Are quantum subsystems invariant?*

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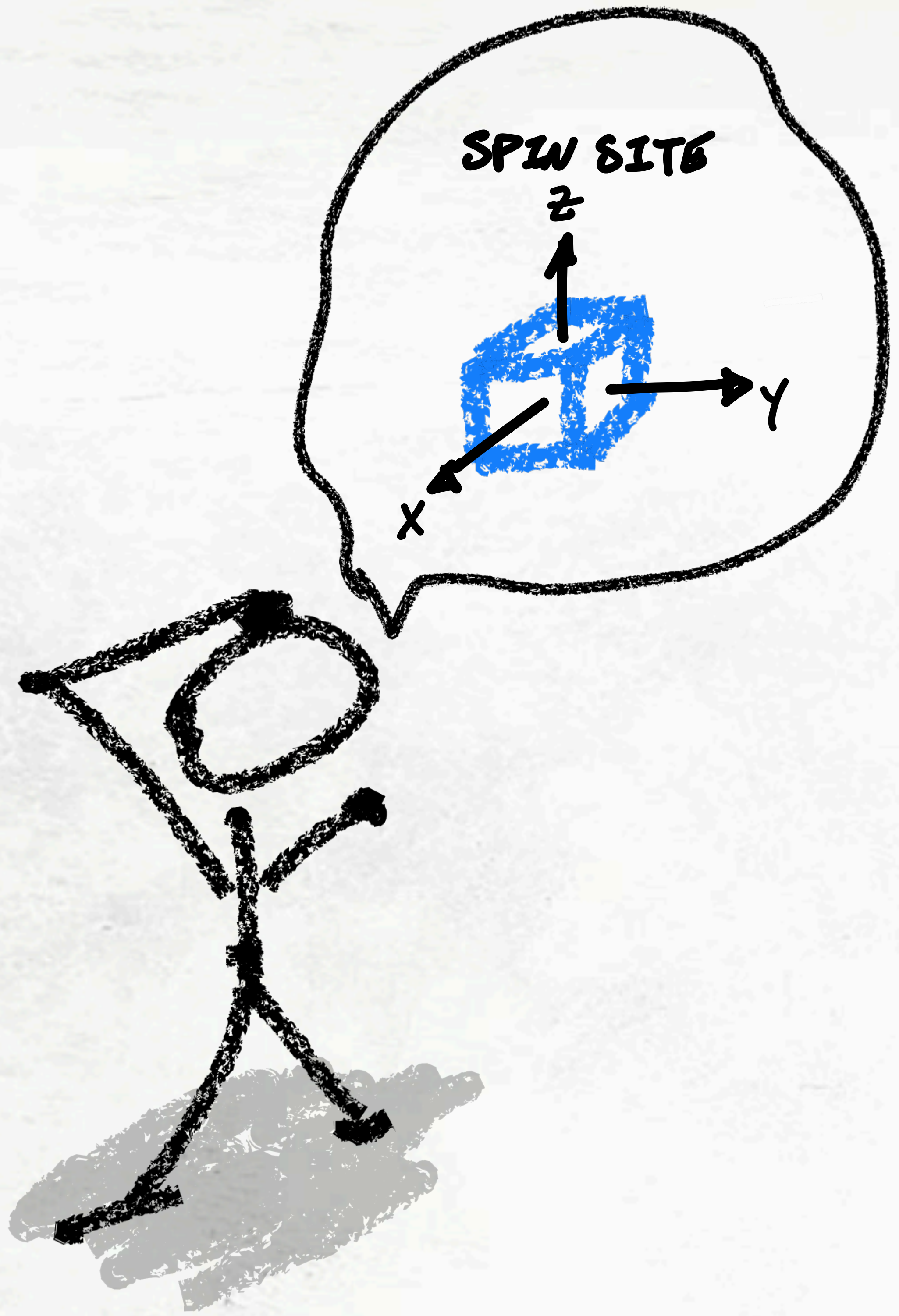
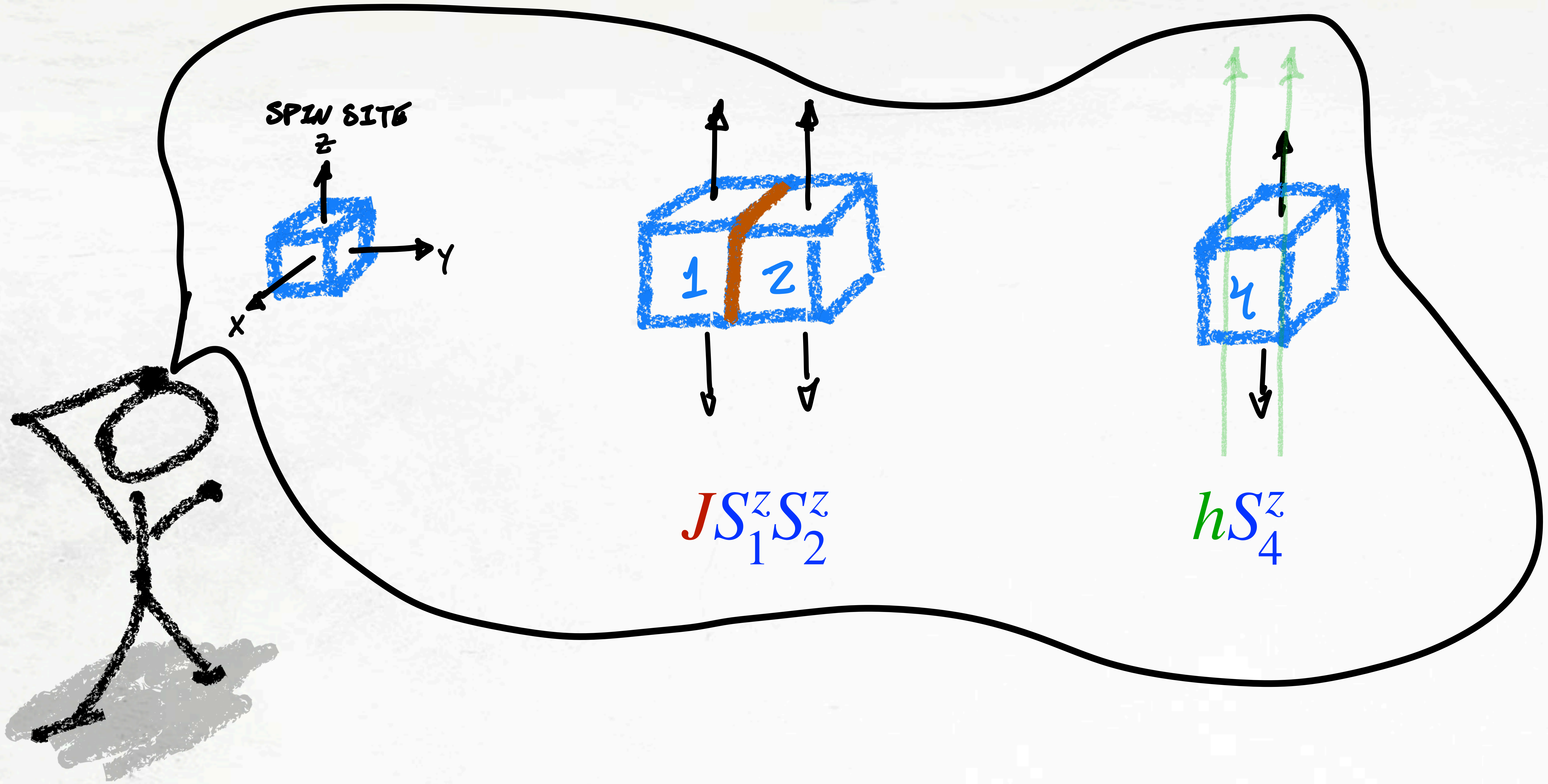
*Based on "*To be or not to be, but where?*" (quant-ph/2405.21031).

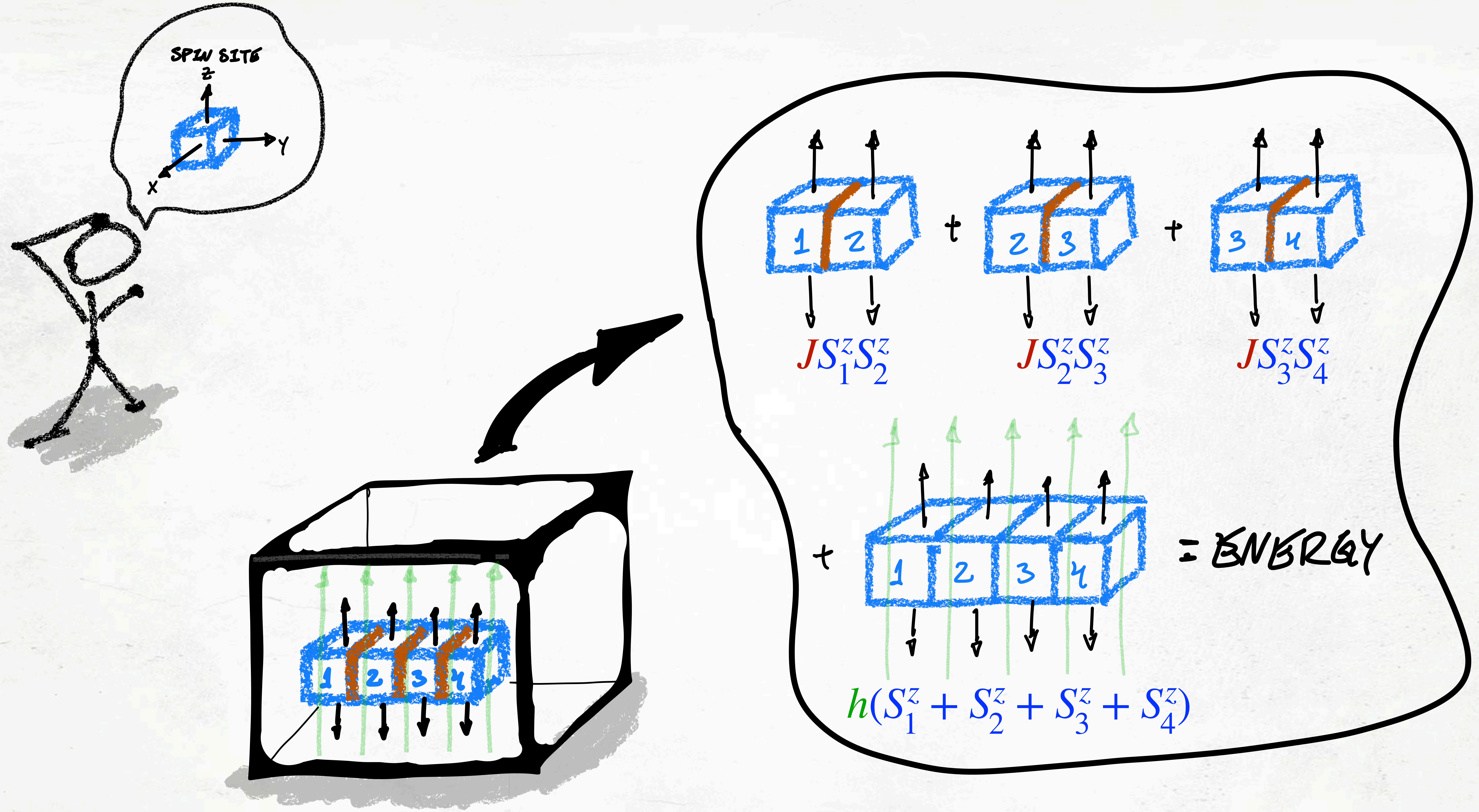
The mysterious box


?
LAB
ENERGY


SPIN SITE
Z
Y
X

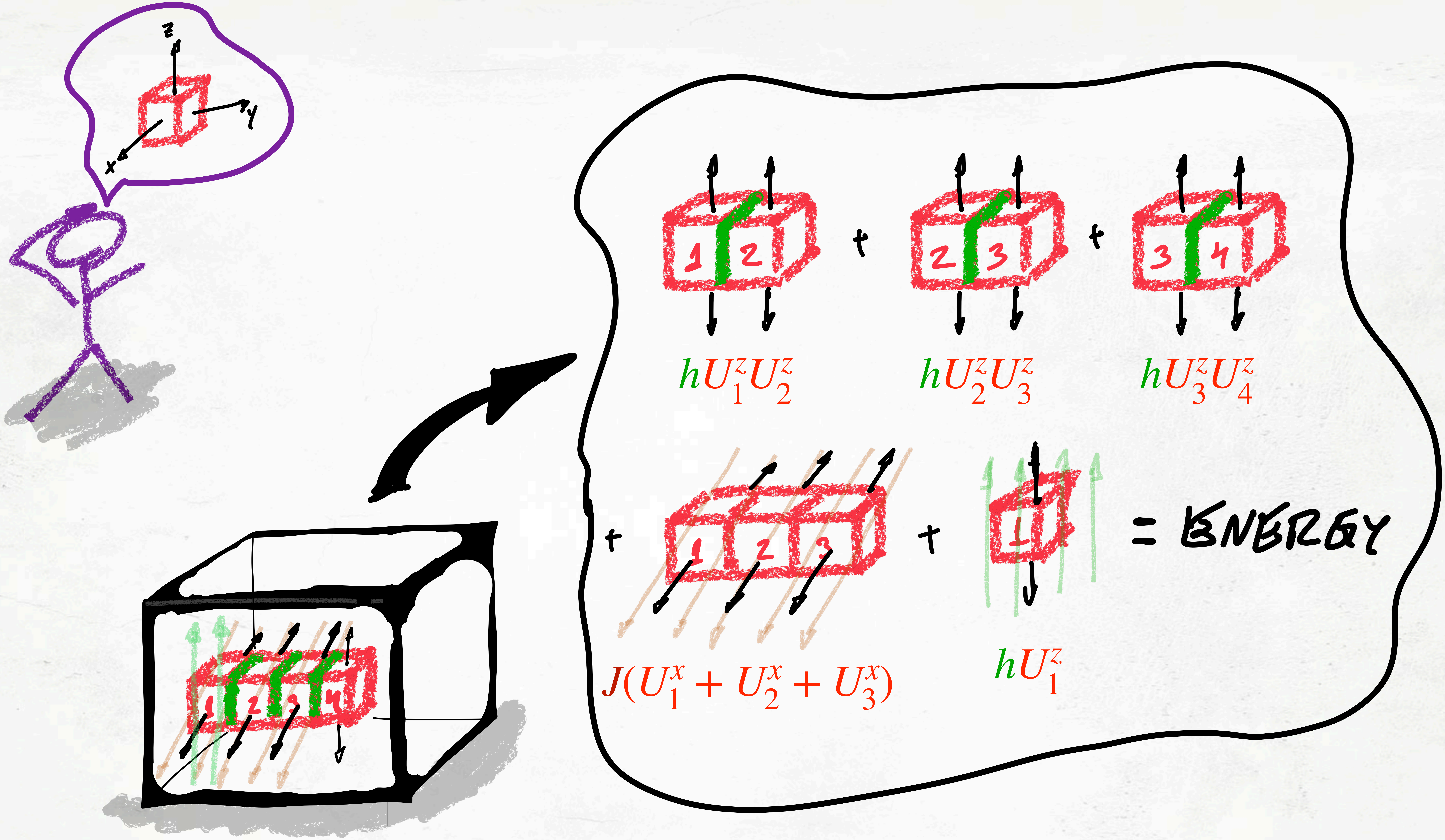


SPIN SITE
Z
Y
X
1
2
4
$JS_1^zS_2^z$
hS_4^z



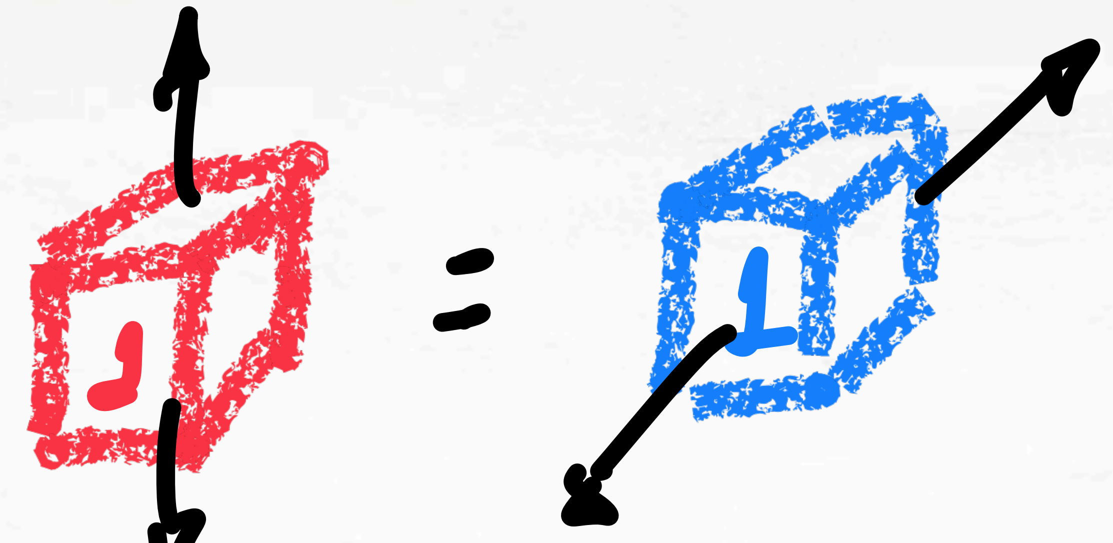
SPIN SITE
Z
Y
X
1 2 3 4
1 2
2 3
3 4
$JS_1^zS_2^z$ + $JS_2^zS_3^z$ + $JS_3^zS_4^z$
+ $h(S_1^z + S_2^z + S_3^z + S_4^z)$ = ENERGY


STOP!
ENERGY
LAB

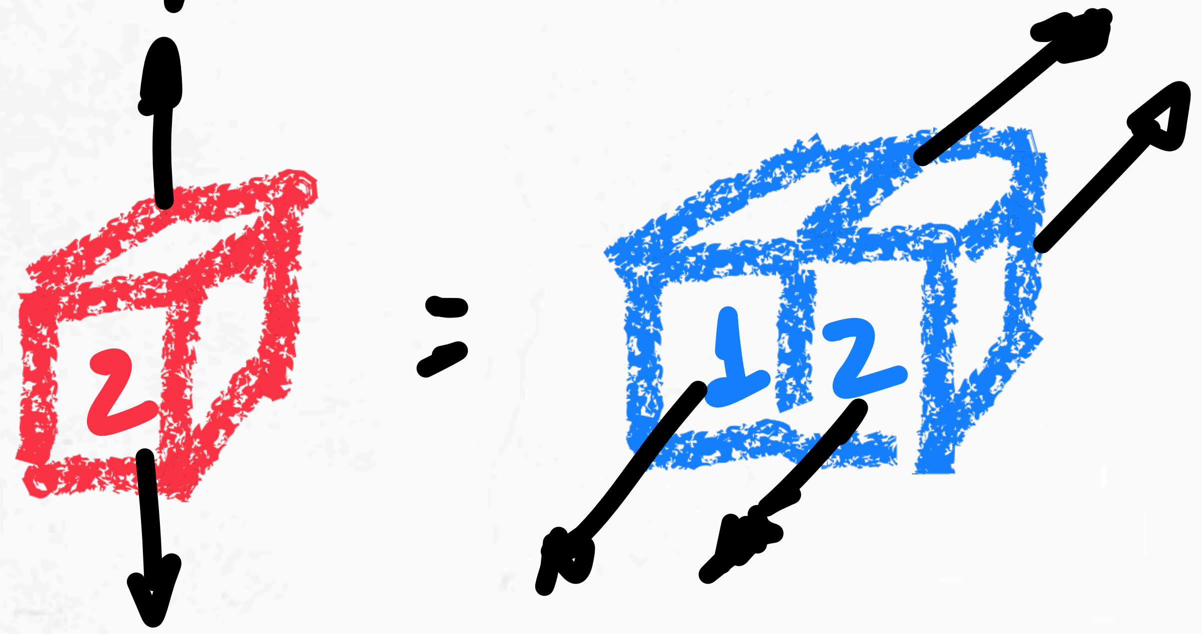

z
y
x
1 2
2 3
3 4
$hU^z_1U^z_2$ + $hU^z_2U^z_3$ + $hU^z_3U^z_4$
+ 1 2 3 + 1
$J(U^x_1 + U^x_2 + U^x_3)$
hU^z_1
= ENERGY
1 2 3 4


