Dynamical frames and relativity of subsystems

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QISS | THE QUANTUM INFORMATION STRUCTURE OF SPACETIME

Based on: quant-ph/2308.09131 with I. Kotecha, P. A. Höhn

& ongoing work with S. Carrozza, P. A. Höhn, J. Kirklin

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INTRODUCTION AND MOTIVATIONS

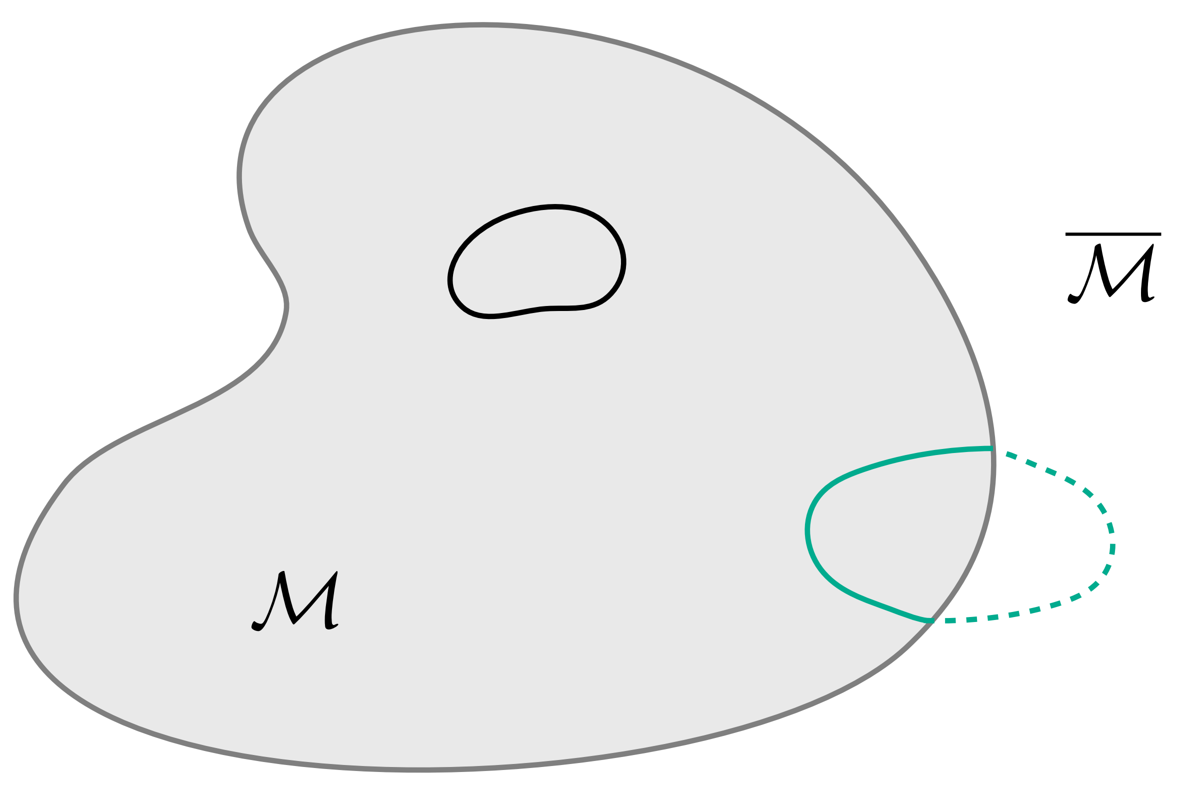
Operationally, subsystems are distinguished by subalgebras of physically accessible observables [Zanardi '01; Zanardi, Lidar, Lloyd '03]

often relative to external frame, e.g. the Lab, or to notion of locality of a background spacetime (external to the fields of interest).

What if no external relatum is available and/or there is tension between locality and gauge-invariance?

In constrained/gauge systems and gravity:

- Kinematical notion of subsystems generically **not** inherited at gauge-inv. level;
- Non-local gauge-invariant observables;
- Partitioning vs. cross-boundary observables. (edge modes) [Donnelly, Freidel, Francois, Geiller, Gomes, Pranzetti, Riello, Speranza, Speziale, Wieland, Carrozza, Eccles, Höhn,…]



