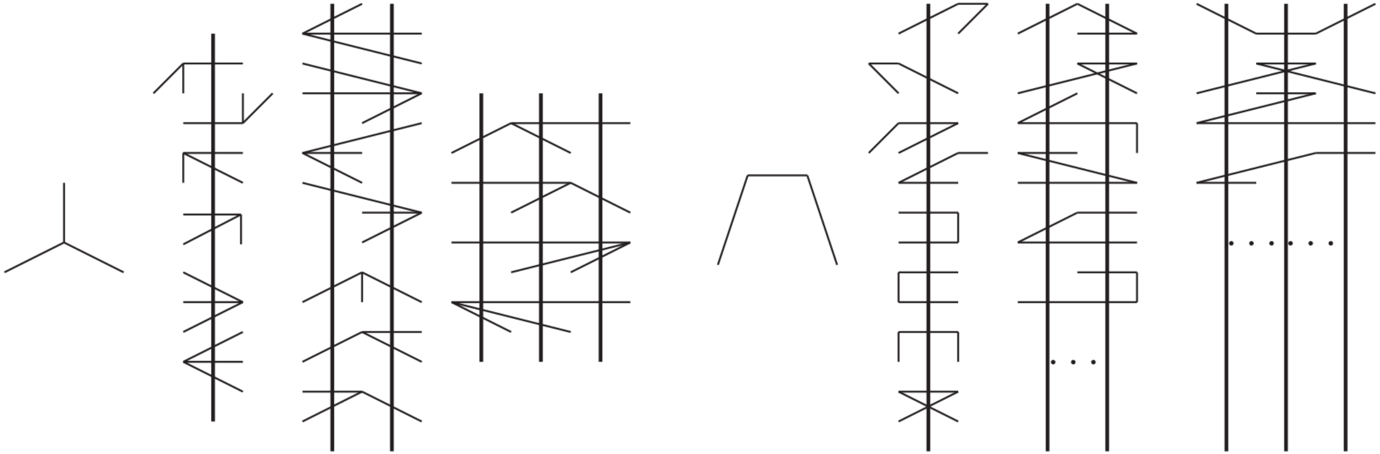
$$\theta(p^0)(2\pi)\delta(p^2-m^2) \xrightarrow{\text{Fourier}} \langle 0|\varphi(x)\varphi(y)|0\rangle \text{ vs } \langle 0|T\varphi(x)\varphi(y)|0\rangle$$

- Draw an arbitrary number of vertical lines (cuts)

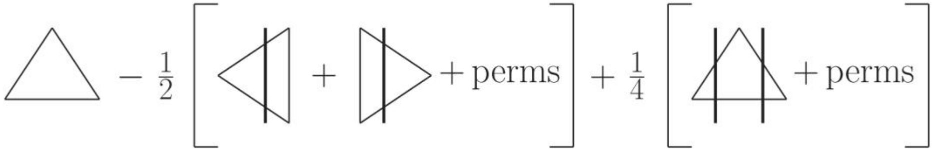
- Distribute the vertices in all possible ways

- Connect the vertices

Three propagators



Triangle



- Sum all possibilities: this gives all the correlation functions you can think of

- There is a magic formula that subtracts all the on-shell contributions of a particle (rendering it a PVP) and satisfies the common diagrammatic properties

- The cut propagators $\theta(p^0)(2\pi)\delta(p^2 - m^2)$, which are purely on shell, are used to define purely off-shell particles by subtracting all the would-be on-shell contributions of normal particles or ghosts

- The magic subtraction formula provides the formulation of PVPs based on (minimally) non-T ordered diagrams

Unitarity, again

$$S^\dagger S = 1 \qquad iT - iT^\dagger = -T^\dagger T \qquad S = 1 + iT$$

More generally, if a theory has ghosts, we have

$$iT - iT^\dagger = -T^\dagger C T$$

$$S = 1 + V$$

$$V + V^\dagger = -V^\dagger C V$$

where C = diag(+1, +1, +1, +1, …, –1, –1, –1, –1, –1, …)

ghosts

Define

A = diag(+1, +1, +1, +1, …, 0, 0, 0, 0, 0, …) # of +s = your choice

and B = C – A

Then $V_{\rm red} = V_\Omega(A,B),$ where Ω is an arbitrary anti-Hermitian matrix, and