?!
LAB
ENERGY

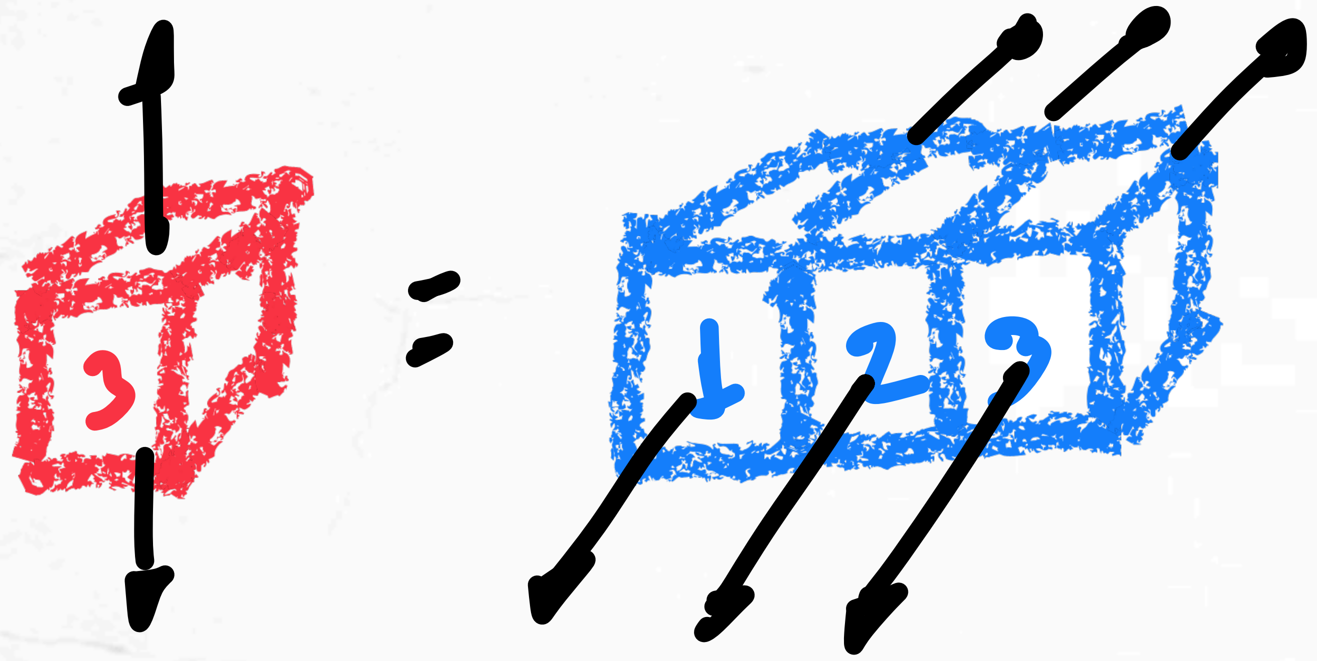
$U_1^z = S_1^x$



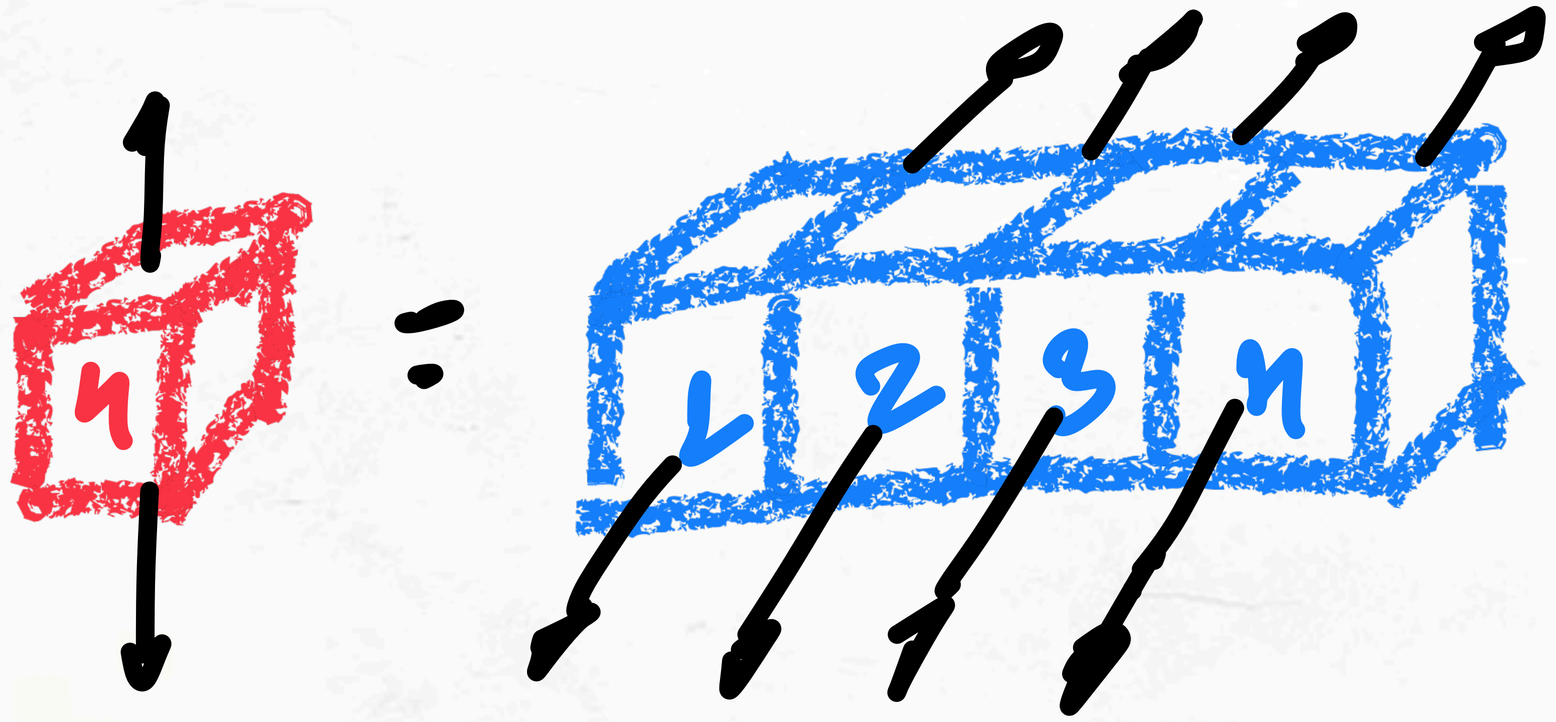
$U_2^z = S_1^x S_2^x$



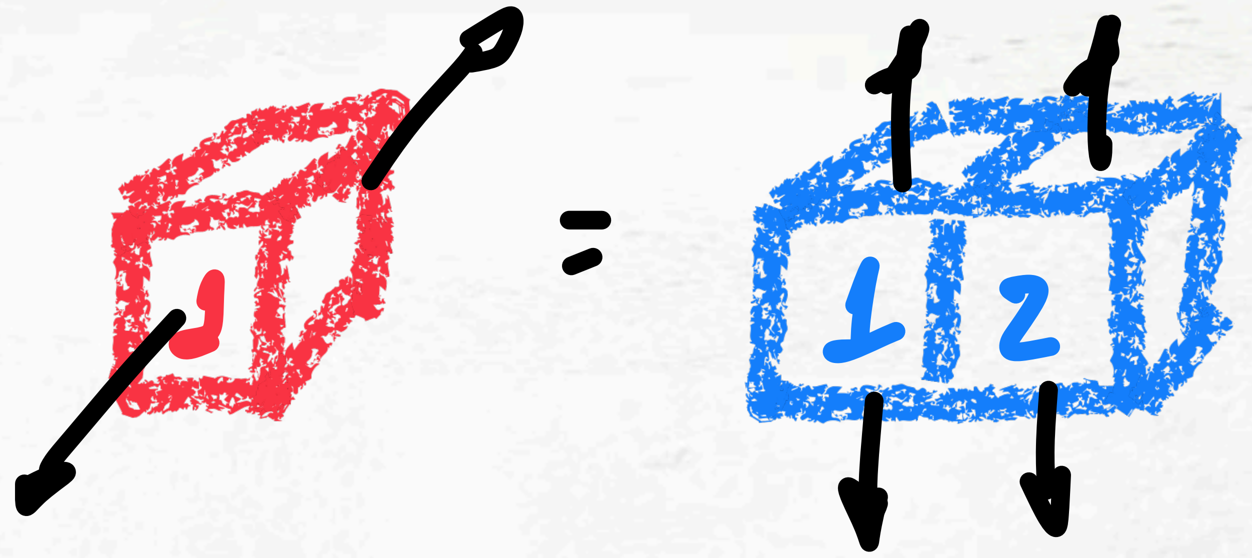
$U_3^z = S_1^x S_2^x S_3^x$



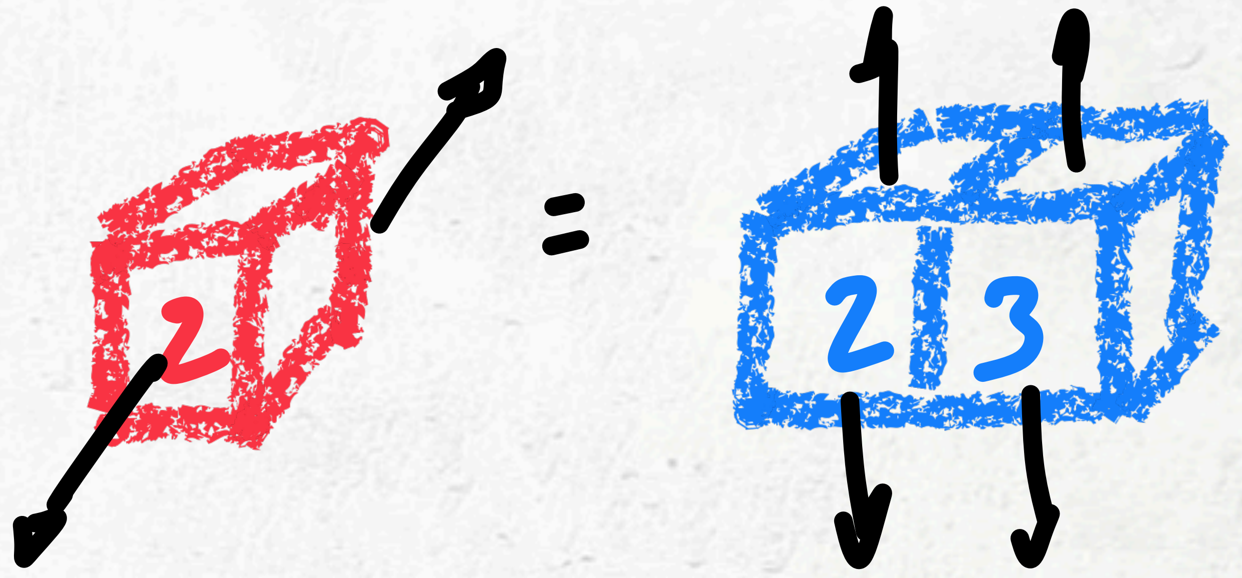
$U_4^z = S_1^x S_2^x S_3^x S_4^x$



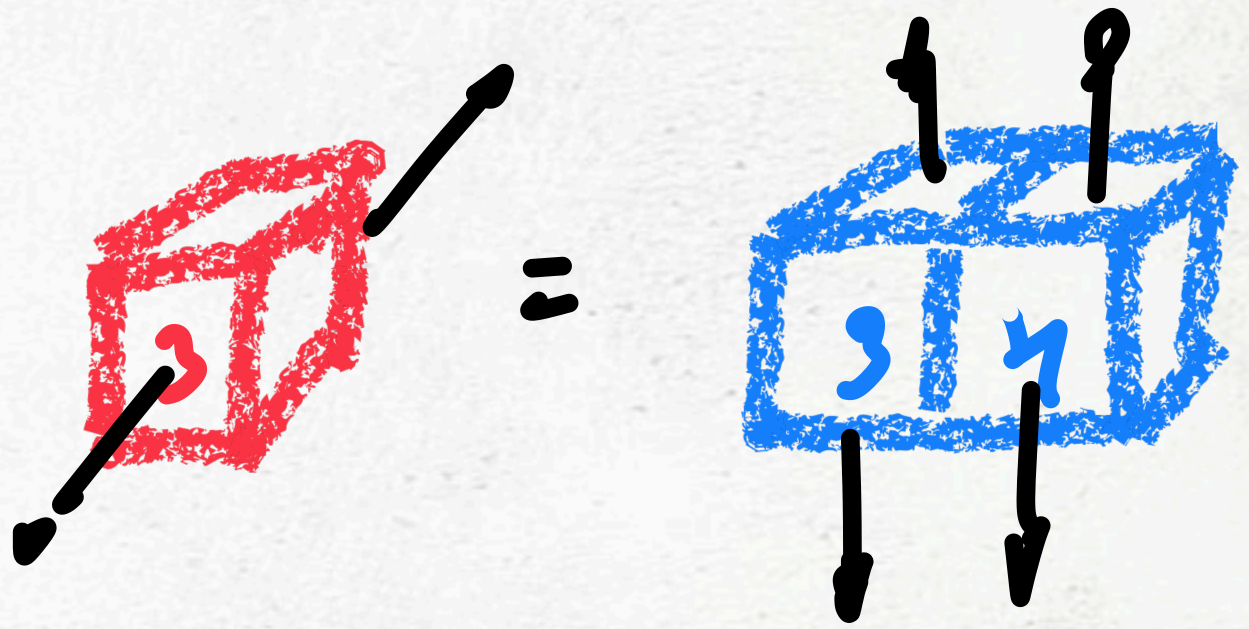
$U_1^x = S_1^z S_2^z$



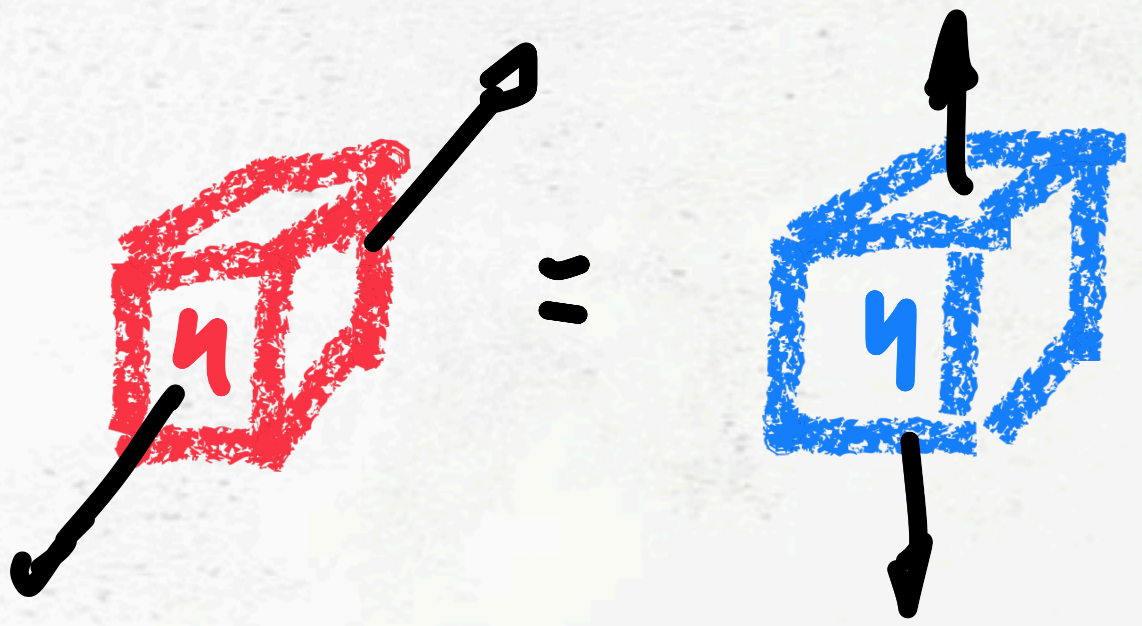
$U_2^x = S_2^z S_3^z$



$U_3^x = S_3^z S_4^z$



$U_1^x = S_4^z$



$U_1^z = S_1^x$

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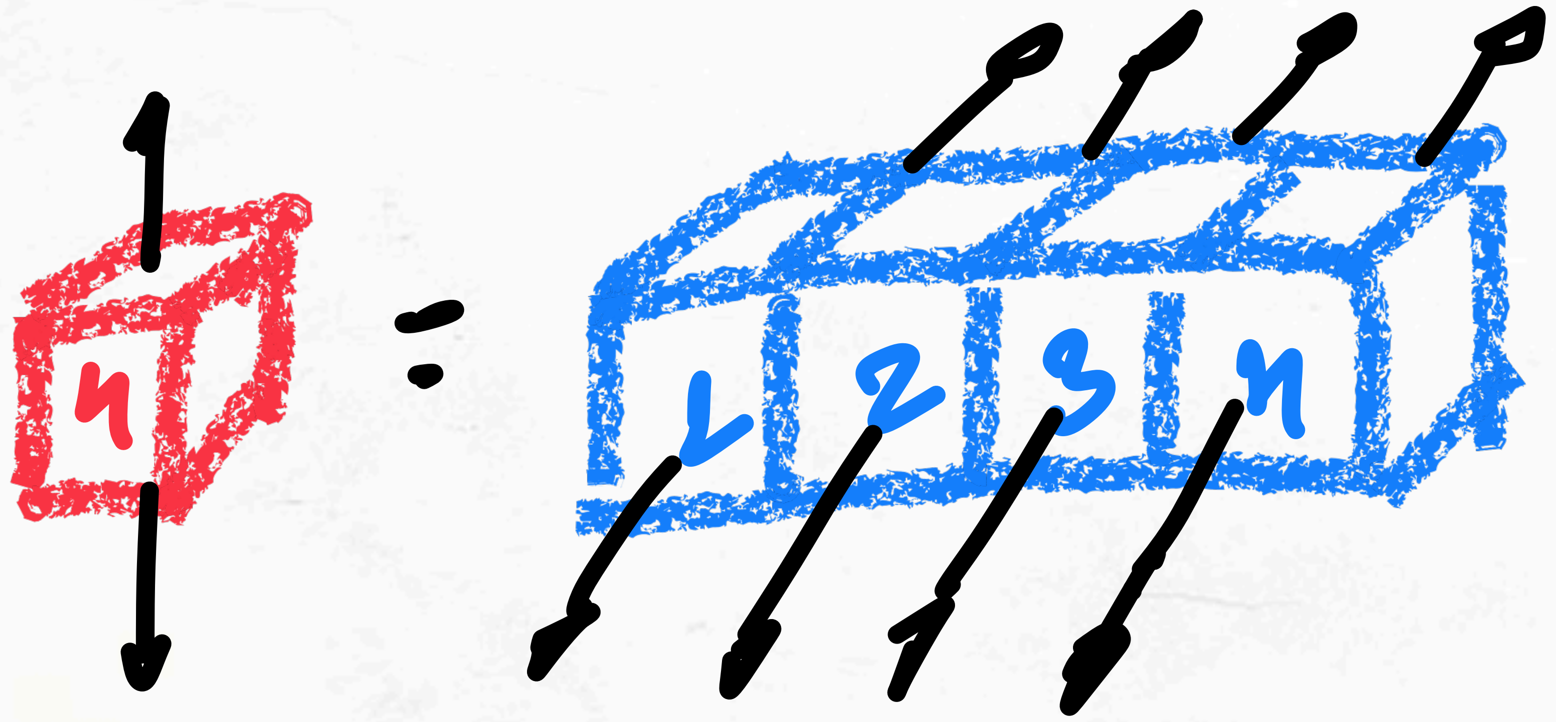
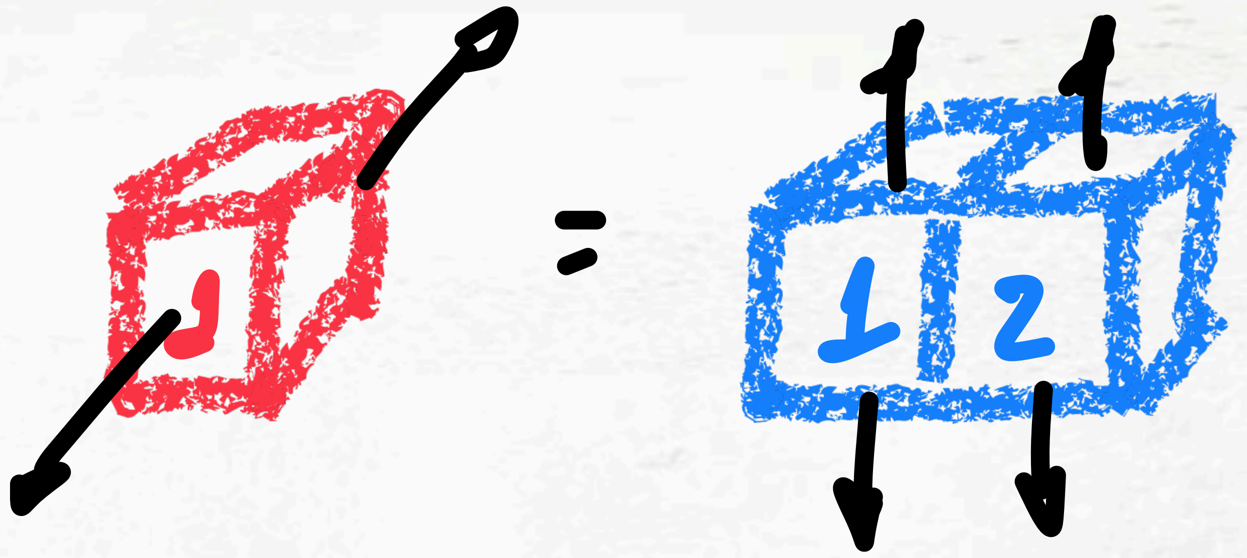