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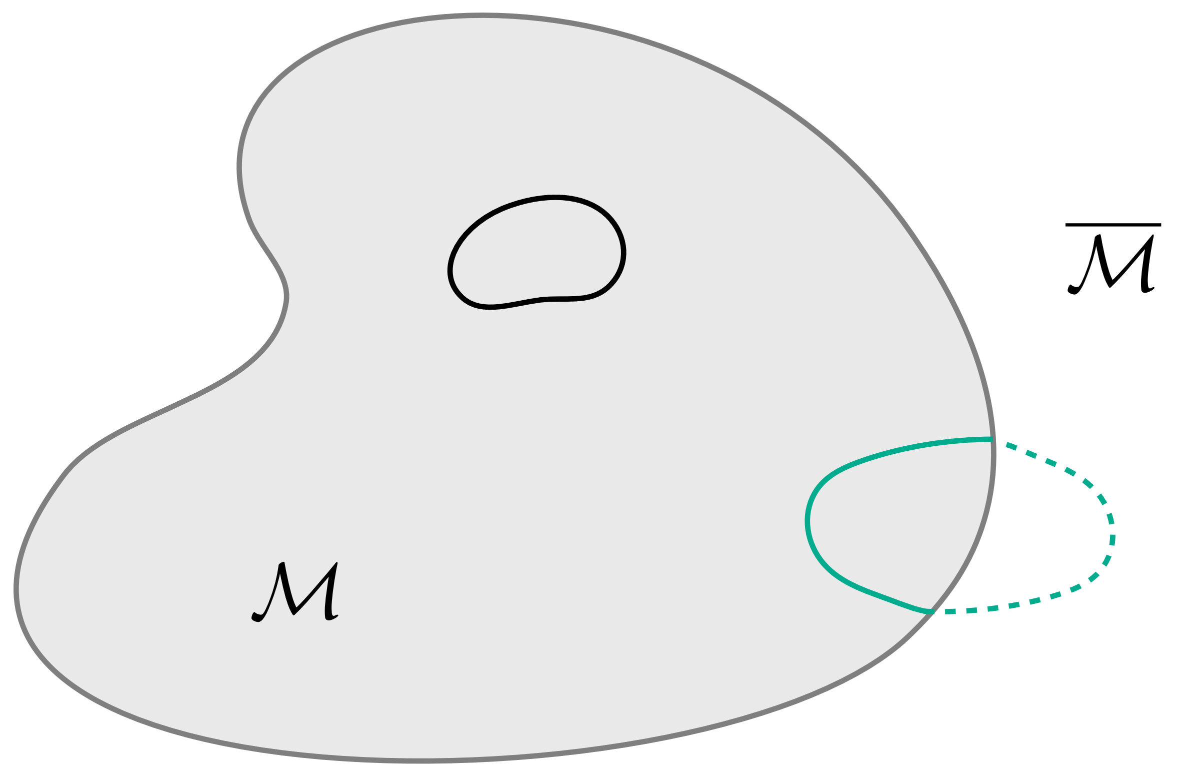

SUBSYSTEM RELATIVITY

Idea: **Use internal reference frames & relational observables** ← MULTIPLE CHOICES → **Gauge-inv. subsystems depend on the relational observables accessible in the chosen internal frame**

[Ahmad Ali, Galley, Höhn, Lock, Smith '21; de la Hamette, Galley, Höhn, Loveridge, Müller '22; Höhn, Kotecha, **FMM** '23; Carrozza, Höhn, Kirklin, **FMM** to appear]

Consequences: frame-dependent gauge-inv. properties of subsystems + alternative proposal for entanglement entropy

EXAMPLES OF SUBSYSTEM RELATIVITY

- Special relativity with tetrads frames [de la Hamette, Galley, Höhn, Loveridge, Müller '21; Höhn, Kotecha, **FMM** '23] ⟹ Relativity of simultaneity from relativity of subsystems [Höhn, Kotecha, **FMM** '23]

- Finite-dim. quantum systems: external-frame independent description of DoF of interest relative to the remaining DoFs (used as internal frame) ⟹ Relativity of correlations, subsystem dynamics, equilibrium and non-equilibrium thermodynamics [Höhn, Kotecha, **FMM** '23]

- Subregions in gauge theories and gravity: edge modes frames [Carrozza, Höhn '21; Carrozza, Eccles, Höhn '22] ⟹ Frame-dependent subregional gauge-inv. algebras [Carrozza, Höhn, Kirklin, **FMM** to appear]

- Regulatisation of gravitational entropy via introduction of observer (from Type III to Type II algebras) [Chandrasekaran, Longo, Pennington, Witten '22; Kudler-Flam, Leutheusser, Satishchandran '23; Jensen, Sorce, Speranza '23; Freidel, Gesteau to appear] ⟹ Observer dependence of gravitational entropy [De Vuyst, Eccles, Höhn, Kirklin '24]

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IN THIS TALK

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PLAN OF THE TALK

Part I Warm up: Special relativity with internal tetrad frames

Part II Finite-dimensional quantum constrained/gauge systems

Illustration via mechanical toy model example;

Basics of quantum reference frames (perspective-neutral formulation);

Quantum relativity of subsystems & its physical consequences;

Part III Comparison with center construction for subsystem entropy in presence of constraints

Different assignment of gauge invariant subalgebras → Different notion of entropy (proper entanglement entropy?)