$$\boxed{\begin{aligned} V_\Omega(A,B) &= \left(1+\tfrac{1}{2}VB\right)^{-1}V \\ &+ \left(1+\left(1+\tfrac{1}{2}VB\right)^{-1}VA\right)^{1/2}\left(1-\tfrac{1}{2}\Omega A\right)^{-1}\Omega\left(1+AV\left(1+\tfrac{1}{2}BV\right)^{-1}\right)^{1/2} \end{aligned}}$$

is the most general solution of the equation

$$V_{\rm red} + V_{\rm red}^\dagger = -V_{\rm red}^\dagger A V_{\rm red}$$

The simplest option, Ω = 0, and the ones where Ω is generic, have undesirable properties (e.g., a product of diagrams is not mapped into the product of the mapped diagrams).
The right Ω is worked out iteratively so as to preserve the common diagrammatic properties.
The solution exists and is unique. It matches the ones of the other formulations of PVPs

Once you have the theory with $V_{\rm red} + V_{\rm red}^\dagger = -V_{\rm red}^\dagger A V_{\rm red}$

and A = diag(+1, +1, +1, +1, ..., 0, 0, 0, 0, 0, ...) = $\Pi_{\rm ph} = {\rm diag}(\overbrace{1, \cdots, 1}^{N_{\rm ph}}, \overbrace{0, \cdots, 0}^{N_{\rm pv}})$

you can view A as the projector onto the physical subspace, which is the "subspace of A pluses", and project out the complementary subspace

This means: you discard all the diagrams with external legs belonging to the complement

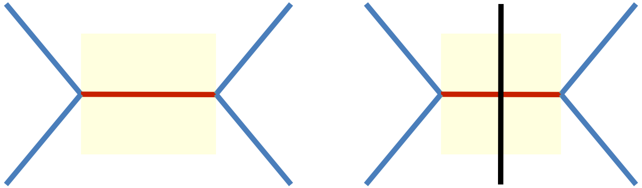
In the physical subspace you have a unitary S matrix:

$$V_{\rm ph} \equiv \Pi_{\rm ph} V_{\rm red} \Pi_{\rm ph} \qquad S_{\rm ph} \equiv \Pi_{\rm ph} + V_{\rm ph}$$

$$V_{\rm ph} + V_{\rm ph}^\dagger = -V_{\rm ph}^\dagger V_{\rm ph} \qquad S_{\rm ph}^\dagger S_{\rm ph} = \Pi_{\rm ph}$$

Examples

Propagator



$\mathcal{P}$ = Cauchy principal value

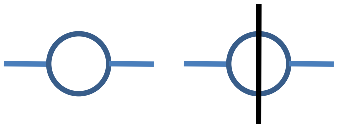
$$\frac{1}{p^2-m^2+i\epsilon}=\mathcal{P}\frac{1}{p^2-m^2}-\cancel{i\pi\delta(p^2-m^2)}\longrightarrow\mathcal{P}\frac{1}{p^2-m^2}$$

Careful! Do NOT use this as a propagator *inside* diagrams, or you obtain “Wheelerons”

C.G. Bollini and M.C. Rocca, The Wheeler propagator, Int. J. Theor. Phys. 37 (1998) 2877 and arXiv:hep-th/9807010

The interest people had in the past for “non-existent particles” shows their intuition that something of this type could indeed “exist”. Although they did not find the right answer, I regard their attempts as very valuable

Bubble diagram



$$\int\frac{\mathrm{d}^D p}{(2\pi)^D}\frac{1}{p^2-m^2+i\epsilon}\frac{1}{(p+k)^2-m^2+i\epsilon'}$$

$$\omega_{\mathbf{p}}=\sqrt{\mathbf{p}^2+m^2}$$

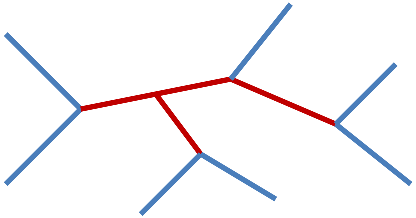
$$\longrightarrow -i\pi\mathcal{P}\int\frac{\mathrm{d}^{D-1}\mathbf{p}}{(2\pi)^{D-1}}\frac{\omega_{\mathbf{p}+\mathbf{k}}+\omega_{\mathbf{p}}}{\omega_{\mathbf{p}+\mathbf{k}}\omega_{\mathbf{p}}}\frac{1}{(k^0)^2-(\omega_{\mathbf{p}+\mathbf{k}}+\omega_{\mathbf{p}})^2}$$

Check the references for triangles, boxes, multiple loops, etc.