$U_1^x = S_1^z S_2^z$



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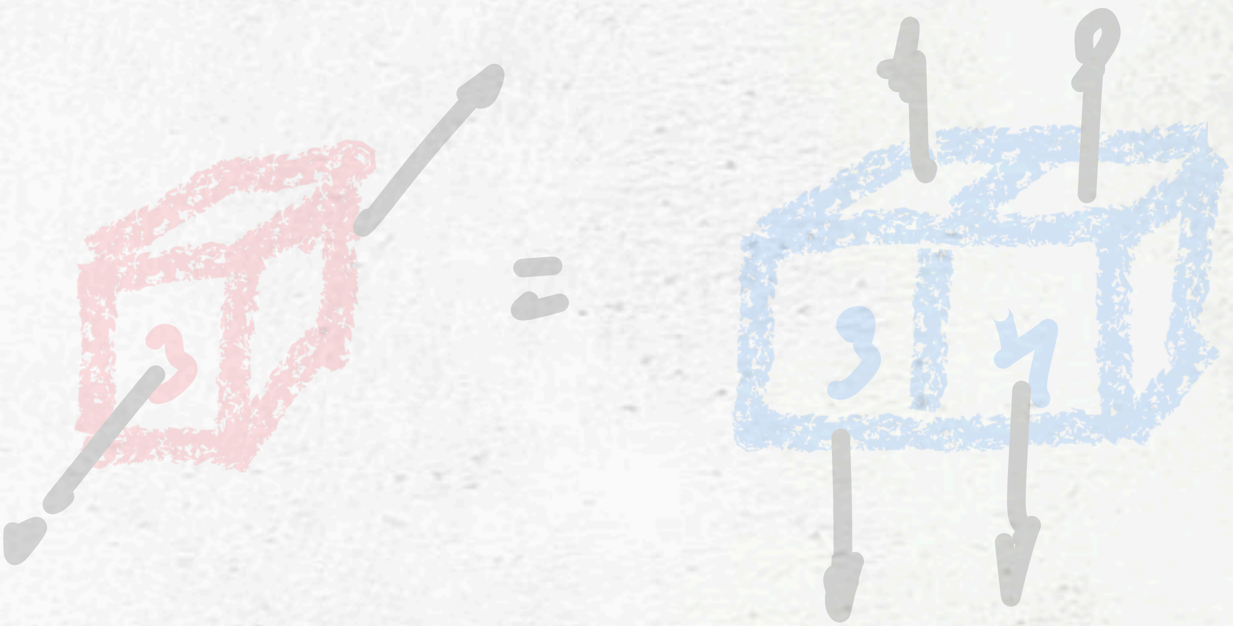
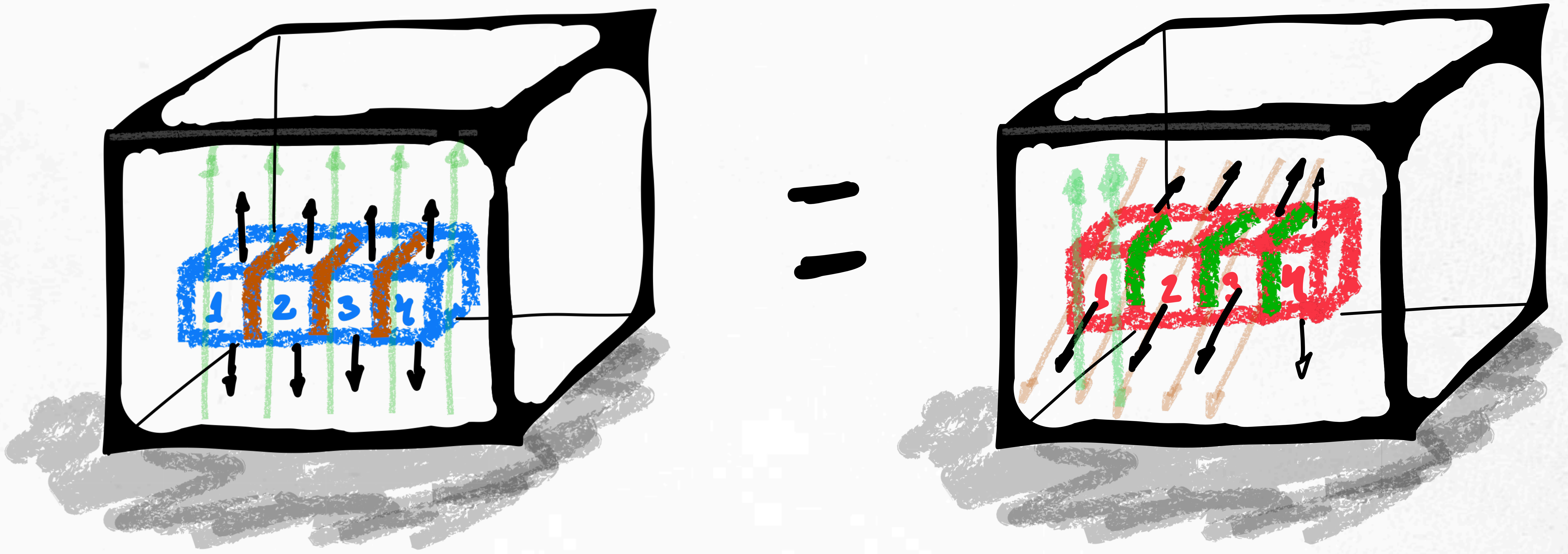


$U_4^x = S_4^z$

Formally, this is called a **duality**.



$$H = J\sum_{i=1}^{n-1} S_i^z S_{i+1}^z + h\sum_{i=1}^{n} S_i^z = J\sum_{i=1}^{n} U_i^x + h\sum_{i=1}^{n-1} U_i^z U_{i+1}^z - JU_n^x + hU_1^z$$

$$U_i^z = \prod_{j\leq i} S_j^x, \quad U_i^x = S_i^z S_{i+1}^z, \quad U_n^x = S_n^z$$

So, which one is it? How do we find out?

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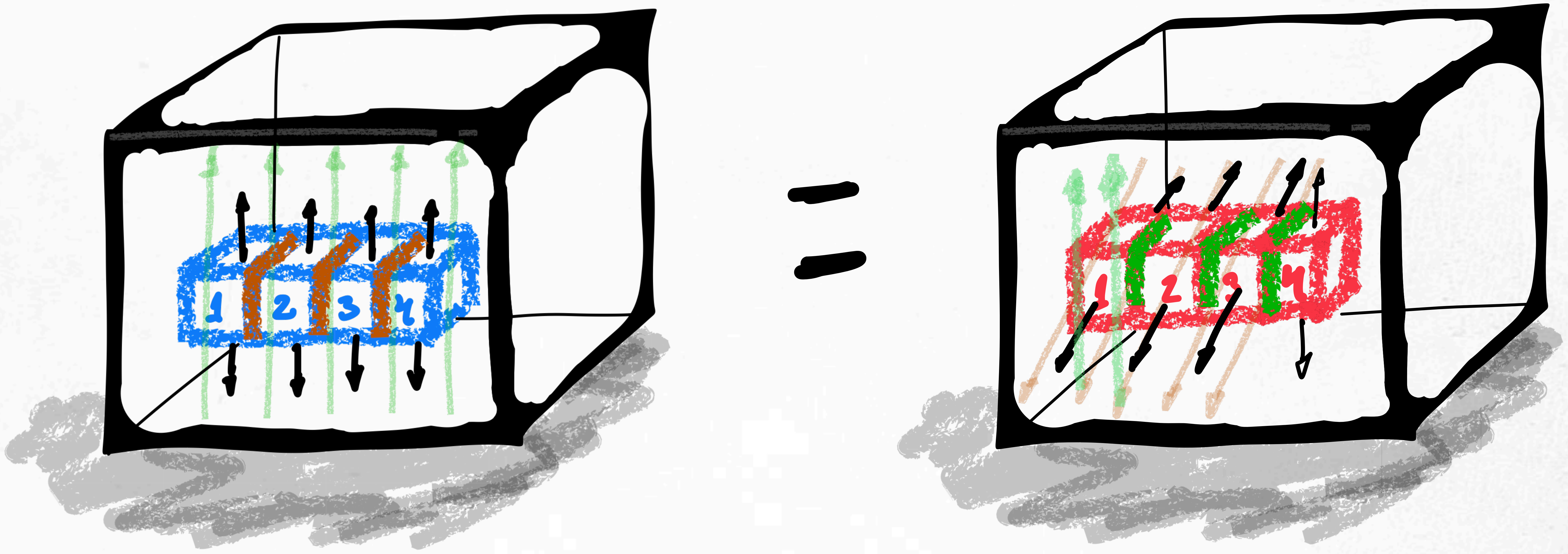

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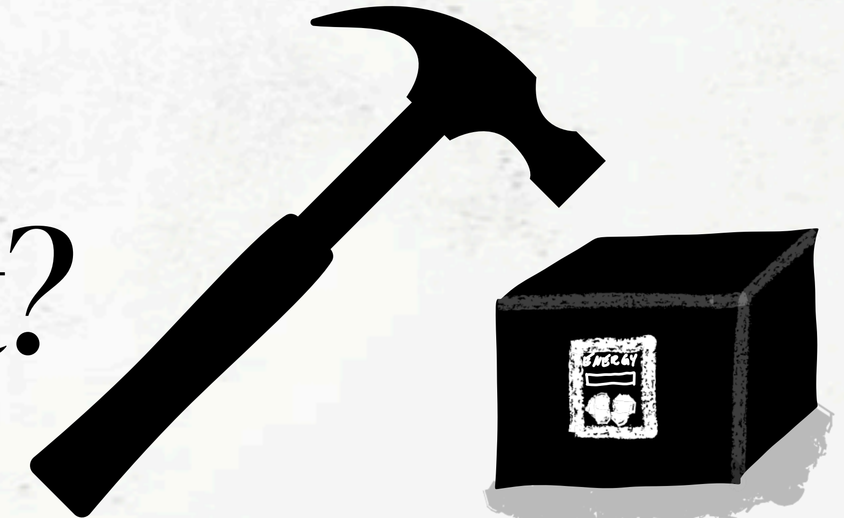
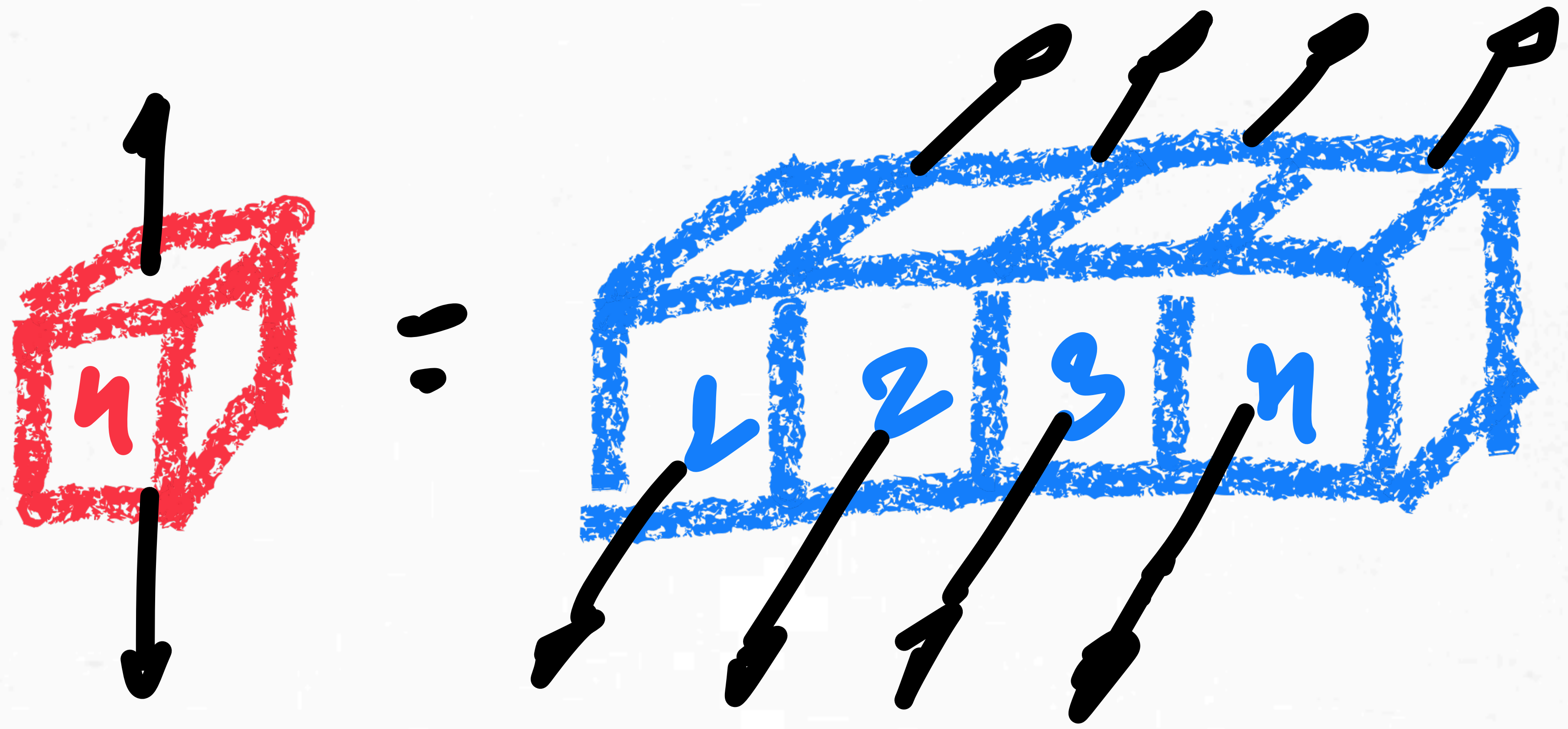


What if you cannot open the box?



We see that each description of what's inside assumes a different notion of the system, different notion of which parts are interacting, and therefore different notion of what's a neighboring site!

How do we make sense of that?

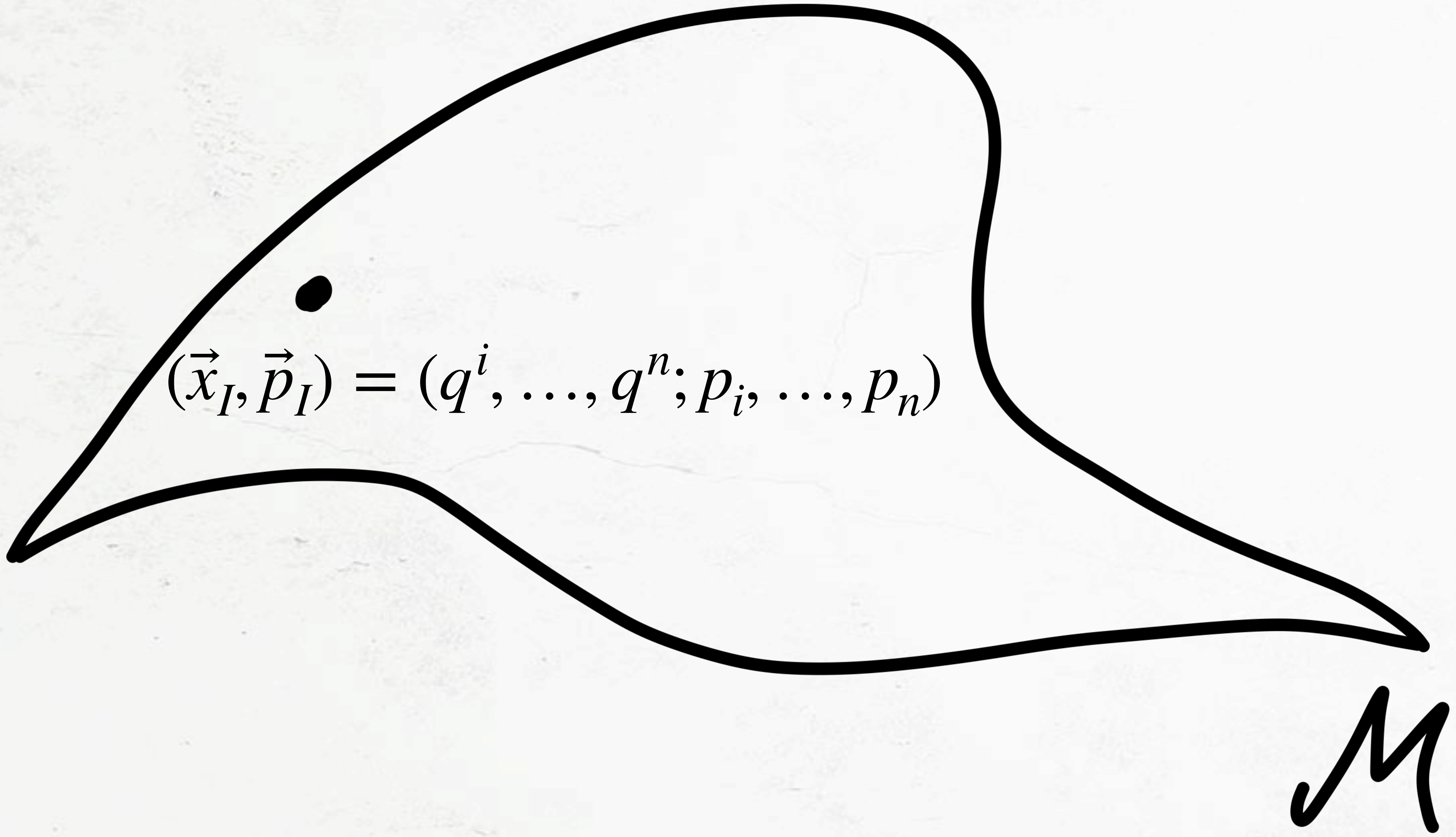
Classical Mechanics

What is a system*?