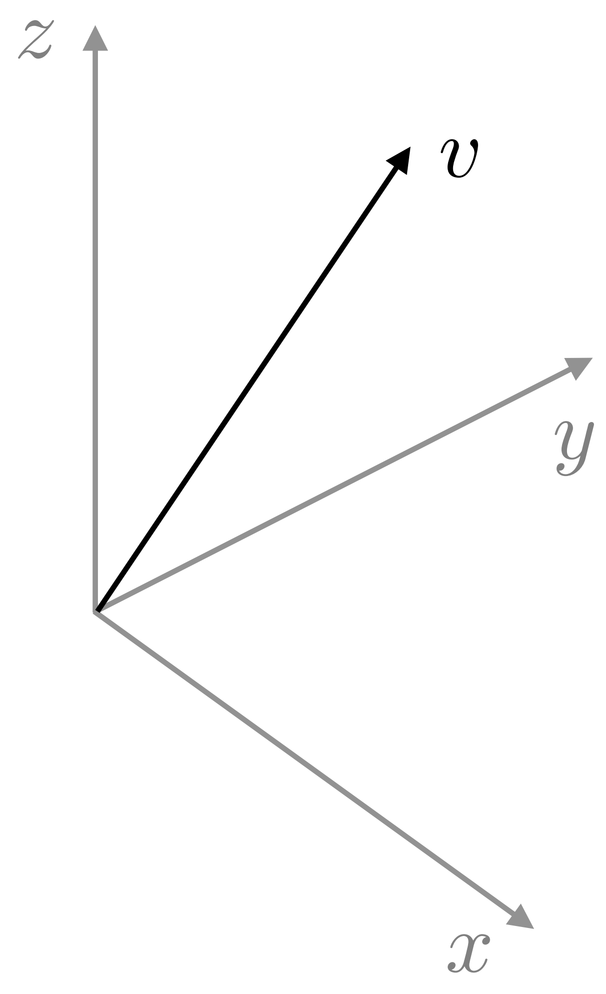
Part IV Subregions in gauge field theory (edge modes as boundary dynamical frames)

Part I

Special relativity with tetrad frames

SPECIAL RELATIVITY WITH INTERNAL FRAMES

[de la Hamette, Galley, Höhn, Loveridge, Müller '22; Höhn, Kotecha, **FMM** '23]



External/background coordinate frame

$v^\mu \mapsto \Lambda^\mu_\nu v^\nu$ $\quad \Lambda \in SO_+(3,1)$

internally indistinguishable

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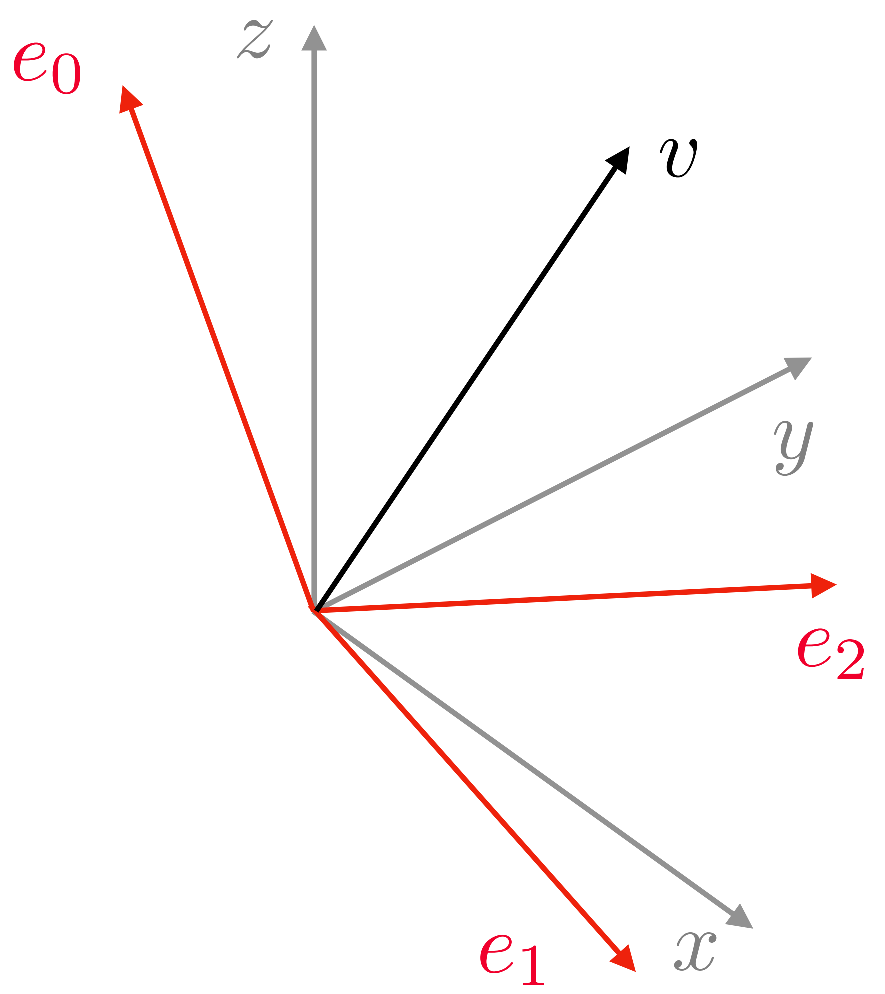