CLASSICIZATION

The projection must be done classically as well!

Classical limit = set of tree diagrams with PVPs propagating inside



The classicization does NOT return the starting classical Lagrangian

The true classical Lagrangian of a theory of PVPs is obtained by collecting the tree diagrams with physical particles outside and PVPs inside

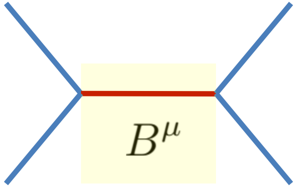
It turns out to be nonlocal. The nonlocality is confined to scales shorter than $1/m_{PVP}$

For example, let us consider Lee-Wick finite QED and turn it into a PVP finite QED

T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033.

$$\mathcal{L}_{\mathrm{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - \frac{M^2}{2}B_\mu B^\mu + \bar{\psi}i\gamma^\mu(\partial_\mu + ieA_\mu + ieB_\mu) - m\bar{\psi}\psi$$

$$\mathcal{L}_{\mathrm{cl}}^{\mathrm{LW}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^\mu(\partial_\mu + ieA_\mu) - m\bar{\psi}\psi + \frac{e^2}{2M^2}\bar{\psi}\gamma^\mu\psi\frac{\eta_{\mu\nu}M^2 + \partial_\mu\partial_\nu}{\Box + M^2 + i\epsilon}\bar{\psi}\gamma^\nu\psi$$



$$\mathcal{L}_{\mathrm{cl}}^{\mathrm{PVP}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}i\gamma^\mu(\partial_\mu + ieA_\mu) - m\bar{\psi}\psi + \frac{e^2}{2M^2}\bar{\psi}\gamma^\mu\psi\mathcal{P}\frac{\eta_{\mu\nu}M^2 + \partial_\mu\partial_\nu}{\Box + M^2}\bar{\psi}\gamma^\nu\psi$$

D.A., Purely virtual particles versus Lee-Wick ghosts: physical Pauli-Villars fields, finite QED and quantum gravity, Phys. Rev. D 105 (2022) 125017 and arXiv: 2202.10483 [hep-th]

Comparison with Lee-Wick theories: those theories have *unstable* ghosts

"Living with ghosts"?
(Hawking, Hertog, PRD 65 (2002) 103515?)
No, thanks!

I would call the option of admitting ghosts in fundamental theories the last-last-last-last resort

And given that alternatives are available, probably we can postpone that drastic option

NEGATIVE METRIC AND THE UNITARITY OF THE *S*-MATRIX

T. D. LEE *
CERN, Geneva

and

G. C. WICK
Columbia University, New York, N. Y. **

Received 22 November 1968

the real states are to be built. Suppose now that after interactions, mass renormalization terms, etc., are introduced, no *stable* abnormal particles are left. This seems perfectly possible (and in fact happens in the simple examples studied later) since we have plenty of parameters at our disposal to make it happen. Since the *S*-matrix is concerned only with asymptotic states which, by definition, consist of only stable particles such as p, e^-, γ, ν_e, ν_μ, $\bar{p}$, e^+, . . . , and since these stable particles are all assumed to be of positive metric, eq. (1.3) then takes the customary, acceptable form

$$S^\dagger S = \text{the unit matrix } I \,. \tag{1.4}$$

they "forgot" to mention the muon!

PVPs FOR QUANTUM GRAVITY

D.A., On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086 and arXiv: 1704.07728 [hep-th]
D.A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energ. Phys. 05 (2018) 27 and arXiv: 1803.07777 [hep-th]
D.A. and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21 and arXiv: 1806.03605 [hep-th]

“Interim” classical action

$$S_{\mathrm{QG}} = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left(2\Lambda + R - \frac{1}{6m_\phi^2}R^2 + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

$C^{\mu\nu\rho\sigma} = \text{Weyl tensor}$

It contains a triplet made of:

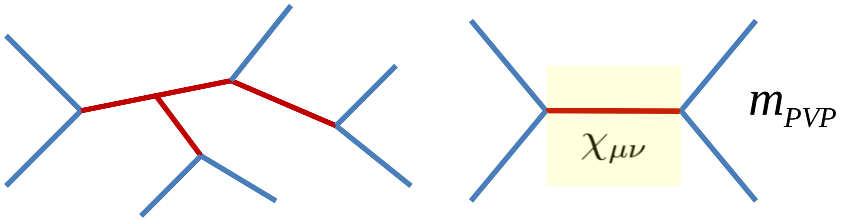
The graviton
The Starobinsky inflaton (spin 0, mass m_ϕ)
The gravity PVP (spin 2, mass m_χ)