System's physical characteristics $f \in \mathscr{F}$:

$$f(q,p) : \mathscr{M} \to \mathbb{R}$$

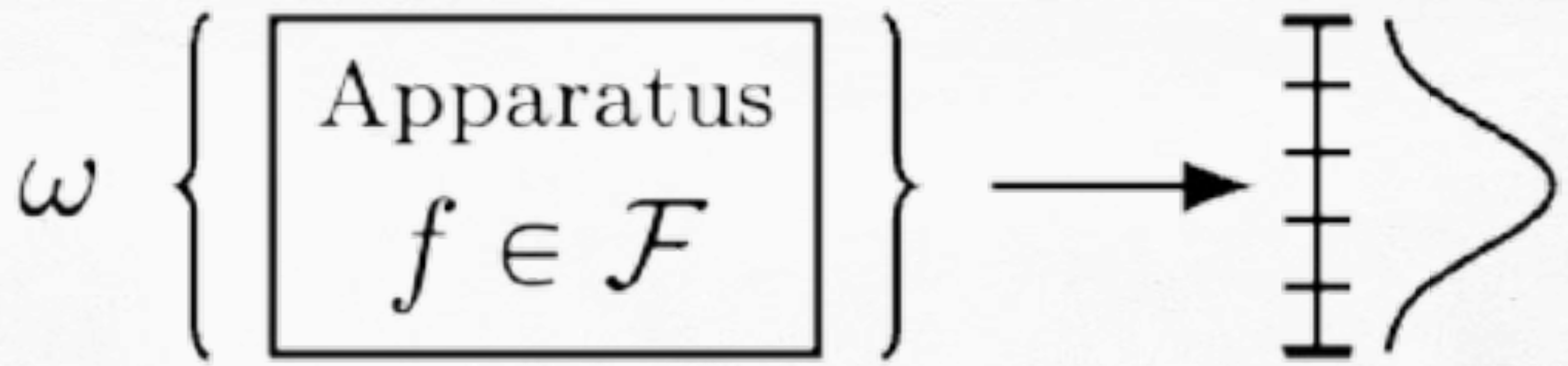
of real infinitely differentiable functions (C^∞).



These physical characteristics are what we call **observables**, thus the algebra of observables in classical mechanics is the algebra of real-valued smooth functions defined on the phase space $\mathscr{M}$.

*We will focus on the Hamiltonian formulation and will avoid constrained systems for now..

What is a state?



Introduce the distribution function $\rho_\omega(p, q)$, making connection to the usual description of the state of a system in statistical mechanics:

Observable's expectation value

$$\langle f \rangle_\omega \equiv \int_{\mathscr{M}} f(p, q)\rho_\omega(p, q)\, dpdq$$

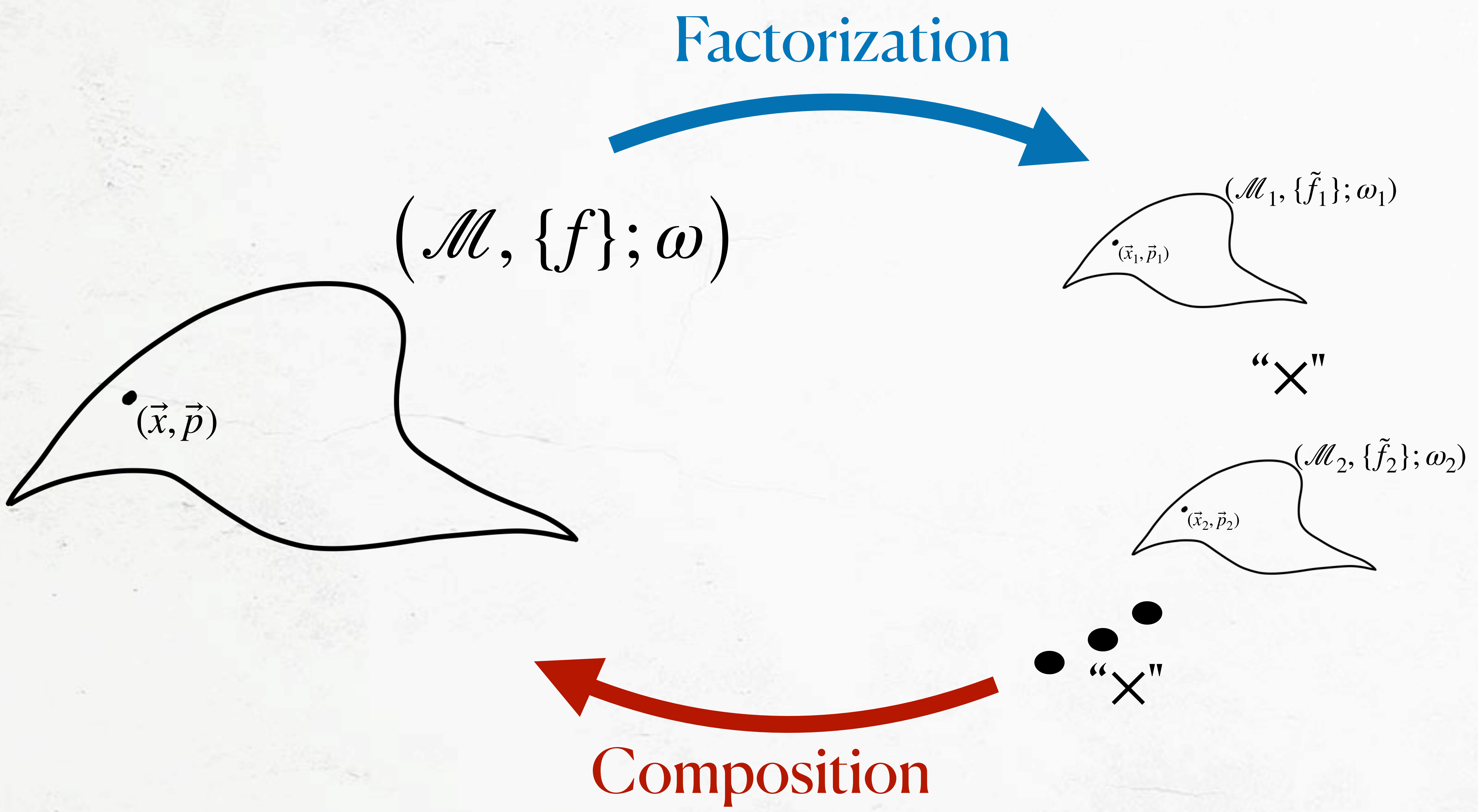
We further require that $f \in L^\infty(\mathscr{M}, \rho_\omega)$, the space of measurable functions that are bounded. Two types of states:

$$\rho_{pure}(p, q) = \delta(q - q^0)\delta(p - p_0)$$

$$\rho_{mix}(p, q) = \sum_{i=1}^{n} a_i \delta(q - q^i)\delta(p - p_i)$$

What is a *sub*-system?

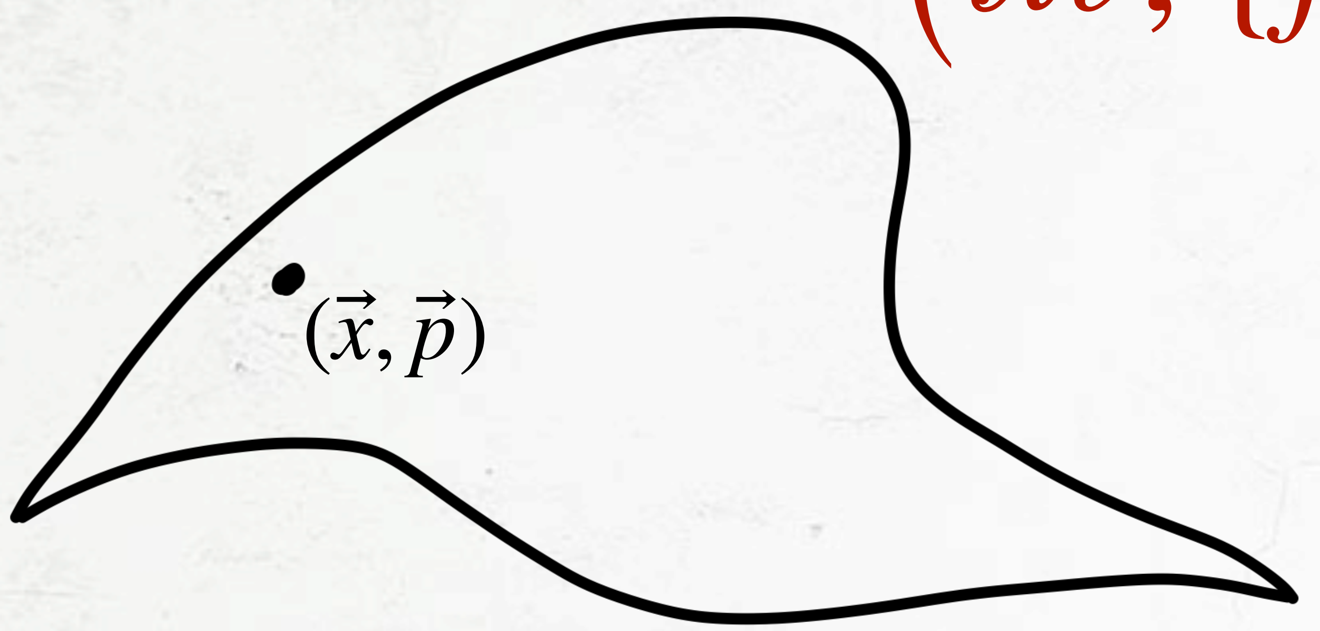
One of the most defining features of quantum mechanics has to do with the notion of **independent subsystems**, which is deeply connected with the notion of classical spacetime locality.



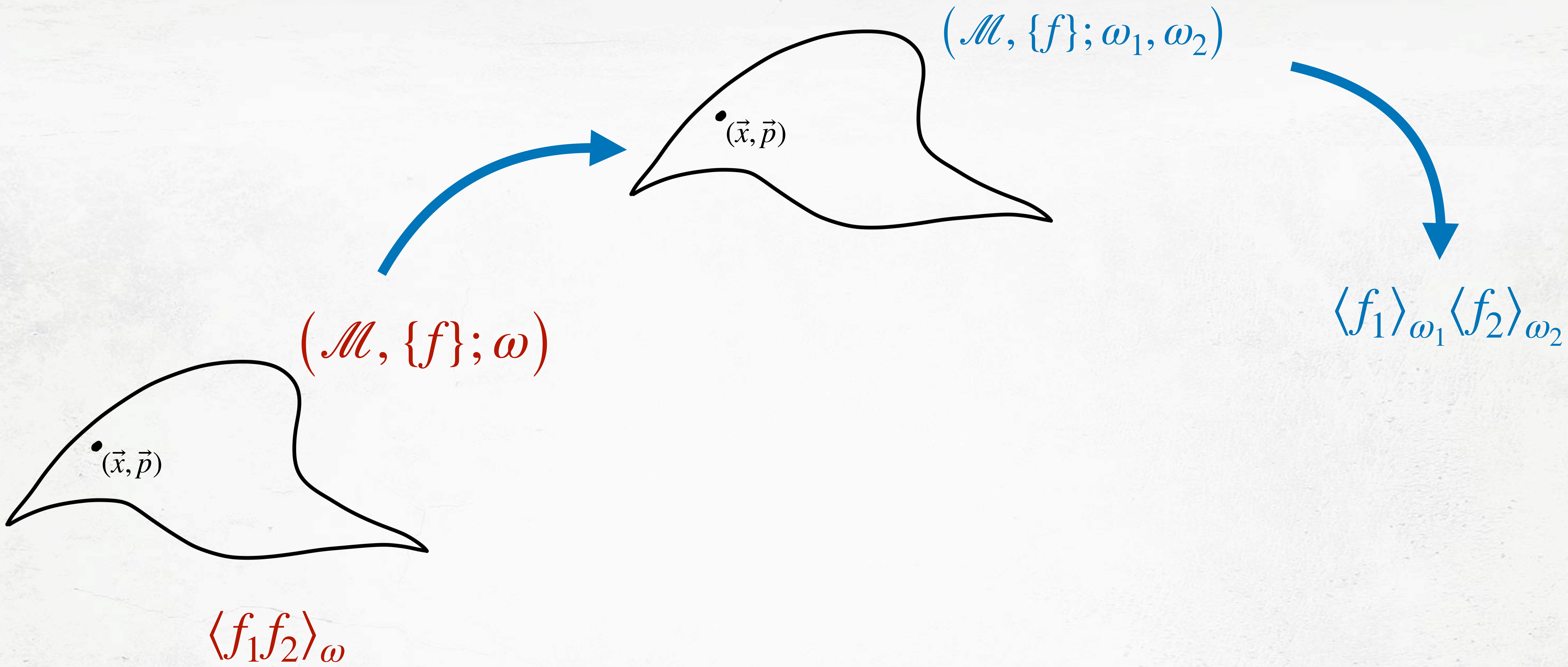
Independent Events:

$$P(A \ and \ B) = P(A) \cdot P(B)$$

$$\left(\mathscr{M}, \{f\}; \omega\right)$$



$$\langle f_1 f_2 \rangle_\omega$$


$(\mathscr{M}, \{f\}; \omega_1, \omega_2)$
$(\vec{x}, \vec{p})$
$\langle f_1 \rangle_{\omega_1} \langle f_2 \rangle_{\omega_2}$
$(\mathscr{M}, \{f\}; \omega)$
$(\vec{x}, \vec{p})$
$\langle f_1 f_2 \rangle_{\omega}$

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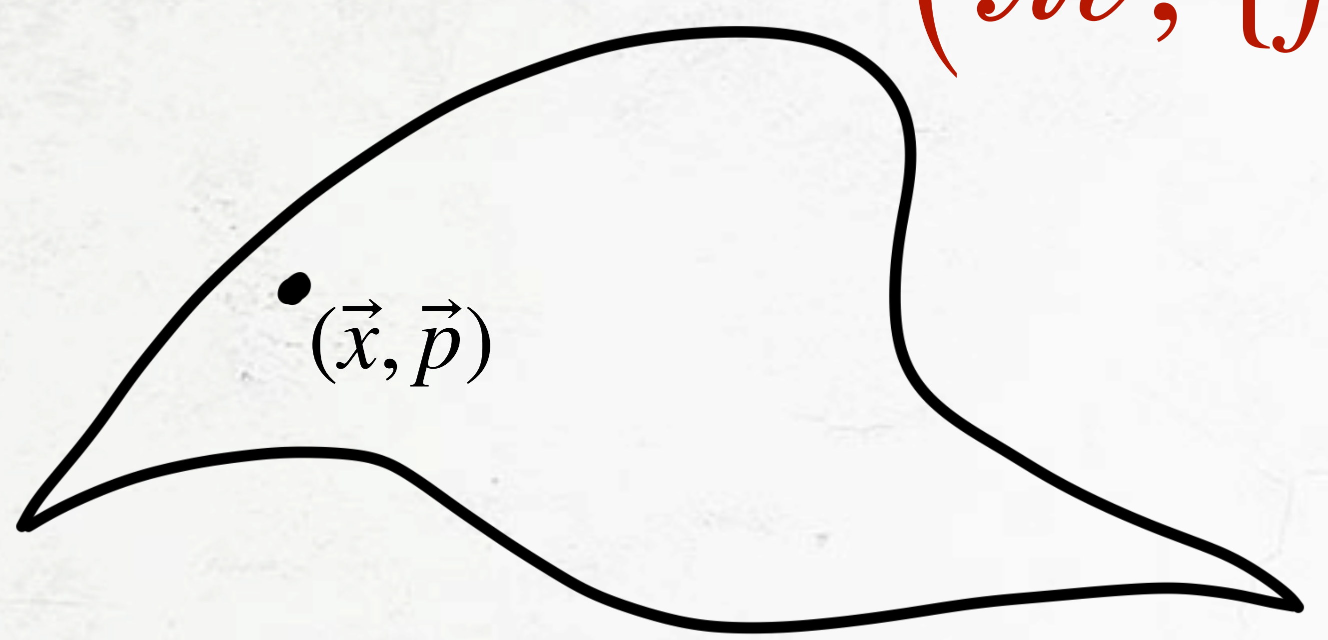

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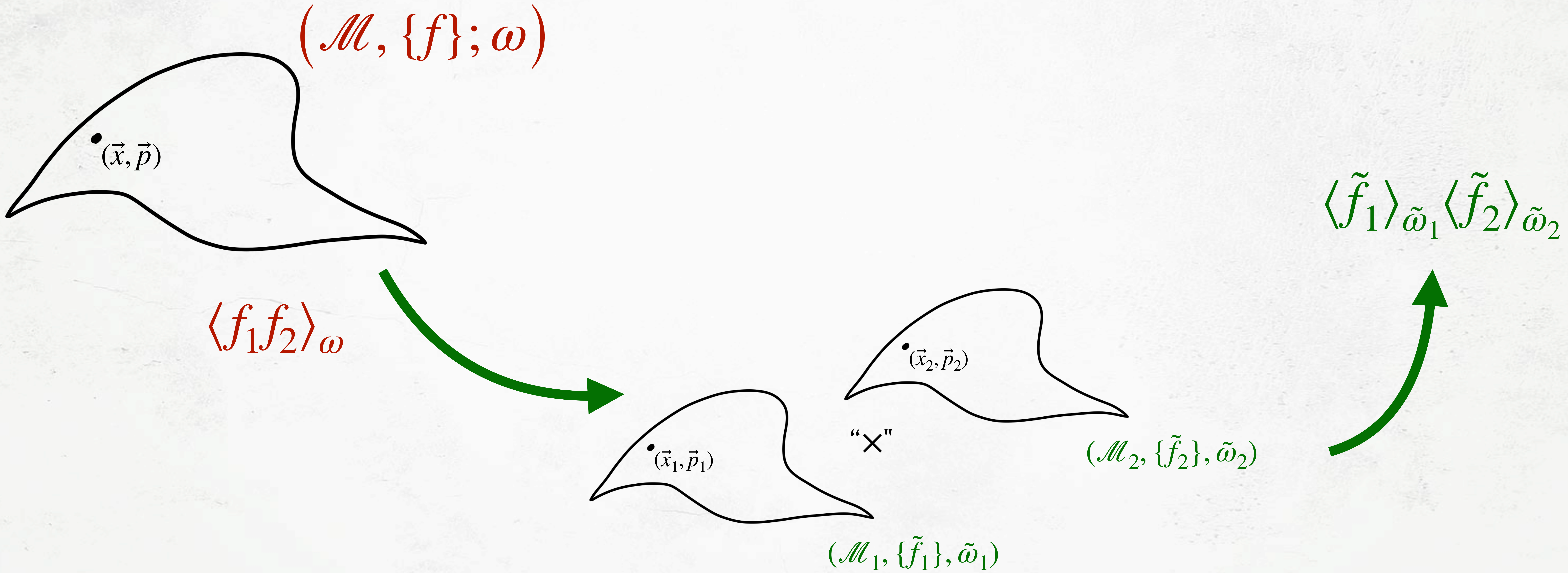
$(\mathscr{M}, \{f\}; \omega)$
$(\vec{x}, \vec{p})$
$\langle f_1 f_2 \rangle_\omega$
$(\vec{x}_1, \vec{p}_1)$
$(\vec{x}_2, \vec{p}_2)$
"×"
$(\mathscr{M}_2, \{\tilde{f}_2\}, \tilde{\omega}_2)$
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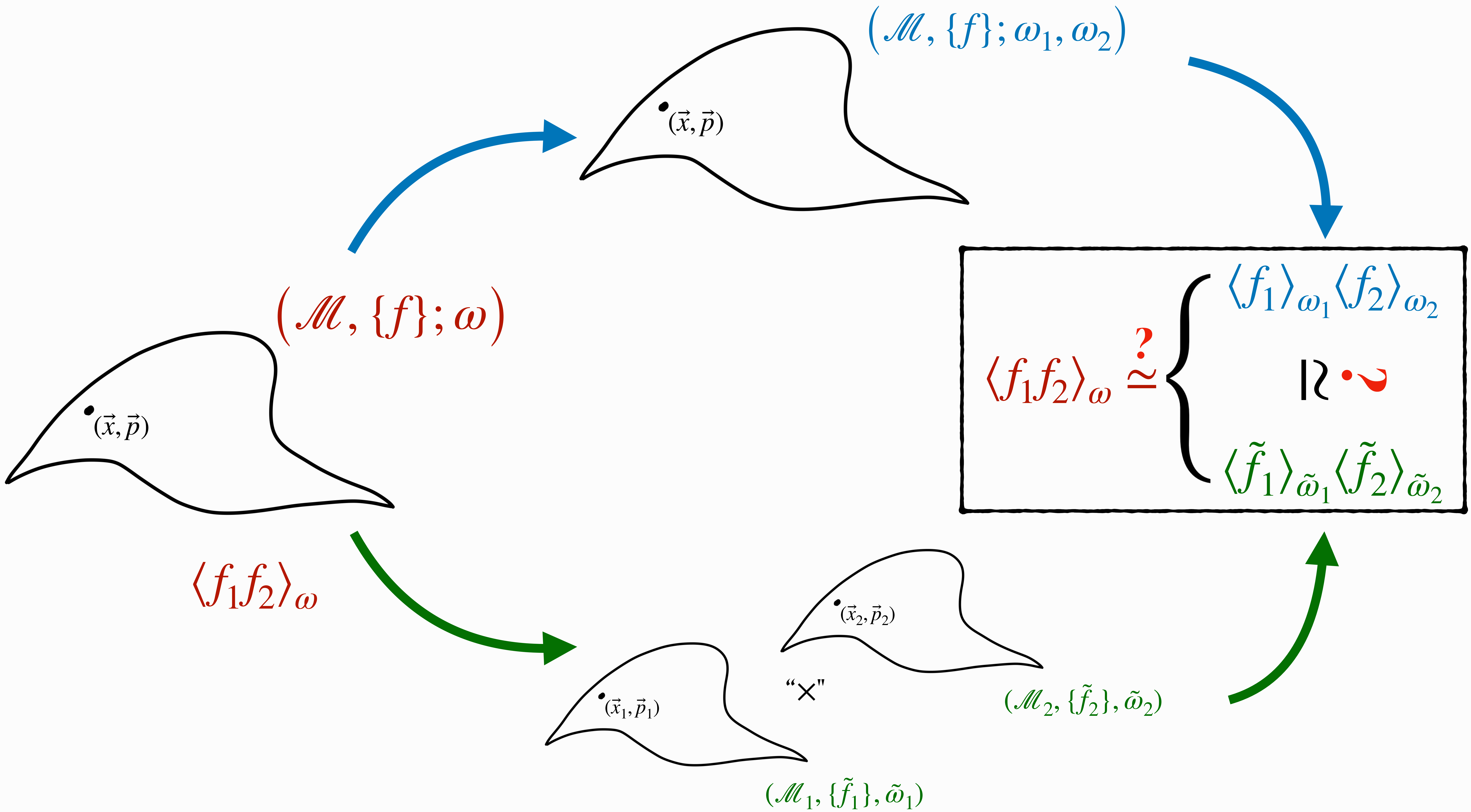
$\langle \tilde{f}_1 \rangle_{\tilde{\omega}_1} \langle \tilde{f}_2 \rangle_{\tilde{\omega}_2}$



