Generalized Noether theorem: Metric from energy-momentum non-conservation

Achim Kempf
University of Waterloo

21 May 2024, Nordita, Stockholm

A. Kempf, Frontiers in Physics, 9, 655857 (2021), arxiv:2110.08278
M. Reitz, B. Šoda, A. Kempf, Phys. Rev. Lett. 131, 211501 (2023), arxiv:2303.01519

What is this talk about?

Spectral Geometry: Hearing the shape of a vibrating object?

New answer: Yes, if you make it ring loud enough.

Why useful? Combines (languages of) GR and Quantum

Also: Sheds new light on QFT locality vs. S-matrix.

Begin with setting the stage

Representations:

- Usual meaning of a representation
- Generalized notion of representation
- Notion of approximate representations

In mathematics, what is a representation?

Traditional "Representation Theory":

A representation is a map $\Phi : A \to M$ from a Lie group or a Lie algebra, to a space M of matrices such that:

$$\Phi(a \cdot b) = \Phi(a) \cdot \Phi(b)$$

Generalized notion of representation

A representation can be any structure-preserving map $\Phi : A \to B$.

Example: Abstract axioms of a vector space. Any concrete vector space is a representation.

Example: In QM, abstract states $|\psi\rangle$ and operators $\hat{f}$ possess position representation, momentum representation, etc, (in this case equivalent by Stone and von Neumann).

Example: In category theory, every morphism, or functor, may be called a representation, e.g., cohomologies.

Approximate representations

Examples:

- Classical phase space is approximate representation of underlying quantum Poisson algebra of QM
- If torsion exists, then torsion-free Lorentzian manifolds are approximate representations.

Notice: Approximate representations can be good in some regimes and bad in other regimes.

QFT on curved spacetime: is it a representation?

If yes, what other representations may the underlying abstract structure have in other (higher energy) regimes?

- E.g., at higher energies have different # of dimensions?
- At Planck energy: maybe no representation in the form of a QFT on curved spacetime?

Emergence of spacetime = emergence of representability?

$\rightarrow$ Search for abstract simple structure underlying QFT-CS.

What's next?

We aim to reconstruct the underlying pre-geometric structure

- Describe matter information theoretically
- Describe spacetime curvature information theoretically
- Describe Matter and Spacetime as representation of an abstract information-theoretic structure that is non-geometric
- Find that this is the completion of spectral geometry

Example: Euclidean gravity + free bosons and fermions

Matter described information theoretically

Observation 1: Matter is describable through correlators

Namely in terms of quantum field theoretic n-point functions $G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$.

Remark: We could now study these information theoretically:

- QFT interactions constitute classical & quantum channels. (See recent papers by my group).
- Question, e.g.: Are the Feynman rules describable entirely through their classical and quantum channel capacities?

But for now, we move on to spacetime curvature:

Gravity described information-theoretically

Observation 2: Spacetime is describable through correlators

The metric is expressible through $G^{(2)}(x,y)$:

$$g_{\mu\nu}(x) = -\frac{1}{2}\left(\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right)^{\frac{2}{D-2}} \lim_{x\to y} \frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\; G^{(2)}(x,y)^{\frac{2}{2-D}}$$

$\Rightarrow$

Spacetimes are expressible as (M, g) and also as $(M, G^{(2)})$.

M. Saravani, S. Aslanbeigi, A. Kempf, Physical Review D93, 045026 (2016)

Summary so far

Both matter, and spacetime, are expressible through the QFT *n*-point functions:

$$G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$$

However:

- The correlators $G^{(n)}(x^{(1)}, x^{(2)}, \ldots, x^{(n)})$ depend on spacetime coordinates, which assumes that there is a differentiable spacetime manifold.

How to obtain abstract, pre-geometric underlying structure?

QFT on curved spacetime, coordinate free

Recall: In QM, a Green's function $G(x, x')$ can be written in any basis, it represents an abstract Hilbert space operator.

Here too: The correlators $G^{(n)}(x_1, ..., x_n)$ represent abstract n-argument Hilbert space operators, $G^{(n)}$.

Is the set of $G^{(n)}$ operators a coordinate-free description of QFT on curved spacetime?

Are the $G^{(n)}$ the underlying abstract structure?

Given the operators $G^{(n)}$, can we get back the field correlators $G^{(n)}(x_1, ..., x_n)$ on a spacetime manifold?

Via $G^{(2)}(x_1, x_2)$, this would then recover also the metric.

Strategy in QM:

In QM, we can use position operators to obtain position bases, but no position operators here.

Strategy here:

Here in QFT, can use "**local** interactions" $G^{(n)}$, $n > 2$ to obtain position bases. Note: we rely on the locality of vertices.

How to re-obtain spacetime representation of the $G^{(n)}$?

If the theory's interactions are local, then:

- The vertices, i.e., $G^{(n)}$ for $n > 2$ are diagonalizable.
- A diagonalizing basis is a coordinate system. We obtain:

$$G^{(n)}(x^{(1)}, x^{(2)}, \dots, x^{(n)}) \qquad \text{for all} \quad n$$

- Now that we have $G^{(2)}(x, y)$, we also obtain the metric:

$$g_{\mu\nu}(x) = -\frac{1}{2}\left(\frac{\Gamma(D/2-1)}{4\pi^{D/2}}\right)^{\frac{2}{D-2}} \lim_{x \to y} \frac{\partial}{\partial x^{\mu}} \frac{\partial}{\partial y^{\nu}} \; G^{(2)}(x, y)^{\frac{2}{2-D}}$$

Notice: This completes spectral geometry!

Completion of Spectral Geometry

If loud enough, the sound of a vibrating manifold tells its shape.

1. Measure harmonic spectrum G.
2. Drive it at the resonance frequencies, record the additional frequencies, in matrix $V_{n,m,r,s}$ (e.g., for ϕ^4).
3. Diagonalize V to obtain a coordinate system x.
4. Express propagator $G^{(2)}$ in that basis: $G^{(2)}(x, x')$.
5. Calculate the metric $g_{\mu\nu}(x)$ from $G(x, x')$.

Conclusion:

- **In QFT:** $V_{n,m,r,s}$ is the S-matrix on curved spacetime.
- **Recall Noether:** Conservation $\Leftrightarrow$ Spacetime symmetry
- **Generalized Noether:** Non-conservation $\Leftrightarrow$ $g_{\mu\nu}(x)$

Do arbitrary $G^{(n)}$ describe a spacetime and matter?

No, because generic $G^{(n)}$, for $n > 2$, are not exactly diagonalizable.

- In "low energy" regimes, the $G^{(n)}$ for $n > 2$ may be approximately diagonalizable.
- In "high energy" regimes, the $G^{(n)}$ generally possess no representation as QFT correlators on a spacetime.

A. Kempf, Front. Phys., Vol.9, 655857 (2021), https://arxiv.org/abs/2110.08278

Summary

- Spacetime and matter are describable by a collection of abstract n-point correlators $G^{(n)}$
- The set of all representations of a set of $G^{(n)}$ could include quantum reference frames and dualities such as AdS/CFT.

However:

- Generic $G^{(n)}$ are at best approximately representable as QFT correlators on a spacetime manifold.

In this way, QFT in curved spacetime could be an approximate representation of abstract n-point correlators of a fundamentally non-geometric theory given by a set of $G^{(n)}$.