Consistency of PVPs in nontrivial background gives the condition:

$$m_\chi > \frac{m_\phi}{4}$$

This is NOT the true classical action, because it is unprojected

True classical action: collection of tree diagrams with no external $\chi_{\mu\nu}$ legs



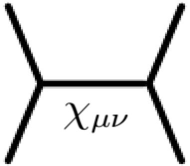
The PVP Green function in flat space $\mathcal{P}\frac{1}{p^2 - m^2}$

Becomes after Fourier transform $\frac{1}{2\omega}\sin(\omega|t-t'|)$ $\omega = \sqrt{\mathbf{p}^2 + m^2}$

Locality is recovered in the large mass limit:

$$\lim_{m\to\infty}\frac{1}{2m}\sin(m|t-t'|) = \delta(t-t')$$

When is the PVP projection consistent?



The PVP Green function in the background field is

$$\hat{G}_{\mathrm{f}}(t,t') = \frac{i\pi \mathrm{sgn}(t-t')}{4H\sinh(n_\chi \pi)} \left[J_{in_\chi}(\check{k}) J_{-in_\chi}(\check{k}') - J_{in_\chi}(\check{k}') J_{-in_\chi}(\check{k}) \right]$$

where $n_\chi = \sqrt{\frac{m_\chi^2}{H^2} - \frac{1}{4}}$, $\quad \check{k} = \frac{k}{a(t)H}$, $\quad \check{k}' = \frac{k}{a(t')H}$ $\quad H = m_\phi/2$

The subhorizon and superhorizon limits give the principal value as expected **for flat space**

$$\hat{G}_{\mathrm{f}} \simeq \frac{1}{2Hk\sqrt{\tau\tau'}} \sin(k|\tau - \tau'|)$$

$$k|\tau|, k|\tau'| \gg 1$$

conformal time

$$\frac{1}{2Hn_\chi} \sin(Hn_\chi|t-t'|)$$

$$k|\tau|, k|\tau'| \ll 1$$

cosmological time

n_χ real and $H = m_\phi/2$ give $m_\chi > \frac{m_\phi}{4}$

The bound makes the theory predictive even before knowing the actual value of m_χ

The prediction of the tensor-to-scalar ratio from quantum gravity with purely virtual particles is given by the formula

$$\boxed{\frac{(n_s-1)^2}{3}} \leqslant r \leqslant \boxed{3(n_s-1)^2}$$

Starobinsky prediction

Bound due to PVPs $m_\chi > \frac{m_\phi}{4}$

- D. A., E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211 and arXiv:2005.10293 [hep-th]

where n_s is the tilt of the scalar spectrum. Inserting its measured value, we find

$$0.4 \lesssim 1000r \lesssim 3.5 \qquad r \simeq \frac{24m_\chi^2}{N^2(m_\phi^2+2m_\chi^2)}$$

Hopefully, we will know its values by the end of the decade (LiteBird and others)

NO OTHER THEORY CAN MAKE SUCH A SHARP PREDICTION!

You can compute the spectra to high orders before the nonlocalities become intrusive:

- D.A., Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, J. Cosmol. Astropart. Phys. 01 (2021) 048 and arXiv: 2007.15023 [hep-th]
- D.A, High-order corrections to inflationary perturbation spectra in quantum gravity, J. Cosmol. Astropart. Phys. 02 (2021) 029 and arXiv: 2010.04739 [hep-th]

Without Weyl squared (Starobinsky model):

$$\mathcal{P}_T(k) = \frac{4Gm_\phi^2}{\pi}\left[1 - 3\alpha_k + (47 - 24\gamma_M)\frac{\alpha_k^2}{4} - \left(\frac{307}{6} + 12\gamma_M^2 - 42\gamma_M - \pi^2\right)\alpha_k^3 + \mathcal{O}(\alpha_k^4)\right],$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{Gm_\phi^2}{12\pi\alpha_k^2}\left[1 + (5 - 4\gamma_M)\alpha_k - \frac{67}{12}\alpha_k^2 + (12\gamma_M^2 - 40\gamma_M + 7\pi^2)\frac{\alpha_k^2}{3} + \mathcal{O}(\alpha_k^3)\right].$$

$$\gamma_M \equiv \gamma_E + \ln 2$$

α_k is a “running coupling replacing the usual slow-roll parameter

$$k_* = 0.05\ \mathrm{Mpc}^{-1} \qquad \alpha_* = 0.0087 \pm 0.0010, \qquad m_\phi = (2.99 \pm 0.37)\cdot 10^{13}\mathrm{GeV}.$$