$(\mathscr{M}, \{f\}; \omega_1, \omega_2)$
$(\vec{x}, \vec{p})$
$(\mathscr{M}, \{f\}; \omega)$
$(\vec{x}, \vec{p})$
$\langle f_1 f_2 \rangle_\omega$
$\langle f_1 f_2 \rangle_\omega \overset{?}{\simeq}$
$\langle f_1 \rangle_{\omega_1} \langle f_2 \rangle_{\omega_2}$
$\langle \tilde{f}_1 \rangle_{\tilde{\omega}_1} \langle \tilde{f}_2 \rangle_{\tilde{\omega}_2}$
$(\vec{x}_2, \vec{p}_2)$
$(\vec{x}_1, \vec{p}_1)$
"×"
$(\mathscr{M}_2, \{\tilde{f}_2\}, \tilde{\omega}_2)$
$(\mathscr{M}_1, \{\tilde{f}_1\}, \tilde{\omega}_1)$

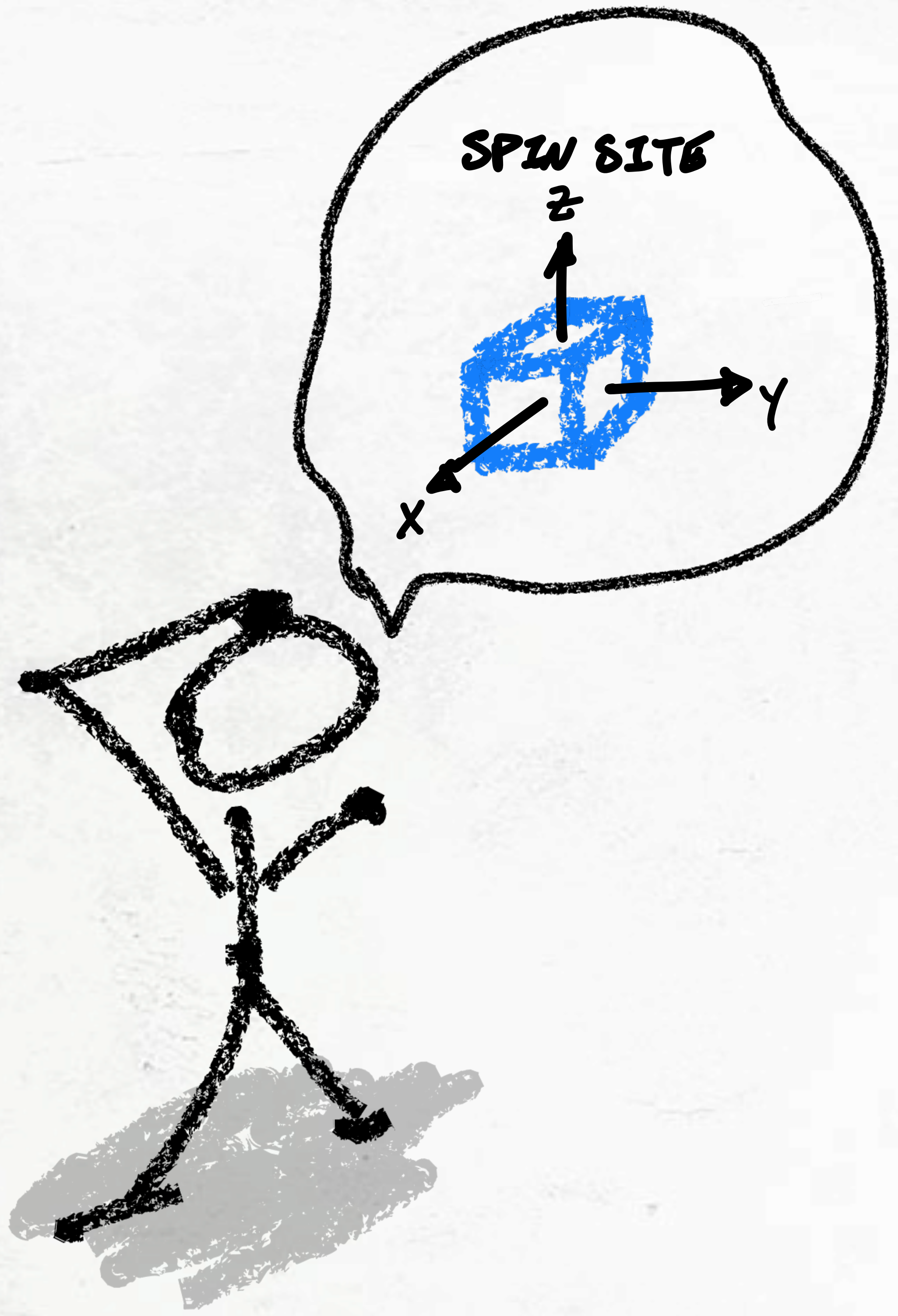
$\langle \mathcal{O}_I \mathcal{O}_J \rangle_\omega = \langle \mathcal{O}_I \rangle_{\omega_I} \langle \mathcal{O}_J \rangle_{\omega_J}$: *measurements* and *state preparations* can be carried out **independently**

	Notion of Separability	Notion of Subsystem
CM	$L_I^\infty(\mathcal{M}) \simeq L^\infty(\mathcal{M}_I) \times \tilde{\mathbf{1}}_J$	$\mathcal{M}_I$ of states for C-subsystem I

Classical Mechanics	$\langle f_1 f_2 \rangle_\omega \simeq \langle f_1 \rangle_{\omega_1} \langle f_2 \rangle_{\omega_2} \simeq \langle \tilde{f}_1 \rangle_{\tilde{\omega}_1} \langle \tilde{f}_2 \rangle_{\tilde{\omega}_2}$
Quantum Mechanics	
Quantum Field Theory	

[Malament, '96]

How space hides in quantum mechanics



The *mysterious box* illustrates that quantum systems only described by their global $\mathscr{H}$, $|\Psi\rangle$, and H, might not fully characterize its classical properties, such as the notion of a "site" - a location in space.

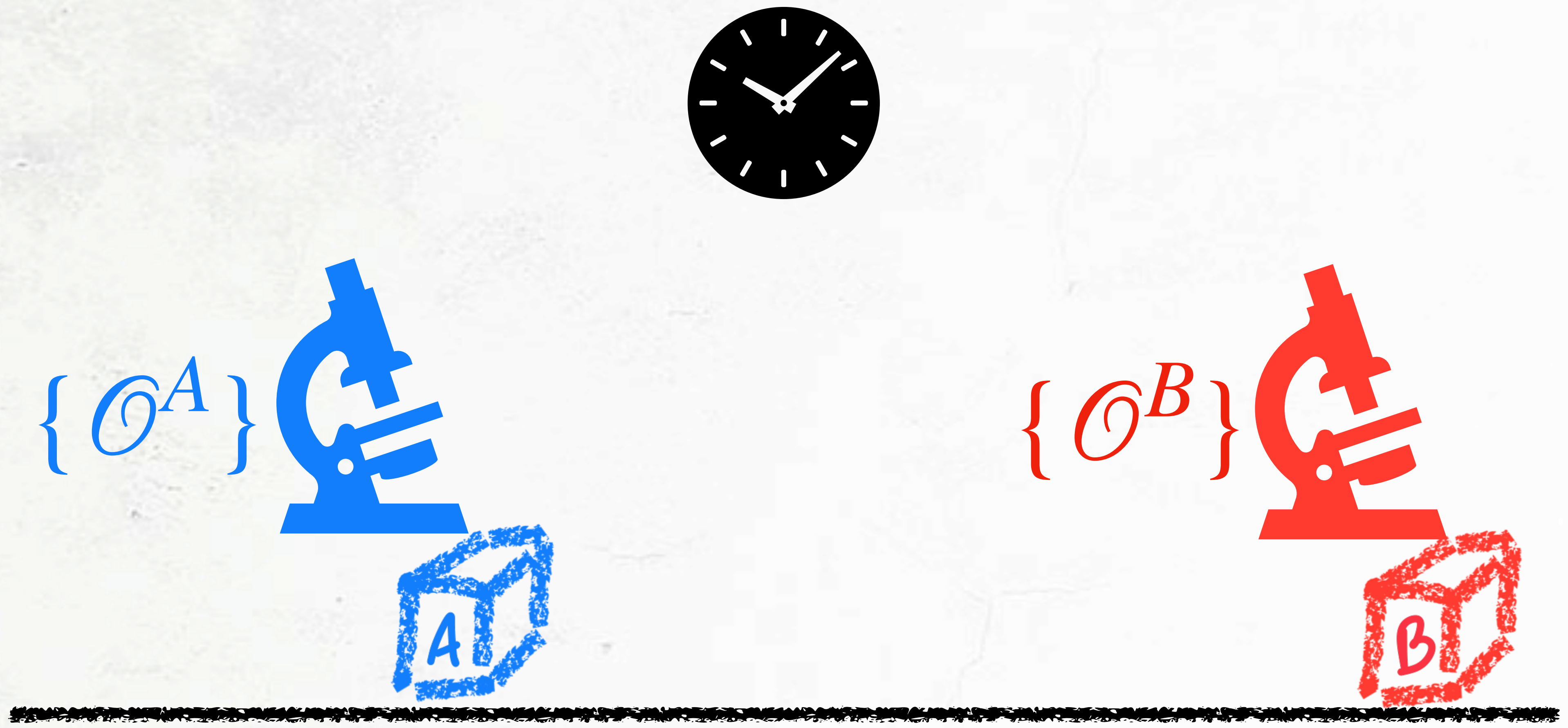
This is puzzling because typically, the situation is reversed: *we impart classical notions of location into our quantum mechanical descriptions*. For example, for every spin site located at position i, we assign a $\mathbb{C}^2$-Hilbert space factor, such that the global Hilbert space is

$$\mathscr{H} = \bigotimes_{i=1}^{n} \mathbb{C}^2.$$

This is just a simple example of a general attitude: we typically assign a Hilbert space $\mathscr{H}_{\text{system}}$ to each classical subsystem in a given spatial (or more generally, spacetime) region. Thus, some notion of space, or at very least, some notion of location in space seems to leak into our quantum-mechanical descriptions.

Let's make that explicit.

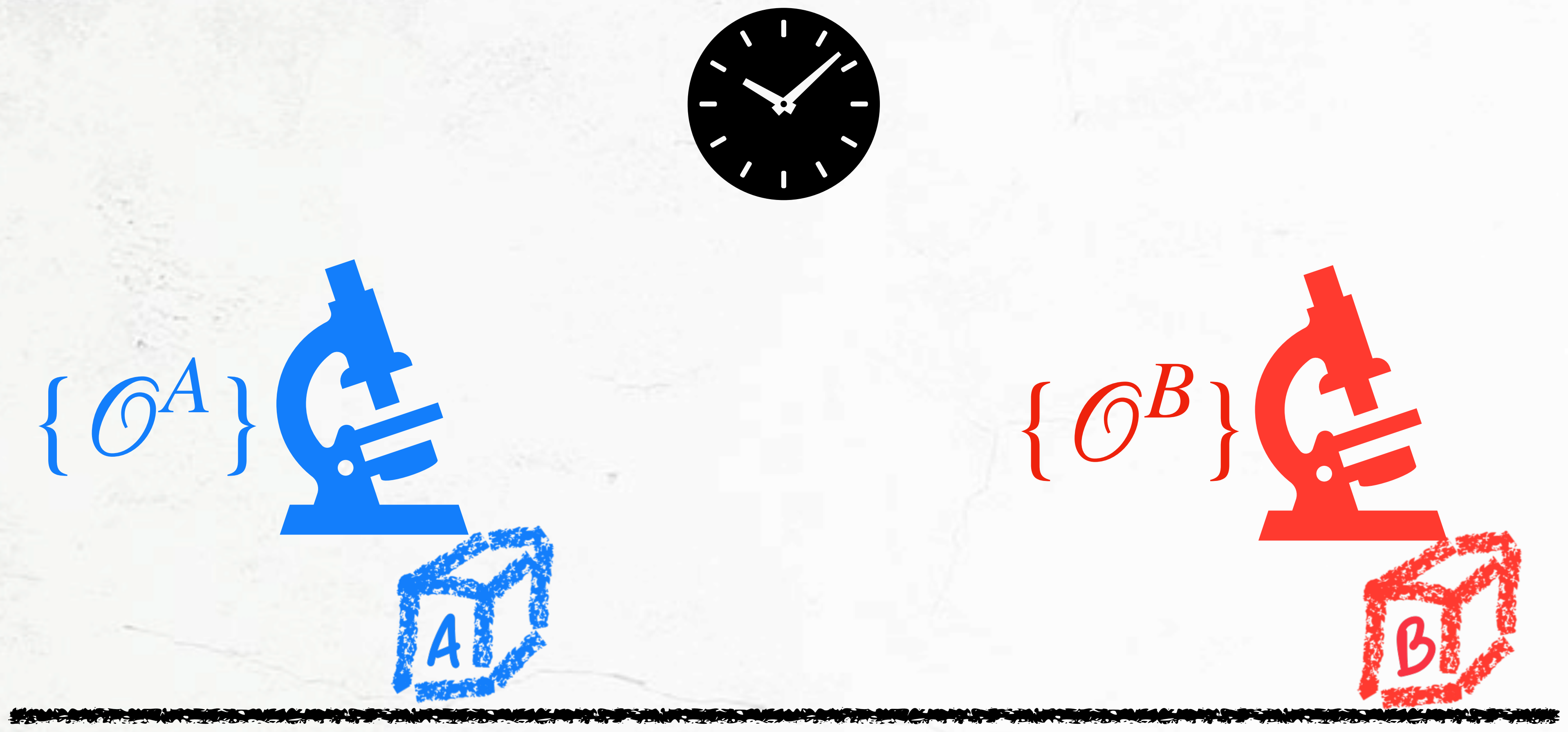
Back at your lab, you consider an extended quantum mechanical system divided into two parts, A and B, with global Hilbert space $\mathscr{H}$. They can be interacting or not, very much like the spin sites in the box.



Each part is surrounded by measuring devices, defining the sets of controlled observables $\{\mathcal{O}^A\}$ and $\{\mathcal{O}^B\}$, while a clock fixed against the wall sets up a reference frame for the entire lab. Thus, at any fixed time in the lab, these parts are spacelike separated, and you expect measurements on A to not affect measurements on B, at least within the time interval light takes to travel between the systems. This is what we call *macrocausality*.

Essentially, you aim to ensure that the statistics of these parts are independent, implementing an operational notion of macrocausality. For example, the statistics associated with any observable in part A are obtained by $\mathrm{tr}\,[\rho\mathcal{O}^A]$, where ρ is the state of the system.

Now consider two measurements, one in each part of the system, defining two spacetime events that are spacelike separated. Each measurement updates the state of the system, but their statistics should be the same whether we consider the original state, ρ, or the updated one, ρ'.



Therefore, we demand $\mathrm{tr}\,[\rho\mathcal{O}^A] = \mathrm{tr}\,[\rho'\mathcal{O}^A]$, and similarly for B. It turns out that this is possible if, *and only if*, the set of observables for each part commute,

$$[\mathcal{O}^A, \mathcal{O}^B] = 0.$$

This argument can be generalized to subalgebras associated with arbitrary causally disconnected spacetime regions, where then goes by the name *microcausality*, or Einstein separability.

[Zanardi et al., '04]
[Cotler et al., '17]

Now that the physical intuition is properly encoded in the math, let's be more precise and general. Consider a finite-dimensional Hilbert space $\mathscr{H}$ and a collection of subalgebras of observables corresponding to each subsystem satisfying the following properties:

- *Local acessibility:* Each $\{\mathscr{O}^I\}$ corresponds to a set of controllable observables, where I labels the different systems;
- *Subsystem independence:* $[\mathscr{O}^I, \mathscr{O}^J] = 0$, where $I \neq J$, now understood as a consequence of operational macrocausality;
- *Completeness:* $\bigvee_I \mathscr{O}^I \cong L(\mathscr{H})$, there is a minimal inclusion of the subalgebras isomorphic to the full algebra.

Once these properties are in place, then the set of subalgebras induces a tensor product structure (TPS):

Definition: A TPS $\mathscr{T}$ of Hilbert space $\mathscr{H}$ is an equivalence class of isomorphisms $T : \mathscr{H} \to \otimes_i \mathscr{H}_i$, where $T_1 \sim T_2$ whenever $T_1 T_2^{-1}$ can be written as a product of local unitaries $\otimes_i U_i$ and permutations of subsystems.

[Fewster, '16]
[EmerGe, 24xx]

Classical Mechanics	$\langle f_1 f_2\rangle_\omega \simeq \langle f_1\rangle_{\omega_1}\langle f_2\rangle_{\omega_2} \simeq \langle \tilde{f}_1\rangle_{\tilde{\omega}_1}\langle \tilde{f}_2\rangle_{\tilde{\omega}_2}$
Quantum Mechanics	$\langle O_A O_B\rangle_\omega \simeq \langle O_A\rangle_{\omega_A}\langle O_B\rangle_{\omega_B} \simeq \langle \tilde{O}_A\rangle_{\tilde{\omega}_A}\langle \tilde{O}_B\rangle_{\tilde{\omega}_B}$
Quantum Field Theory	

QFT

1. QED coupled with scalar field

[Donelly&Giddings, '16]
[Fewster, '16]

1. QED coupled with scalar field

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - |(\partial_\mu - iqA_\mu)\phi|^2 - \frac{1}{2}m^2|\phi|^2}$$

$$\begin{cases} A_\mu(x) \to A_\mu(x) + \partial_\mu \Lambda(x) \\ \phi(x) \to e^{-iq\Lambda(x)}\phi(x) \end{cases}$$

$[\phi(x), \phi(y)] = 0,$ for x and y spacelike, but not gauge invariant.

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$[\phi(x), \phi(y)] = 0,$ for x and y spacelike, but not gauge invariant.

Dressed Operator: **Local-gauge invariant observable**

$$\mathscr{D}(x,y) = \phi(x)\ \exp\left\{ \mathrm{i}q \int_{z_x}^{z_y} \mathrm{d}z\ A_z \right\} \phi^*(y)$$

[Fewster, '16]
[EmerGe, 24xx]

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	Notion of Separability	Notion of Subsystem
CM	$L^\infty_I(\mathcal{M}) \simeq L^\infty(\mathcal{M}_I) \times \tilde{\mathbf{1}}_J$	$\mathcal{M}_I$ of states for C-subsystem I
QM	$\mathcal{B}_A(\mathcal{H}) \simeq \mathcal{B}(\mathcal{H}_A) \otimes \tilde{\mathbf{1}}_B$	$\mathcal{H}_A$ of states for Q-subsystem A
QFT	**Einstein Separability** $[\mathcal{R}_U, \mathcal{R}_V] = 0$	$\mathcal{R}_U(\mathcal{H}) \simeq \mathcal{B}(\mathcal{H}_U) \otimes \mathbf{1}_{\bar{U}}$
	Split Property If $U \subset U'$, then $\exists$ a type I factor $\mathcal{N}$ s.t. $\mathcal{R}_U \subset \mathcal{N} \subset \mathcal{R}_{U'}$	$\mathcal{H}_U$ of states *__associated__* with region U

[Fewster, '16]
[EmerGe, 24xx]

Intuition goes a long way...

Independent Events:

$$P(A \ and \ B) = P(A) \cdot P(B)$$

Classical Mechanics	$\langle f_1 f_2 \rangle_\omega \simeq \langle f_1 \rangle_{\omega_1} \langle f_2 \rangle_{\omega_2} \simeq \langle \tilde{f}_1 \rangle_{\tilde{\omega}_1} \langle \tilde{f}_2 \rangle_{\tilde{\omega}_2}$
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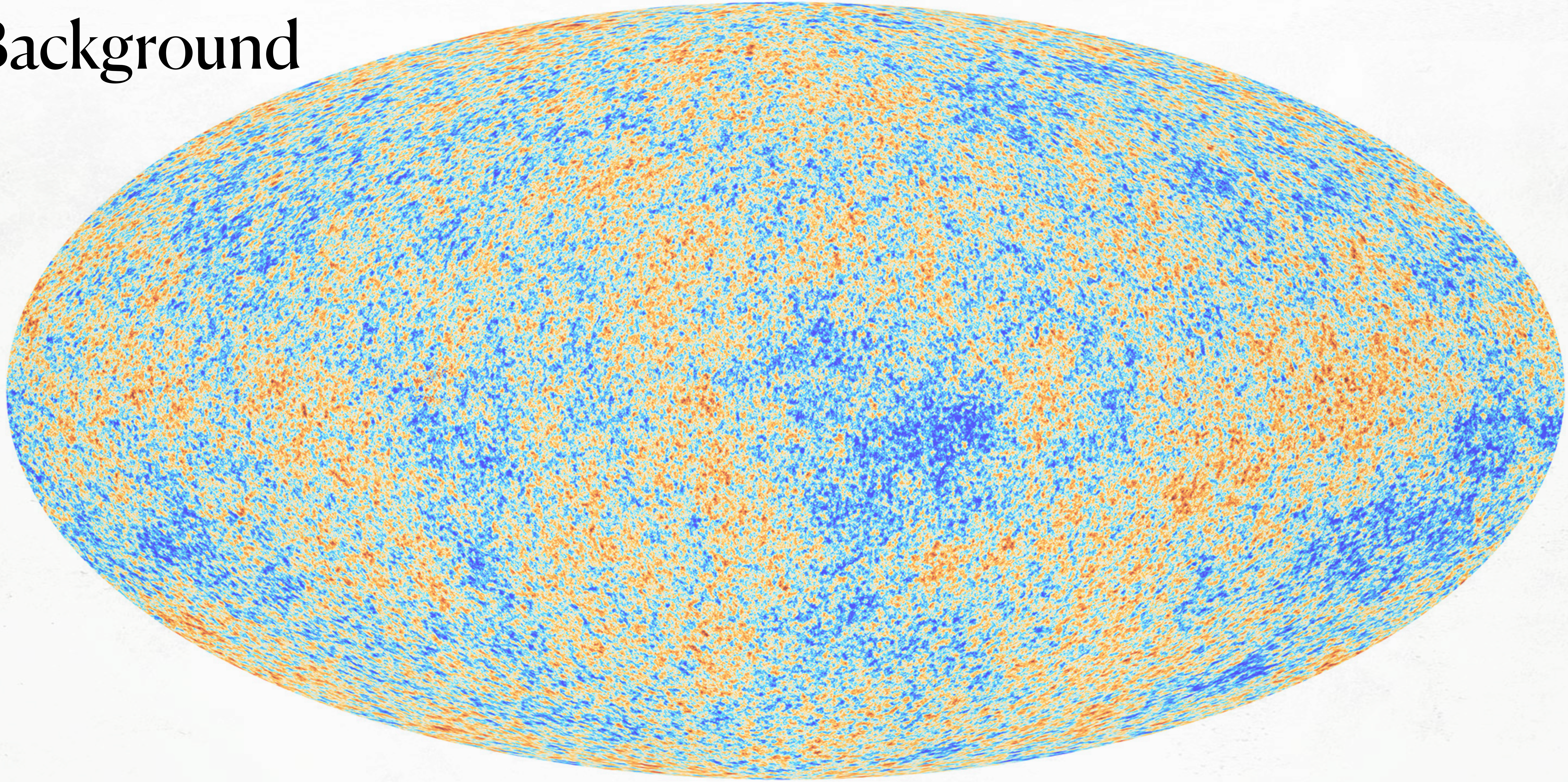
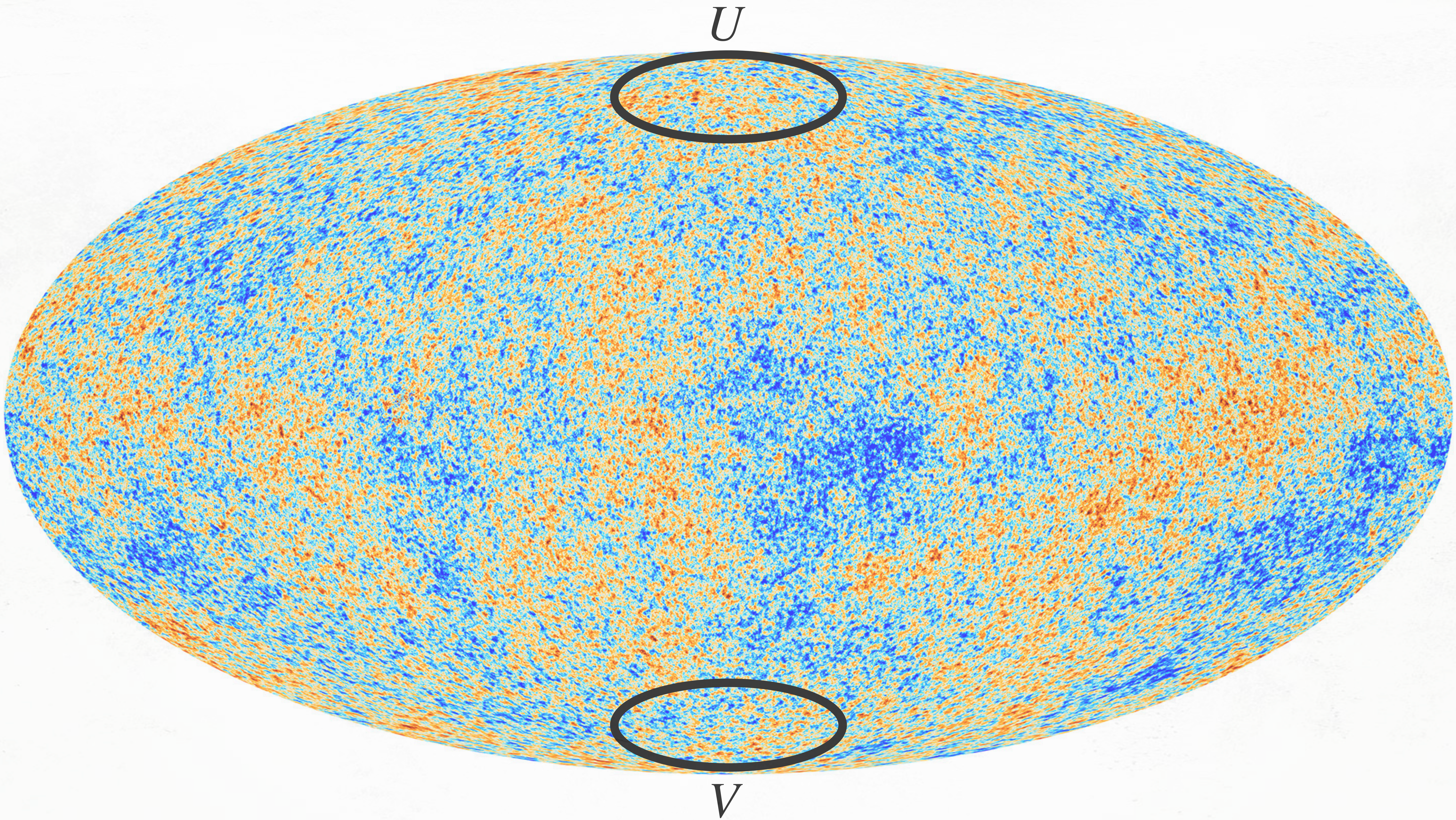
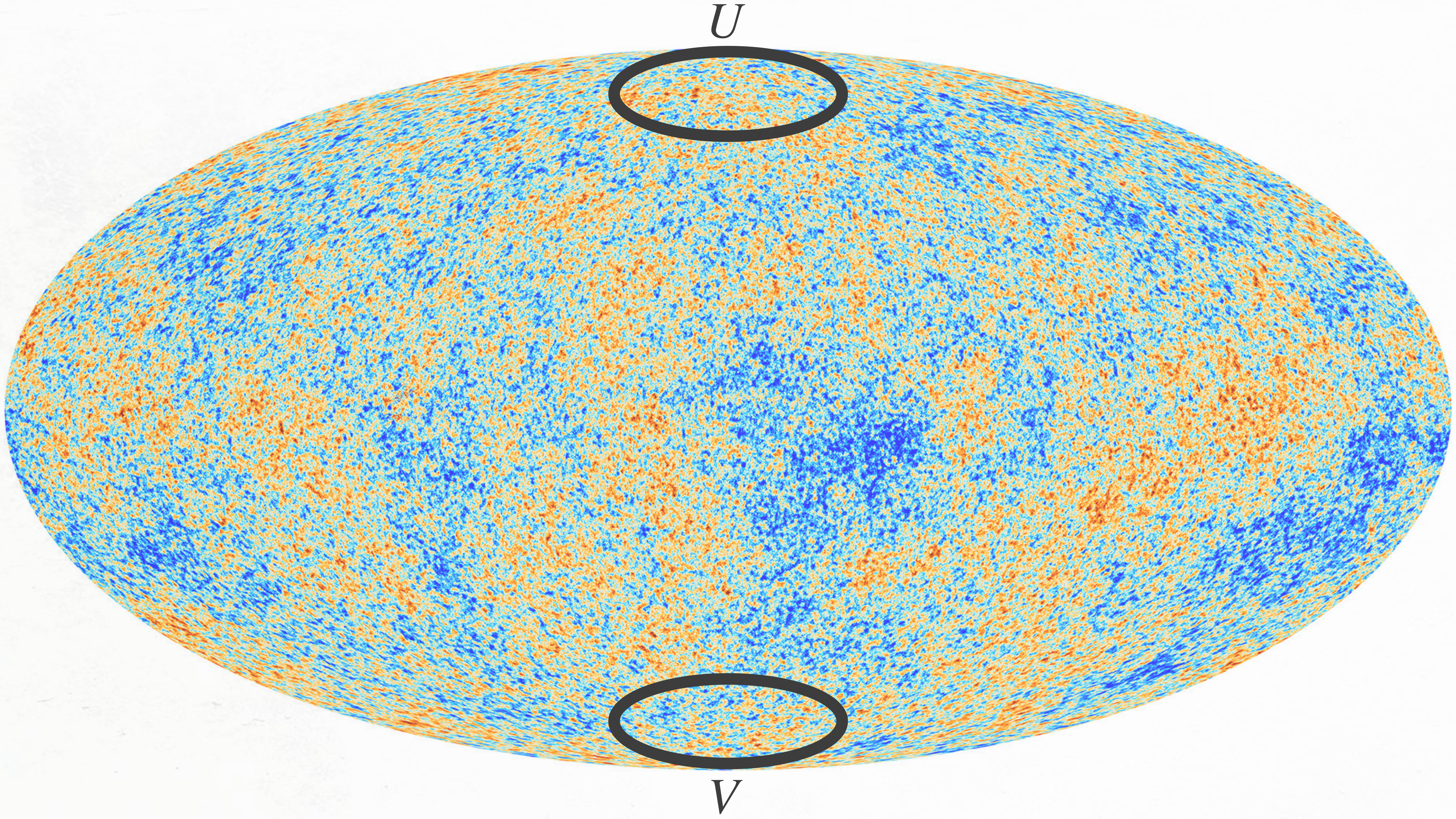
Quantum Field Theory	$\langle \mathscr{R}_U \mathscr{R}_V \rangle_\omega \simeq \langle \mathscr{R}_U \rangle_{\omega_U} \langle \mathscr{R}_V \rangle_{\omega_V} \simeq \langle \mathscr{B}(\mathscr{H}_U) \rangle_{\tilde{\omega}_U} \langle \mathscr{B}(\mathscr{H}_V) \rangle_{\tilde{\omega}_V}$

But what do we care about?

1. Cosmic Microwave Background




U
V



Einstein Separability

$$[\mathscr{R}_U, \mathscr{R}_V] = 0$$

[Bose et al., '17]
[Marletto&Vedral, '17]

2. Gravitationally-Induced-Entanglement

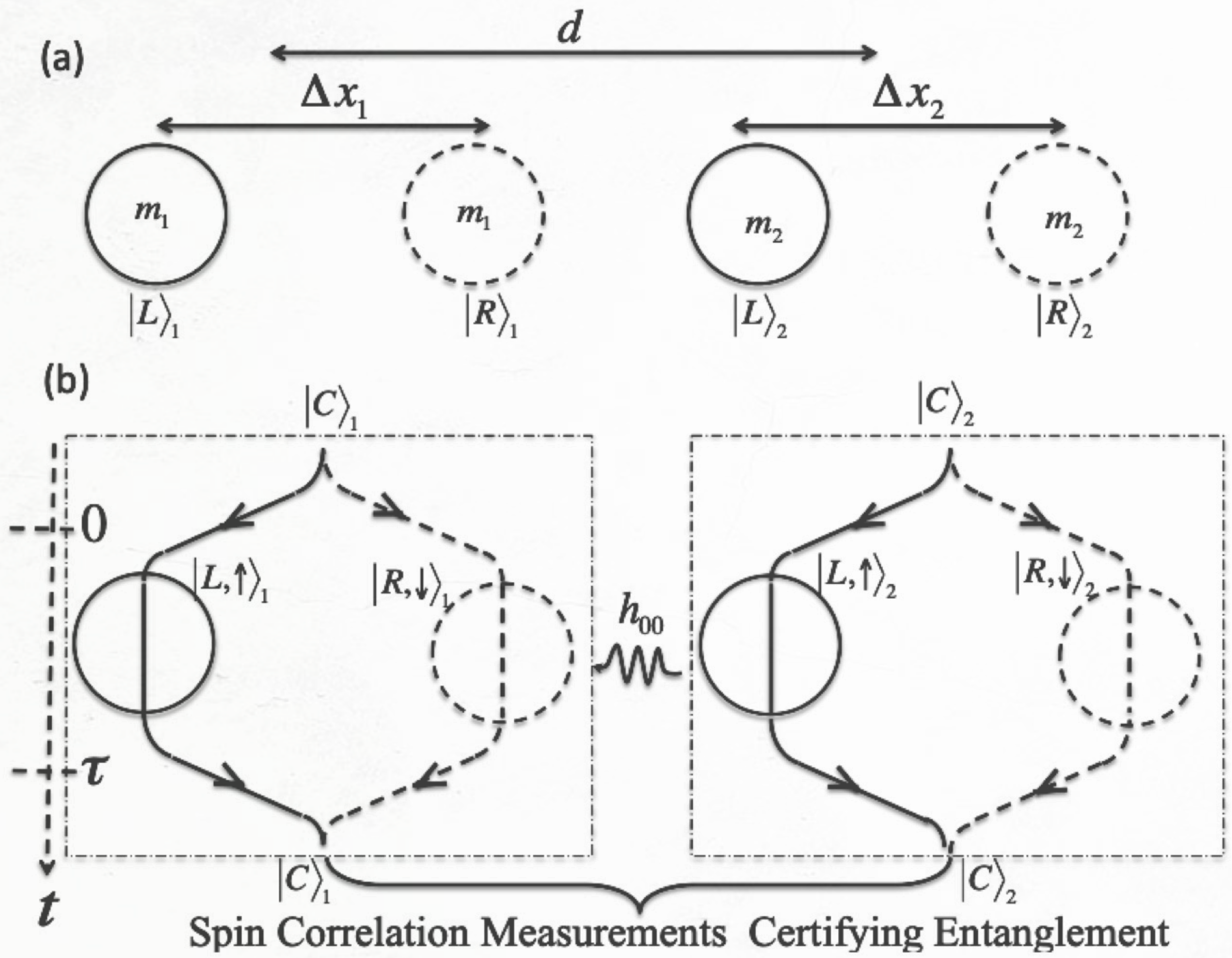
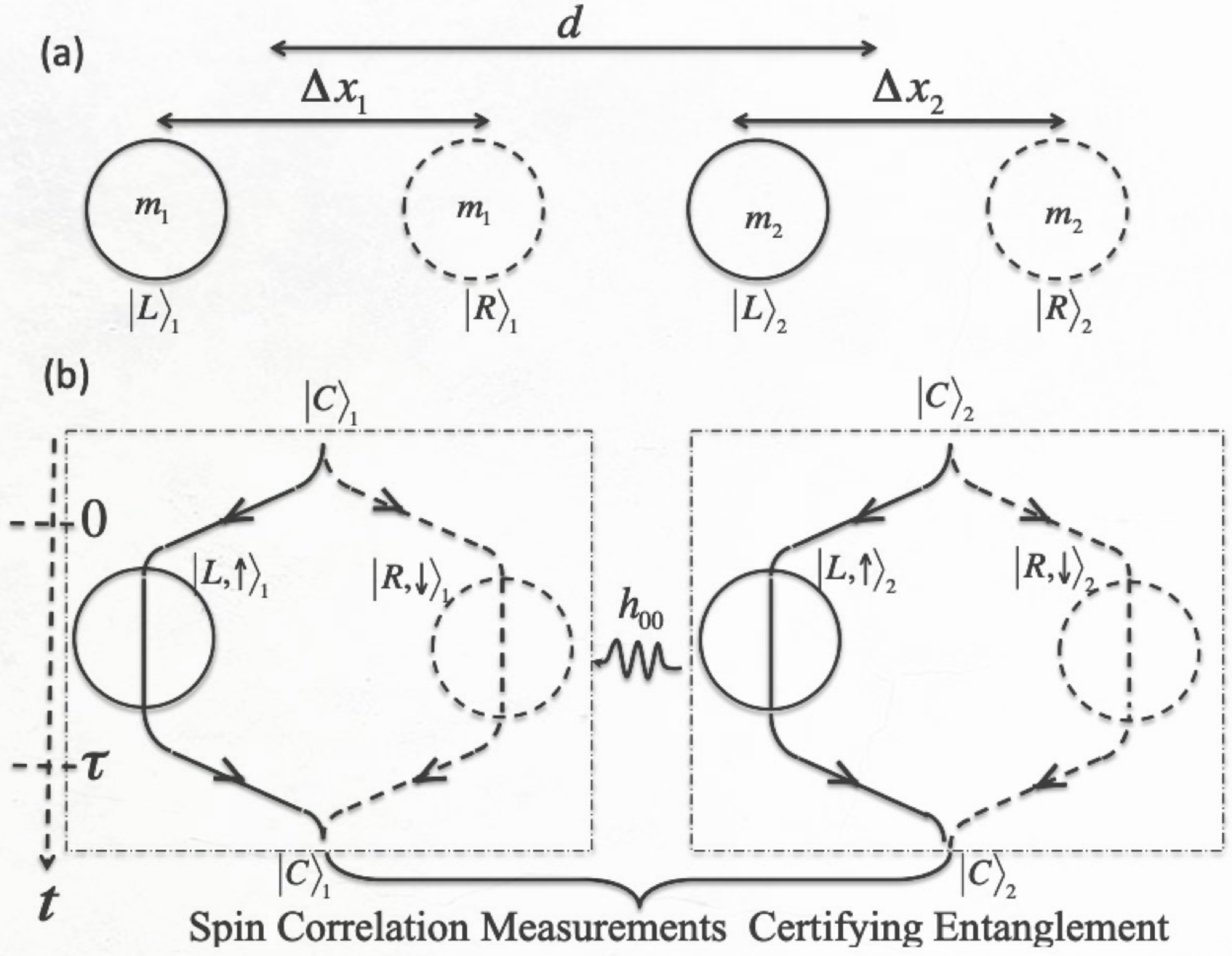