$\alpha_* \simeq \dfrac{1}{115}$ = “fine structure constant of inflation”

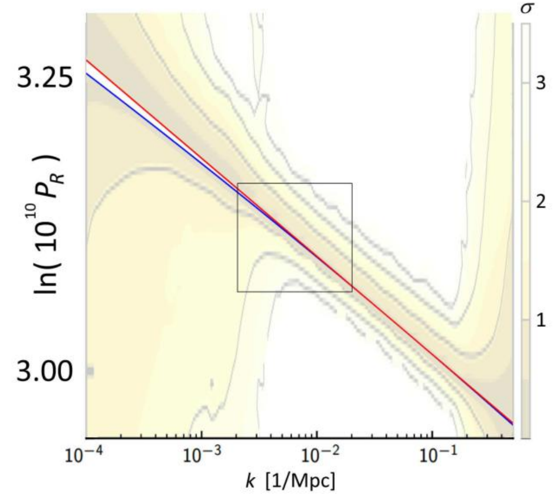
Quantum gravity with PVP $\chi_{\mu\nu}$

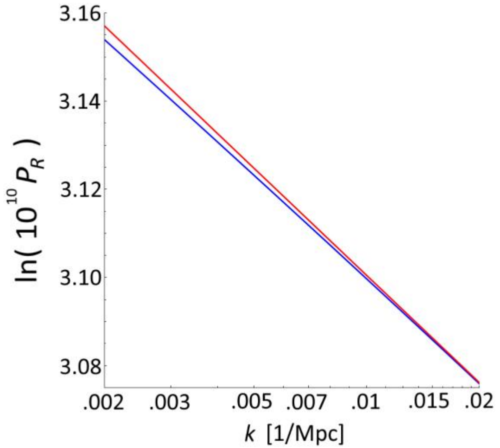
$$\mathcal{P}_T(k) = \frac{4m_\phi^2\zeta G}{\pi}\left[1 - 3\zeta\alpha_k\left(1 + 2\alpha_k\gamma_M + 4\gamma_M^2\alpha_k^2 - \frac{\pi^2\alpha_k^2}{3}\right) + \frac{\zeta^2\alpha_k^2}{8}(94 + 11\xi)\right.$$
$$\left. + 3\gamma_M\zeta^2\alpha_k^3(14 + \xi) - \frac{\zeta^3\alpha_k^3}{12}(614 + 191\xi + 23\xi^2) + \mathcal{O}(\alpha_k^4)\right].$$

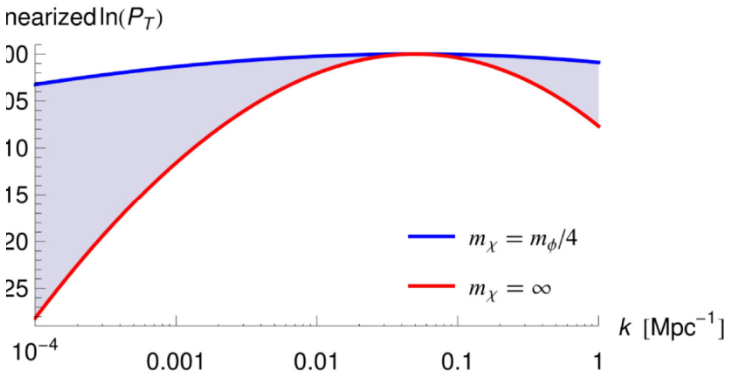
$$\mathcal{P}_\mathcal{R}(k) = \frac{Gm_\phi^2}{12\pi\alpha_k^2}\left[1 + (5 - 4\gamma_M)\alpha_k + \left(4\gamma_M^2 - \frac{40}{3}\gamma_M + \frac{7}{3}\pi^2 - \frac{67}{12} - \frac{\xi}{2}F_s(\xi)\right)\alpha_k^2 + \mathcal{O}(\alpha_k^3)\right]$$

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}$$

$$F_s(\xi) = 1 + \frac{\xi}{4} + \frac{\xi^2}{8} + \frac{\xi^3}{8} + \frac{7\xi^4}{32} + \frac{19}{32}\xi^5 + \frac{295}{128}\xi^6 + \frac{1549}{128}\xi^7 + \frac{42271}{512}\xi^8 + \mathcal{O}(\xi^9)$$