FIG. 1. Adjacent interferometers to test the quantum nature of gravity: **(a) Two test masses held adjacently in superposition of spatially localized states $|L\rangle$ and $|R\rangle$. (b) Adjacent Stern-Gerlach interferometers in which initial motional states $|C\rangle_j$ of masses are split in a spin dependent manner to prepare states** $|L,\uparrow\rangle_j + |R,\downarrow\rangle_j$. Evolution under mutual gravitational interaction for a time τ entangles the test masses by imparting appropriate phases to the components of the superposition. **This entanglement can only result from the exchange of quantum mediators – if all interactions aside gravity are absent, then this must be the gravitational field.**



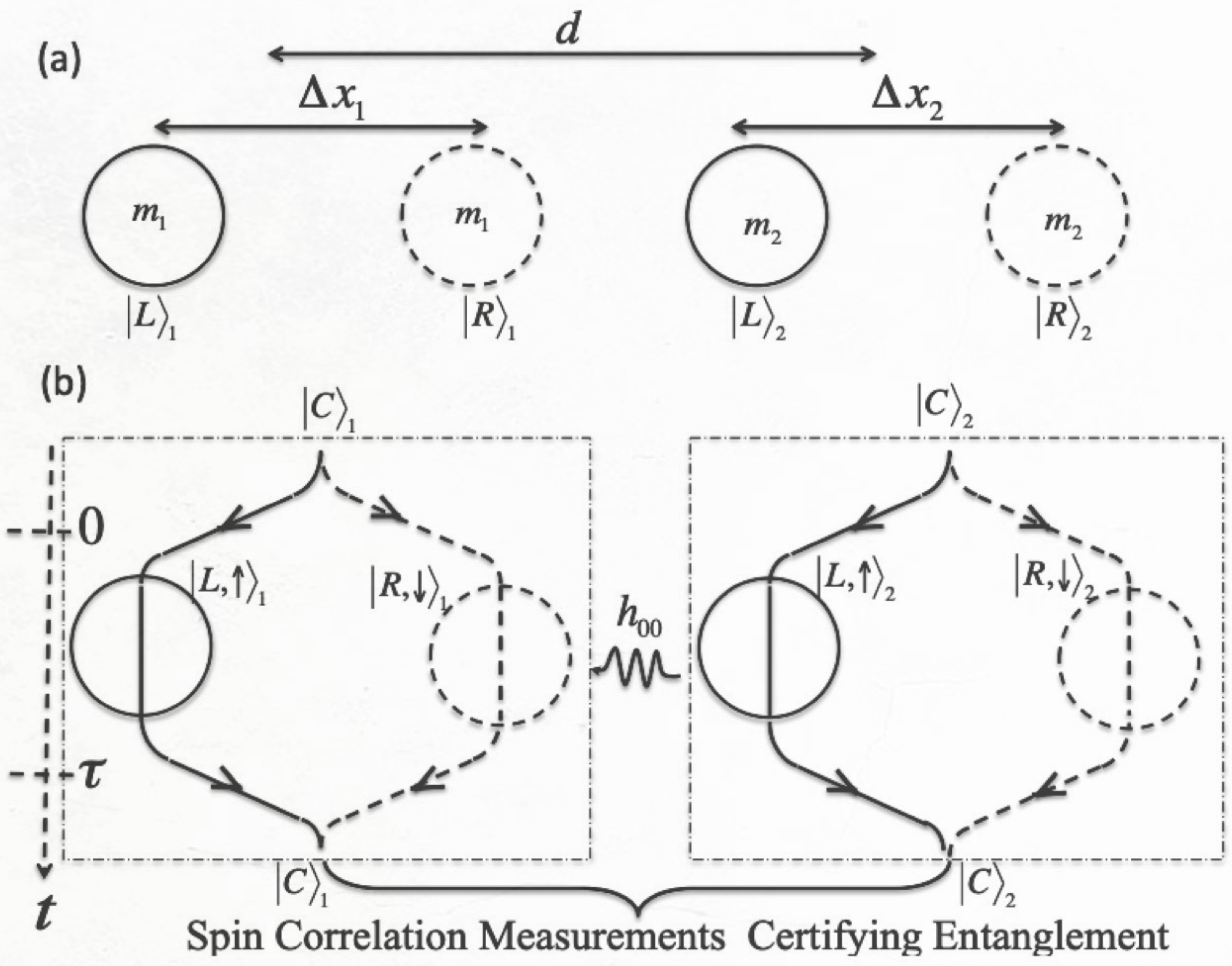
$$|\Psi(t=0)\rangle_{12} = \frac{1}{\sqrt{2}}(|L\rangle_1 + |R\rangle_1)\frac{1}{\sqrt{2}}(|L\rangle_2 + |R\rangle_2) \tag{1}$$

$$\rightarrow |\Psi(t=\tau)\rangle_{12} = \frac{e^{i\phi}}{\sqrt{2}}\{|L\rangle_1 \frac{1}{\sqrt{2}}(|L\rangle_2 + e^{i\Delta\phi_{LR}}|R\rangle_2) + |R\rangle_1 \frac{1}{\sqrt{2}}(e^{i\Delta\phi_{RL}}|L\rangle_2 + |R\rangle_2)\} \tag{2}$$

where $\Delta\phi_{RL} = \phi_{RL} - \phi$, $\Delta\phi_{LR} = \phi_{LR} - \phi$, and

$$\phi_{RL} \sim \frac{Gm_1m_2\tau}{\hbar(d-\Delta x)}, \quad \phi_{LR} \sim \frac{Gm_1m_2\tau}{\hbar(d+\Delta x)}, \quad \phi \sim \frac{Gm_1m_2\tau}{\hbar d}.$$

[Bose et al., '17]
[Marletto&Vedral, '17]



$$\otimes$$

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[D. Wallace, '21]
[V. Fragkos et al., '22]

Despite some level of contention (*as usual...*),

these are examples of **low-energy quantum-gravity (LEQG) phenomenology**:

cosmological perturbations are defined in terms of the matter and geometrical degrees of freedom, while GIE experiments are *locally* modeled in terms of the exchange of gravitational quanta.

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these are examples of **low-energy quantum-gravity (LEQG) phenomenology**:

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Question: How can we meaningfully talk about **independent** *gravitational-quantum* systems?

2. LEQG coupled with scalar field

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[Donelly&Giddings, '16]

$$\boxed{\mathscr{L} = \frac{2}{\kappa^2}R - \frac{1}{2}\left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + m^2\phi^2\right)}$$

$$\begin{cases} \delta h_{\mu\nu} = -2\partial_{(\mu}\xi_{\nu)} + \mathcal{O}(\kappa) \\ \delta\phi = -\kappa\xi^\mu\partial_\mu\phi + \mathcal{O}(\kappa^2) \end{cases}$$

$[\phi(x), \phi(y)] = 0,$ for x and y spacelike, but not gauge invariant.

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Dressed Operator: ***NOT* local-gauge invariant observable!**

$$\Phi(x) = e^{\mathrm{i}V^\mu(x)P_\mu}\phi(x)e^{-\mathrm{i}V^\mu(x)P_\mu} \quad \text{e.g.: } V_\mu(x) = \frac{\kappa}{2}\int_x^\infty d\tilde{x}^\nu \left[h_{\mu\nu}(\tilde{x}) + 2\int_{\tilde{x}}^\infty dx'^\lambda\, \partial_{[\mu}h_{\nu]\lambda}(x')\right]$$

The algebraic approach is obstructed in gravity because $\Phi(x)$ does not commute with itself at all spacelike separations.

The *intuition* is that the gravitational strings of any two operators $\Phi(x)$ and $\Phi(y)$ can intersect no matter how far apart these points are. We cannot screen the gravitational field of a particle as there is no notion of a negatively "charged" particle (or any Poincaré charge for that matter), preventing us from defining localized observables.

We only used the local symmetries to make such an argument, thus remains valid for any diff-invariant theory at the linear level.

[EmerGe, 24xx]

Is there a formal reason why this is happening?!

$$\boxed{\log V^{(\dots)}(x) = = iq_G \int d^4x' f^{(\dots)}(x,x')\, A_{(\dots)}(x')}$$

Connection!

For normal gauge symmetries:

$$[A, \Pi_A] = i$$

Connection!

For canonical low-energy QG:

$$[g, \Pi_g] = i$$

NOT

Connection!

[D. Wallace, '21]

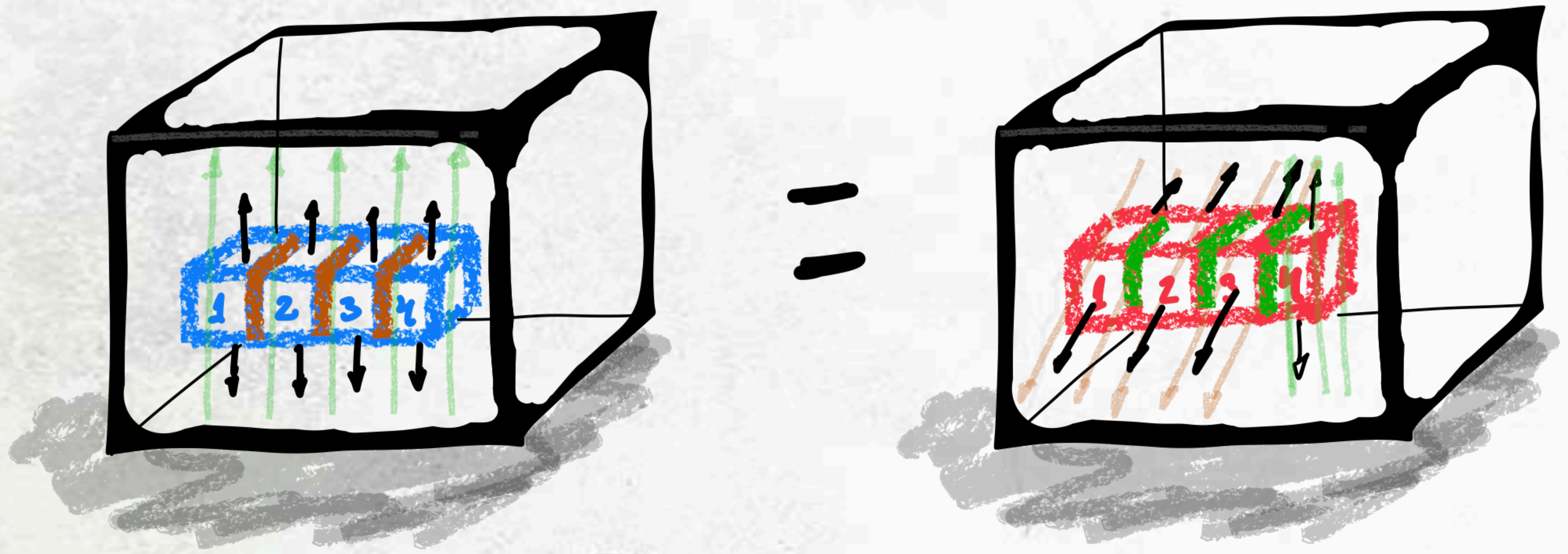
From an EFT perspective, we do live in the regime in which LEQG applies.

"… bars and clocks are the tools that measure *ds* thanks to their coupled with the gravitational field…"

(Rovelli 2021, p. 70)

	Notion of Separability	Notion of Subsystem
LEQG	**Einstein Separability** $[\mathscr{R}_U, \mathscr{R}_V] = 0$	$\mathscr{R}_U(\mathscr{H}) \simeq \mathscr{B}(\mathscr{H}_U) \otimes \mathbf{1}_{\bar{U}}$
	Split Property If $U \subset U'$, then $\exists$ a type I	$\mathscr{H}_U$ of states

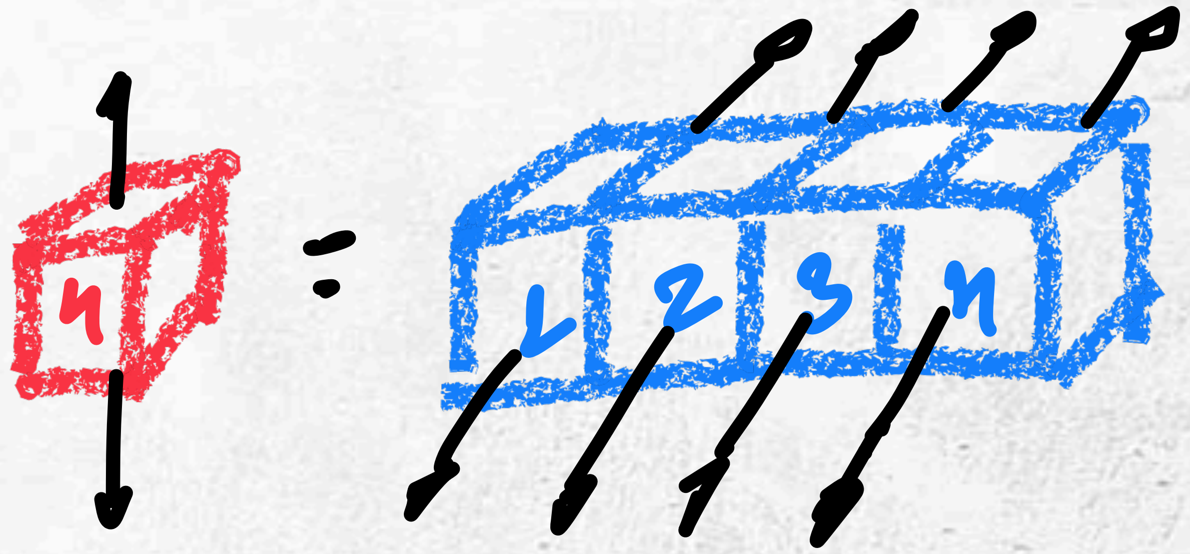
$$H = J\sum_{i=1}^{n-1}\sigma_i^z\sigma_{i+1}^z + h\sum_{i=1}^{n}\sigma_i^z = J\sum_{i=1}^{n}\mu_i^x + h\sum_{i=1}^{n-1}\mu_i^z\mu_{i+1}^z - J\mu_n^x + h\mu_1^z$$



$$\mu_i^z = \prod_{j\le i}\sigma_j^x,\quad \mu_i^x = \sigma_i^z\sigma_{i+1}^z,\quad \mu_n^x = \sigma_n^z$$

So, which one is it? How do we find out?

What if you cannot open the box?