Running of the tensor spectrum

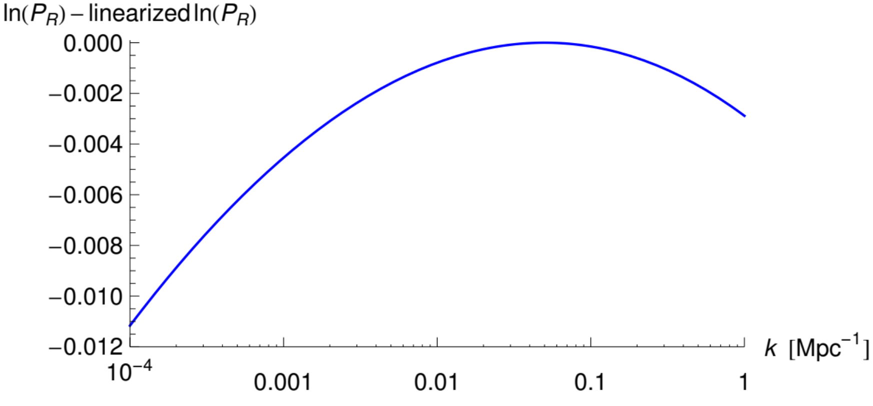


Figure 3: Running of the scalar spectrum

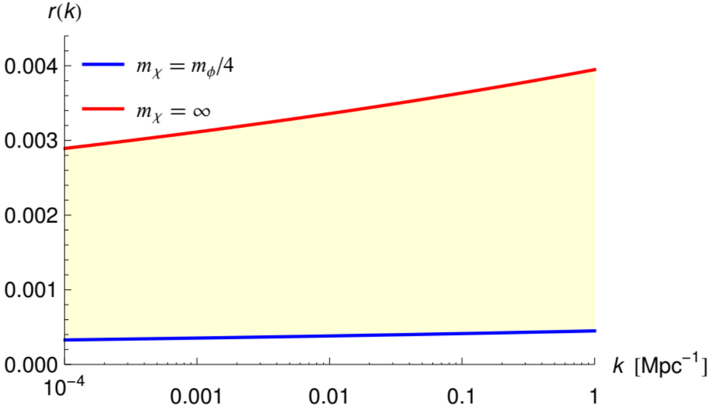


Figure 5: Running of the tensor-to-scalar ratio $r(k)$

“Prices” to pay for the quantization with PVPs:

- at high energies (larger than the smallest PVP mass) temporal ordering (past, present, future) is lost

One could say that microcausality is violated at times intervals shorter than 1 divided by the smallest PVP mass, but to make this statement precise one would need to define causality first, which is not straightforward, especially in quantum field theory

- a new, potentially observable phenomenon is predicted: the peak uncertainty (see later), which gives the physical meaning of the PVP “width” Γ

Comparison with gauge theories

$$S_{\mathrm{gf}}(\Phi) = -\frac{1}{4}\int F^a_{\mu\nu}F^{\mu\nu a} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2\lambda}\int (\partial^\mu A^a_\mu)^2 - \int \bar{C}^a\partial^\mu D_\mu C^a$$

QUANTIZATION: gauge theories are quantized as local quantum field theories by means of Faddeev-Popov ghosts, which compensate for the unphysical components (longitudinal, temporal) of the gauge fields

Projection: diagrams with unphysical external legs are dropped

Unitarity is a bonus, thanks to the gauge symmetry: unphysical degrees of freedom do not propagate on-shell inside the diagrams

With PVPs, the theories are still quantized as local theories

Projection: diagrams with PVPs on the external legs are dropped

Unitarity is not a bonus, unless the very definitions of diagrams are changed. Switching to the special non-T ordered diagrammatics mentioned earlier, PVPs do not propagate on-shell inside the diagrams and unitarity is guaranteed

CLASSICIZATION: the gauge-fixed Lagrangian you use for the diagrammatics is NOT the true classical Lagrangian. Once projected onto the physical modes only, the gauge-fixed classical Lagrangian is nonlocal

$$S_{\mathrm{gf}}(\Phi) = -\frac{1}{4}\int F^a_{\mu\nu}F^{\mu\nu a} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{2\lambda}\int (\partial^\mu A^a_\mu)^2 - \int \bar{C}^a \partial^\mu D_\mu C^a$$

Alternative quantization method for gauge theories: treat the gauge-trivial modes (Faddeeev-Popov ghosts and longitudinal/temporal components of gauge fields) as PVPs

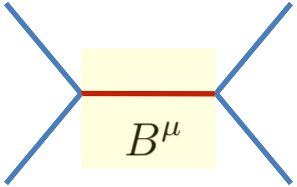
Same physical quantities, proof of unitarity straightforward

The trick can be used to define perturbation theory with a nonvanishing cosmological constant, as well as massive gravitons and massive gauge fields

D.A., Fakeons, unitarity, massive gravitons and the cosmological constant
JHEP 12 (2019) 027 and arXiv: 1909.04955 [hep-th]

The fate of causality

$$\mathcal{L}_{\rm cl}^{\rm PVP} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi i\gamma^\mu(\partial_\mu + ieA_\mu) - m\bar\psi\psi + \frac{e^2}{2M^2}\bar\psi\gamma^\mu\psi\mathcal{P}\frac{\eta_{\mu\nu}M^2 + \partial_\mu\partial_\nu}{\Box + M^2}\bar\psi\gamma^\nu\psi$$



$$\mathcal{P}\frac{1}{p^2 - m^2} \longrightarrow \frac{1}{2\omega}\sin(\omega|t - t'|)$$

$$\omega = \sqrt{\mathbf{p}^2 + m^2}$$

Locality is recovered in the large mass limit:

$$\lim_{m\to\infty}\frac{1}{2m}\sin(m|t - t'|) = \delta(t - t')$$

Feynman anti Feynman

$$\mathcal{P}\frac{1}{p^2-m^2} = \frac{1}{2}\left(\frac{1}{p^2-m^2+i\epsilon}+\frac{1}{p^2-m^2-i\epsilon}\right)$$

$$= \frac{1}{2}\left(\frac{1}{(p^0+i\epsilon)^2-\omega^2}+\frac{1}{(p^0-i\epsilon)^2-\omega^2}\right)$$

Retarded Advanced

The contribution of the advanced potentials is the origin of the violation of microcausality

Note that the Green funcion vanishes for spacelike separated points
This may be not true beyond the tree level

$$\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}(v^2 - \tau^2 a^2) - V(x,t)$$

$$V(x,t) = -xF_{\mathrm{ext}}(t)$$

Consider another toy model, with Lagrangian

$$\mathcal{L}(x,Q,t) = \frac{m}{2}\dot{x}^2 - m\dot{x}\dot{Q} + \frac{m}{2\tau^2}Q^2 + xF_{\mathrm{ext}}(t),$$

the equations of motion being

$$\ddot{x} = -\frac{Q}{\tau^2}, \qquad m\ddot{Q} + \frac{m}{\tau^2}Q = -F_{\mathrm{ext}}(t).$$

If *Q* is quantized as a purely virtual particle, the unique solution of its equation is

$$mQ = -\mathcal{P}\frac{\tau^2}{1+\tau^2\frac{\mathrm{d}^2}{\mathrm{d}t^2}}F_{\mathrm{ext}}(t) = -\frac{\tau}{2}\int_{-\infty}^{\infty}\mathrm{d}u\,F_{\mathrm{ext}}(t-u)\sin\left(\frac{|u|}{\tau}\right)$$

Inserted into the *x* equation, it gives the projected *x* equation:

$$m\ddot{x} = \frac{1}{2\tau}\int_{-\infty}^{\infty}\mathrm{d}u\,F_{\mathrm{ext}}(t-u)\sin\left(\frac{|u|}{\tau}\right)$$

Violation of microcausality: $|\Delta u| \simeq \tau$

This is reminiscent of Dirac's method for converting the Abraham-Lorentz force in classical electrodynamics into an equation with no higher derivatives, which however violates microcausality (please check Jackson's book on this)

The Larmor formula $$P = m\tau a^2, \qquad \tau = \frac{2e^2}{3mc^3},$$

For the radiation power emitted by an accelerated particle in the adiabatic approximation can be encoded into the higher-derivative equation

$$ma(t) = m\tau\dot{a}(t) + F(t),$$

where $F(t)$ is an external force. Higher-derivative equations have runaway solutions. Dirac proposed to eliminate the runaway solution as follows.

Abraham-Lorentz force

J.D. Jackson, *Classical electrodynamics*, John Wiley and Sons, Inc. (1975), chap. 17.