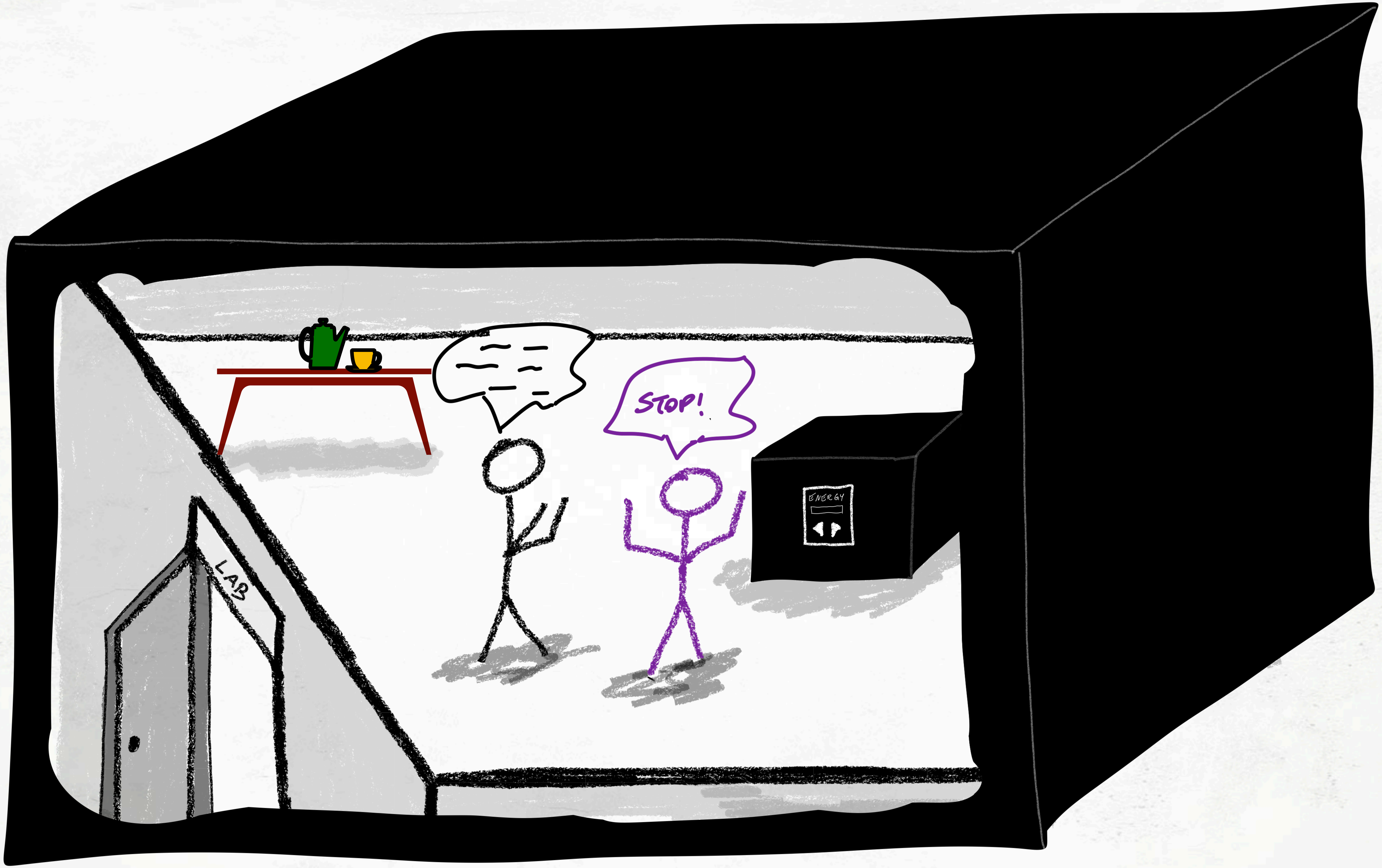


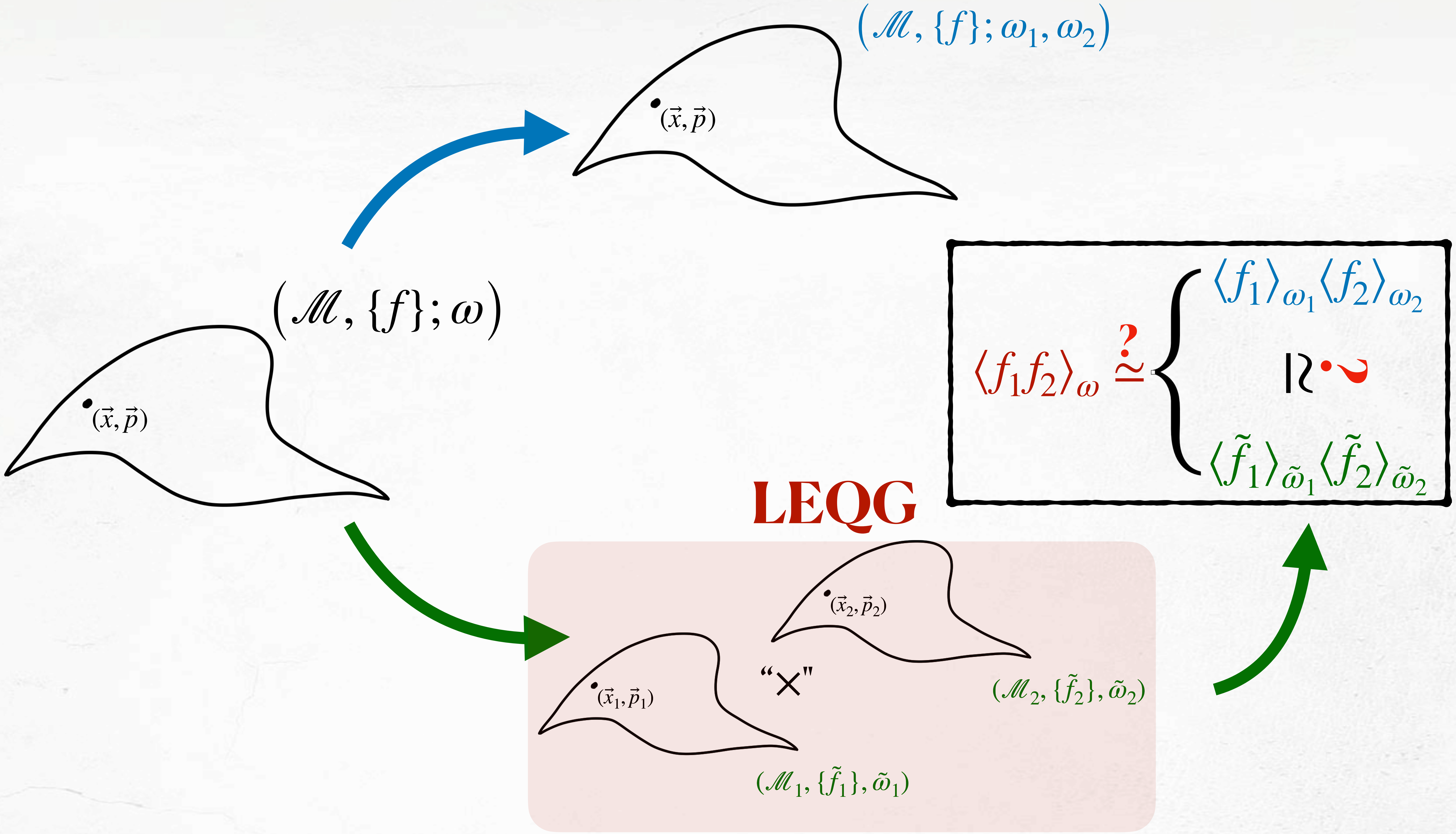
We see that each description of what's inside assumes a different notion of the system, different notion of which parts are interacting, and therefore different notion of what's a neighboring site!

How do we make sense of that?

You thought these were the questions all along, but...

[Dawid, Franzmann, '24]





Should *you* care? It flies in the face of fully funded, implemented, *paradigmatic* research programs such as inflationary (early universe) cosmology and table-top GIE (and QRFs more generally).

But really, how does *anything* work?

Is that all?

'The Copenhagen interpretation of quantum theory starts from a paradox. Any experiment in physics, whether it refers to the phenomena of daily life or to atomic events, is to be described in the terms of classical physics. **The concepts of classical physics form the language by which we describe the arrangement of our experiments and state the results.** We cannot and should not replace these concepts by any others.' (Heisenberg 1958, p. 44)

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The physically relevant aspects of the non-commutative operator algebras of quantum-mechanical observables are only accessible through commutative algebras. (Landsman, 2017, p. 10)

[Fewster, '16]
[EmerGe, '24]

$\langle \mathcal{O}_I \mathcal{O}_J \rangle_\omega = \langle \mathcal{O}_I \rangle_{\omega_I} \langle \mathcal{O}_J \rangle_{\omega_J}$: measurements and state preparations can be carried out independently

	Notion of Separability	Notion of Subsystem
CM	$L_I^\infty(\mathscr{M}) \simeq L^\infty(\mathscr{M}_I) \times \tilde{\mathbf{1}}_J$	$\mathscr{M}_I$ of states for C-subsystem I
QM	$\mathscr{B}_A(\mathscr{H}) \simeq \mathscr{B}(\mathscr{H}_A) \otimes \tilde{\mathbf{1}}_B$	$\mathscr{H}_A$ of states for Q-subsystem A
QFT	**Einstein Separability** $[\mathscr{A}_U, \mathscr{A}_V] = 0$	$\mathscr{A}_U \simeq \mathscr{B}(\mathscr{H}_U) \otimes \mathbf{1}_V$
	Split Property If $U \subset U'$, then $\exists$ a type I factor $\mathscr{N}$ s.t. $\mathscr{A}_U \subset \mathscr{N} \subset \mathscr{A}_{U'}$	$\mathscr{H}_U$ of states ***associated*** with region U

[Fewster, '16]
[EmerGe, '24]

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Why should we expect that classically-induced quantum factorizations should be preserved over time, including over measurements? [Franzmann, '24]

[Franzmann, '24]

Single-world unitary (SWU) quantum mechanics

[Franzmann, '24]

The breakdown of microcausality in LEQG, and the subsequent loss of a well-defined tensor product structure, undermines the concept of gravitational-quantum subsystems, which due to the universal nature of gravity applies to *everything — all systems*.

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This may offer a new perspective on the measurement problem: quantum subsystems are not invariant under measurements or generic Hamiltonian evolution.

Different than *classical subsystems — anchored in spacetime* and mostly preserved under time evolution — their quantum counterparts would move freely within the global Hilbert space.

→ The situation would be analogous to having a fixed thermodynamical macrostate while the system’s microstate roams freely in the region of phase space definining that macrostate.

[Franzmann, '24]

SWU-QM Sketch

Let's consider a global finite-dimensional* Hilbert space, $\mathscr{H}$, with its $B(\mathscr{H})$, a Hamiltonian H, and an initial pure state ρ_0. Then, for each observable $\mathcal{O} \in B(\mathscr{H})$, we have:

$$v_{\mathcal{O}}(\tau) = \operatorname{tr} \rho(\tau)\mathcal{O}, \quad \text{where} \quad \rho(\tau) = e^{iH\tau}\rho_0 e^{-iH\tau}.$$

For any fixed τ_0, the set $\{v_{\mathcal{O}}\}_{\tau_0}$ fully parametrizes all the data of the theory, as the theory evolves *deterministically*.

The set $\{v_{\mathcal{O}}\}_{\tau_0}$ is invariant under the following unitary map

$$T : \mathscr{H} \longrightarrow \bigotimes_i \mathscr{H}^i,$$

that transforms operators and states as:

$$\mathcal{O} \longrightarrow T\mathcal{O}T^{-1} \quad \text{and} \quad |\Psi\rangle \longrightarrow T|\Psi\rangle$$

which is one of the maps used to define a tensor product structure.

[Franzmann, '24]

Let's now consider a subalgebra App $\in B(\mathscr{H})$ associated with our experimental devices: $\mathcal{O}_I^{\mathrm{app}} \in \mathrm{App}$, where $\{\mathcal{O}_I^{\mathrm{app}}\}$ form an orthogonal basis with $d^2 - 1$ elements for the apparatus.

Moreover, let's assume, *for now*, that our apparatuses are independent quantum systems (microcausality holds) such that $B(\mathscr{H})$ is isomorphic to $B(\mathscr{H}^{\mathrm{app}}) \otimes B(\mathscr{H}^{\mathrm{EE}})$, where EE stands for everything else, $\dim \mathscr{H}^{\mathrm{app}} = d$ and $\dim \mathscr{H}^{\mathrm{EE}} = D$, and $d/D \lll 1$.

This is good since we typically do not have access to the whole Hilbert space, so we want to single out our apparatus.

[Franzmann, '24]

This implies that a TPS $\mathscr{T}$ is being considered, such that:

$$\mathscr{H} \xrightarrow{\mathscr{T}} \mathscr{H}^{\text{app}} \otimes \mathscr{H}^{\text{EE}}$$

$$\mathscr{O}_I^{\text{app}} \xrightarrow{\mathscr{T}} \tilde{O}_I^{\text{app}} \otimes I^{\text{EE}}$$

$$\rho \xrightarrow{\mathscr{T}} \sum_{n,m}^{d} \sqrt{p_n p_m} \, | u_n^{\text{app}} \rangle \langle u_m^{\text{app}} | v_n^{\text{EE}} \rangle \langle v_m^{\text{EE}} | \, ,$$

where $\tilde{O}_I^{\text{app}} \in B(\mathscr{H}^{\text{app}})$ and we used the Schmidt decomposition for a generic state ρ. The Hamiltonian also factorizes:

$$H \xrightarrow{\mathscr{T}} a_0 I_{d+D} + \sum_{I=1}^{d^2-1} a_I \tilde{\mathscr{O}}_I^{\text{app}} + \sum_{J=1}^{D^2-1} b_J \tilde{\mathscr{O}}_J^{\text{EE}} + \sum_{I=1}^{d^2-1} \sum_{J=1}^{D^2-1} c_{IJ} \tilde{\mathscr{O}}_I^{\text{app}} \tilde{\mathscr{O}}_J^{\text{EE}} \, .$$

Crucially, note that from the quartet $(\mathscr{H}, B(\mathscr{H}), H, \rho_0)$, now we have factorized each of these elements, and associated with the factorization $\mathscr{T}$ are the coefficients,

$$\{p_n, a_0, a_I, b_J, c_{IJ}\} = \{p_n(\mathscr{T}), a_0(\mathscr{T}), a_I(\mathscr{T}), b_J(\mathscr{T}), c_{IJ}(\mathscr{T})\} \, ,$$

which parametrize the states and the Hamiltonian, thus kinematics and dynamics of the theory, in a given TPS.

Under a given TPS, we can rewrite the global invariants as

$$
\begin{aligned}
v_{\mathcal{O}_I^{\text{app}}} &= \sum_{m,n=1}^{d} \sqrt{p_n p_m}\,\text{tr}_{\text{app}}\left(\tilde{\mathcal{O}}_I^{\text{app}}\,|\,u_n^{\text{app}}\rangle\langle u_m^{\text{app}}\,|\right)\text{tr}_{\text{EE}}\left(\,|\,v_n^{\text{EE}}\rangle\langle v_m^{\text{EE}}\,|\,\right) \\
&= \sum_{n=1}^{d} p_n \text{tr}_{\text{app}}\left(\tilde{\mathcal{O}}_I^{\text{app}} P_n^{\text{app}}\right), \quad \text{where} \quad P_n^{\text{app}} := |\,u_n^{\text{app}}\rangle\langle u_n^{\text{app}}\,| \\
&= \sum_{n=1}^{d} p_n \tilde{v}_{\mathcal{O}_{I,n}^{\text{app}}},
\end{aligned}
$$

where $\tilde{v}_{\mathcal{O}_{I,n}^{\text{app}}}$ are the local invariants associated with the local observable $\tilde{O}_I^{\text{app}}$ in the state $|\,u_n^{\text{app}}\rangle$. Thus, generically, global invariants can be decomposed as a convex sum of local invariants.

[Franzmann, '24]

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- There is a set of TPS, $\{\mathcal{T}_M\}$, where the state of the system factorizes: $\rho \xrightarrow{\mathcal{T}_M} \rho^{\mathrm{app}'} \otimes \rho^{\mathrm{EE}'}$, where $'$ indicates that whatever quantity being considered is in such a TPS;

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- Due to violations of microcausality in quantum gravity, imposing a fixed TPS in any quantum-gravitational system is ill-posed and can only be considered an approximation. As the system evolves under the global Hamiltonian, the approximate TPS changes continuously and can be parametrized by time, $\mathscr{T}(\tau)$. This can be understood in terms of the TPS coefficients evolving, e.g., $a_I = a_I(\tau)$.

[Franzmann, '24]

By putting together these assumptions we can arrive at the following conclusion:

Single outcomes in any measurement correspond to a composition of a change of TPS together with a local unitary transformation of the local algebra assigned to the apparatus.

Whenever $\mathscr{T}(\tau_0) \in \{\mathscr{T}_M\}$, we have a product state between what we call apparatus and everything else:

$$|\Psi(\tau_0)\rangle \xrightarrow{\mathscr{T}_M} \sum_{n=1}^{d} c_n |u_n^{\text{app}'}\rangle \otimes \sum_{m=1}^{D} d_m |v_m^{\text{EE}'}\rangle ,$$

where these are bases in one of the $\mathscr{T}_M$. Now, we can simply *align* our apparatus by a local unitary transformation,

$$|\Psi(\tau_0)\rangle = |o_{I,n}^{\text{app}'}\rangle \otimes \sum_{m}^{D} d_m |v_m^{\text{EE}'}\rangle \quad \text{where} \quad \bar{\mathcal{O}}_I^{\text{app}'} |o_{I,n}^{\text{app}'}\rangle = o_{I,n}^{\text{app}'} |o_{I,n}^{\text{app}'}\rangle ,$$

and $\bar{\mathcal{O}}_I^{\text{app}'} = U^{-1} \tilde{\mathcal{O}}_I^{\text{app}} U$. Thus, the local state is one of the rotated apparatus' eigenstates with eigenvalue $o_{I,n}^{\text{app}'}$. Crucially, we do not know which one. Then, it is easy to show that

$$\begin{aligned} v_{\mathcal{O}_I^{\text{app}}} &= \operatorname{tr}_{\text{app}'} \left(|o_{I,n}^{\text{app}'}\rangle\langle o_{I,n}^{\text{app}'}| \, \bar{\mathcal{O}}_I^{\text{app}'} \right) \operatorname{tr}_{\text{EE}'} \rho^{\text{EE}'} \\ &= o_{I,n}^{\text{app}'} . \end{aligned}$$

Thus, by identifying the appropriate TPS and locally aligning the apparatus, we see that local measurement outcomes are the global invariants that we started with.

[Franzmann, '24]

We expect that the probabilistic nature of the theory arises from our lack of knowledge about the specific set of unitaries required to reach this point, *such that probabilities will be epistemic*. In any case, given that all considerations involved only unitary transformations (both for TPS updates and time evolution) and we still arrive at a description of the apparatus with single outcomes related to the global invariants, this would amount to a single-world unitary quantum mechanics.

[Franzmann, '24]

Comparison with GR:

As one moves through spacetime and wants to compare measurements at different points along their trajectory, it is essential to update their local inertial frames (tetrads) based on the local metric of spacetime.

To achieve this, one must:

- Determine the spacetime metric by solving the Einstein equations, which, despite being local equations, often require *global* boundary conditions for a solution;
- Establish the family of local inertial frames using the metric;
- Align the timelike tetrad with the observer's 4-velocity, which is a local process requiring measurements of the observer's relative velocity to reference points.

If the timelike tetrad vector is not aligned with the observer's 4-velocity, it represents a boosted frame relative to the observer's rest frame, leading to different local measurements of time and space, akin to special relativity. Therefore, it is crucial for the observer to measure their relative velocity in relation to some reference points to determine the appropriate tetrad to use.

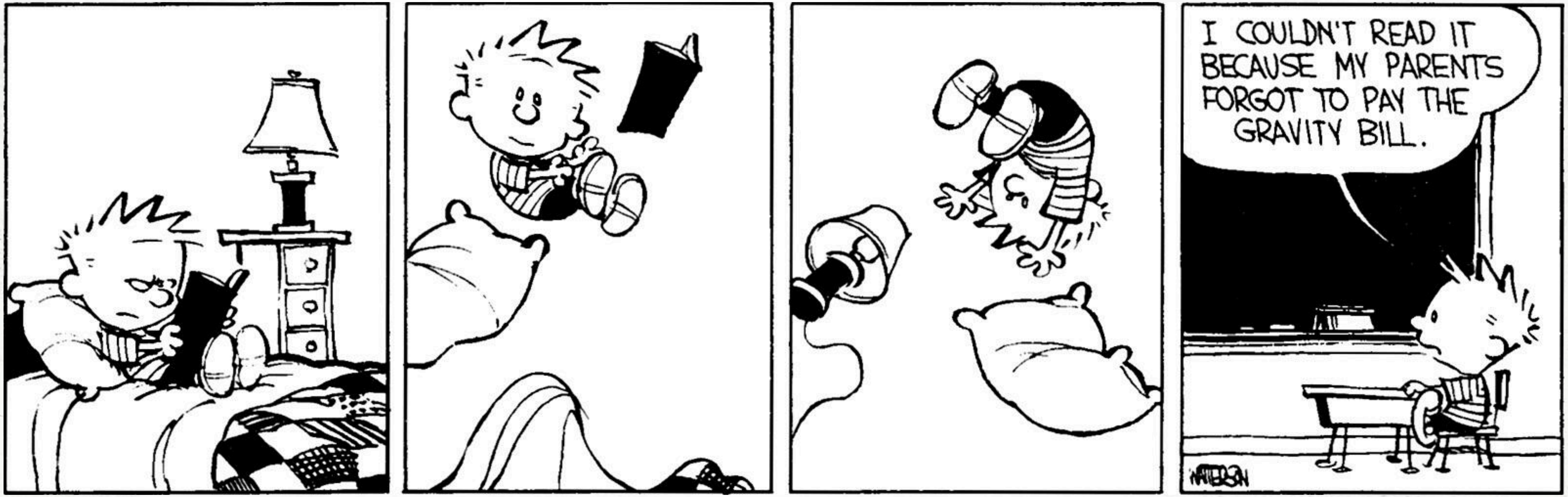
Conclusion

Progress can be typically made when *obvious* notions are finally put into question.

Among the historical remarkable examples, perhaps Einstein's scrutinizing of the nature of simultaneity leading to the theory of relativity and Planck's debunking of the radiation energy spectrum continuum giving rise to quantum mechanics stand out the most.

Another seemingly obvious concept is the idea of independent physical systems. Combining relativity and quantum mechanics challenges the immediate naive understanding of how to independently describe different interacting physical systems.

This is a notion that, despite permeating all the physics we do,
remains to be fully understood.


I COULDN'T READ IT BECAUSE MY PARENTS FORGOT TO PAY THE GRAVITY BILL.
WATTERSON

Thank you!