First, write the equation as

$$\left(1 - \tau \frac{\mathrm{d}}{\mathrm{d}t}\right) ma = F$$

Then invert the operator to obtain

$$ma = \frac{1}{1 - \tau \frac{\mathrm{d}}{\mathrm{d}t}} F \equiv \langle F \rangle$$

Remove the arbitrariness of the inverse by requiring analitycity in τ

The answer is

$$ma(t) = \frac{1}{\tau} \int_t^{\infty} \mathrm{d}t' \, \mathrm{e}^{(t-t')/\tau} F(t') = \langle F \rangle$$

The parameter τ is the duration of the causality violation

Differences and similarities with respect to the causality violations due to PVPs are evident

In our case,

$$\left(1 + \frac{\Box}{m^2}\right)\phi(x) = J(x),$$

is solved by

$$\phi(x) = \langle J \rangle_{\mathrm{f}}(x) \equiv \int G_{\mathrm{f}}(x-y) J(y) \, \mathrm{d}^4 y$$

where

$$\langle J \rangle_{\mathrm{f}} = \left.\frac{m^2}{\Box + m^2}\right|_{\mathrm{f}} J \equiv \frac{m^2}{2}\left(\left.\frac{1}{\Box + m^2}\right|_{\mathrm{ret}} + \left.\frac{1}{\Box + m^2}\right|_{\mathrm{adv}}\right) J$$

$$G_{\mathrm{f}}(x) = \frac{m^4}{8\pi^2}\left[\frac{K_1\left(im\sqrt{x^2 - i\epsilon}\right)}{m\sqrt{x^2 - i\epsilon}} + \frac{K_1\left(-im\sqrt{x^2 + i\epsilon}\right)}{m\sqrt{x^2 + i\epsilon}}\right]$$

Since

$$G_{\mathrm{f}}(x) \sim \frac{m^{5/2}}{4\sqrt{2}\pi^{3/2}(x^2)^{3/4}} \cos\left(m\sqrt{x^2} + \frac{\pi}{4}\right), \qquad m\sqrt{x^2} \gg 1$$

the rapid oscillations ensure that only the contributions close to the light cone effectively matter ($x^2 \approx 1/m^2$)

Moreover, when the mass goes to zero, the whole light cone is interested by the violation:

$$G_{\mathrm{f}}(x) \underset{|x^2| \ll 1/m^2}{\longrightarrow} \frac{im^2}{8\pi^2}\left(\frac{1}{x^2 + i\epsilon} - \frac{1}{x^2 - i\epsilon}\right) = \frac{m^2}{4\pi}\delta(x^2)$$

In other words, what is a violation of microcausality in a reference frame is a violation of macrocausality in another boosted enough reference frame

How can we reconcile this with the data? By putting numbers in…

The microcausality violations concern intervals of time $\approx 1/m$

Assume you have a source emitting light with a frequency ω

In the rest frame of the source, the violations cannot be appreciated if the period $1/\omega$ is much larger than $1/m$

The shortest amount of time ever measured is $\approx 10^{-17}\,\mathrm{sec}$

which gives a mass m around 100eV

Larger PVP masses are fine. In gravity we expect PVP masses around 10^{13} GeV

The gap is a factor 10^{20}

However, we can boost our reference frame with respect to the one of the source

Can we amplify the violation enough?

The frequency is enhanced by factors, $\sqrt{\frac{1-\beta}{1+\beta}}, \qquad \sqrt{\frac{1+\beta}{1-\beta}}$

which require a velocity $\beta \approx 1 - 10^{40}$ insanely close to the one of light

We cannot boost macroscopic objects this much

In the universe, there are galaxies moving away from us at even higher velocities, but we cannot communicate with them

Objection, your honor! Even if the violation of microcausality cannot be enhanced in practice, it can be in principle, so in suitable reference frames your theory predicts absurd behaviors

Your predictions should not be in contradiction with logic!
(S. Giddings, Stockholm, June 2024)

The hidden assumption behind this objection is that there is or must be a logic in the Universe, e.g., in the form of the cause-effect relation. That *something*, called “logic”, pre-exists the Universe!

Answer: What is absurd is this assumption! The only evidence we have is that there is an *appearance* of logic in our portion of the Universe, the visible one. And certainly logic does not pre-exists the Universe, given that it is our own tool (as human beings) to describe the Universe in ways that suit us. *Names* (and concepts, mathematics, formulas, physical laws, etc.) DO NOT come BEFORE *things*!

Thanks