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$v^\mu \mapsto \Lambda^\mu_\nu v^\nu$ $\Lambda \in SO_+(3,1)$ internally indistinguishable

Internal frame (tetrad): e^μ_a

- spacetime index $\mu = t, x, y, z$
- frame index $a = 0, 1, 2, 3$ (frame orientation)

$\Rightarrow$ 2 commuting group actions:

- "gauge" transformations $\Lambda^\mu_\nu e^\nu_a$, $\Lambda^\mu_\nu \in SO_+(3,1)$
- "symmetries" (frame reorientations) $\Lambda^b_a e^\mu_b$, $\Lambda^b_a \in SO_+(3,1)$

acts on frame only

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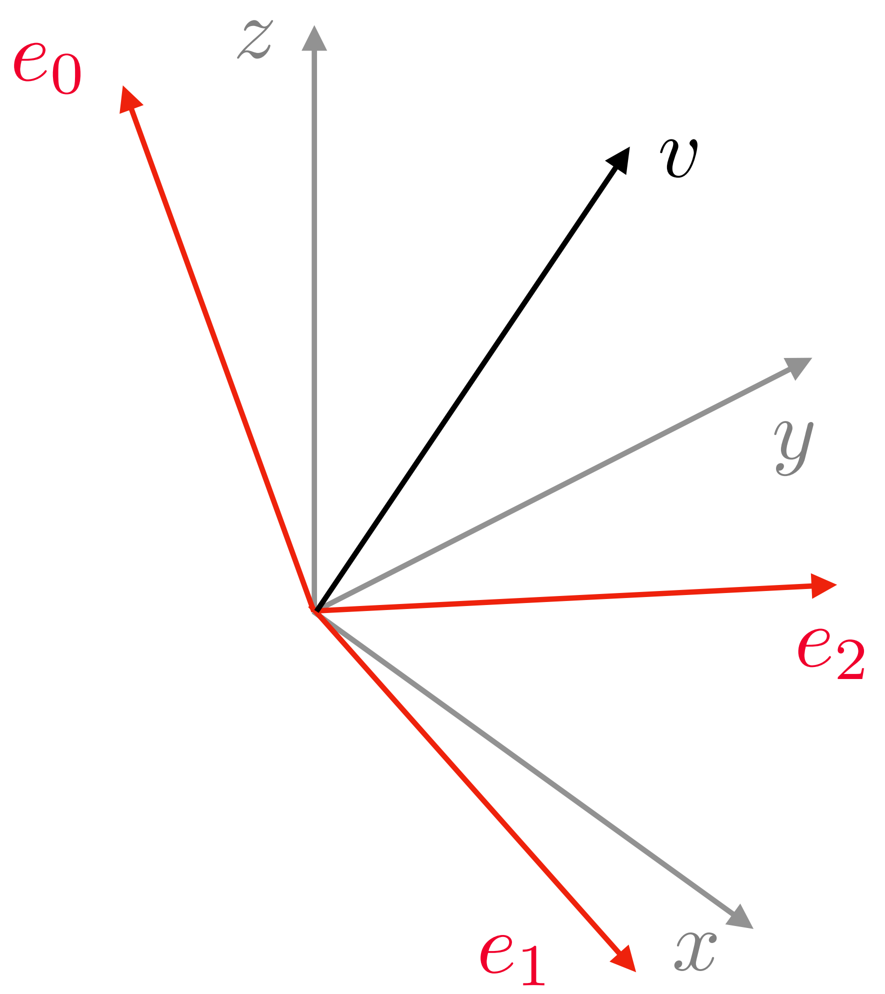

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acts on frame only

- $\eta_{ab} = e^\mu_a e^\nu_b \eta_{\mu\nu}$ $\quad \Rightarrow \quad$ $e^\mu_a \in SO_+(3,1)$ $\quad$ group-valued reference frame (G-frames, more later)

- "Gauge-invariant" description of v: $\quad v_a = e^\mu_a v_\mu$ $\quad$ "relational / frame dressed observables"

  (indep. of external coordinates) $\quad$ (description of v relative to frame)

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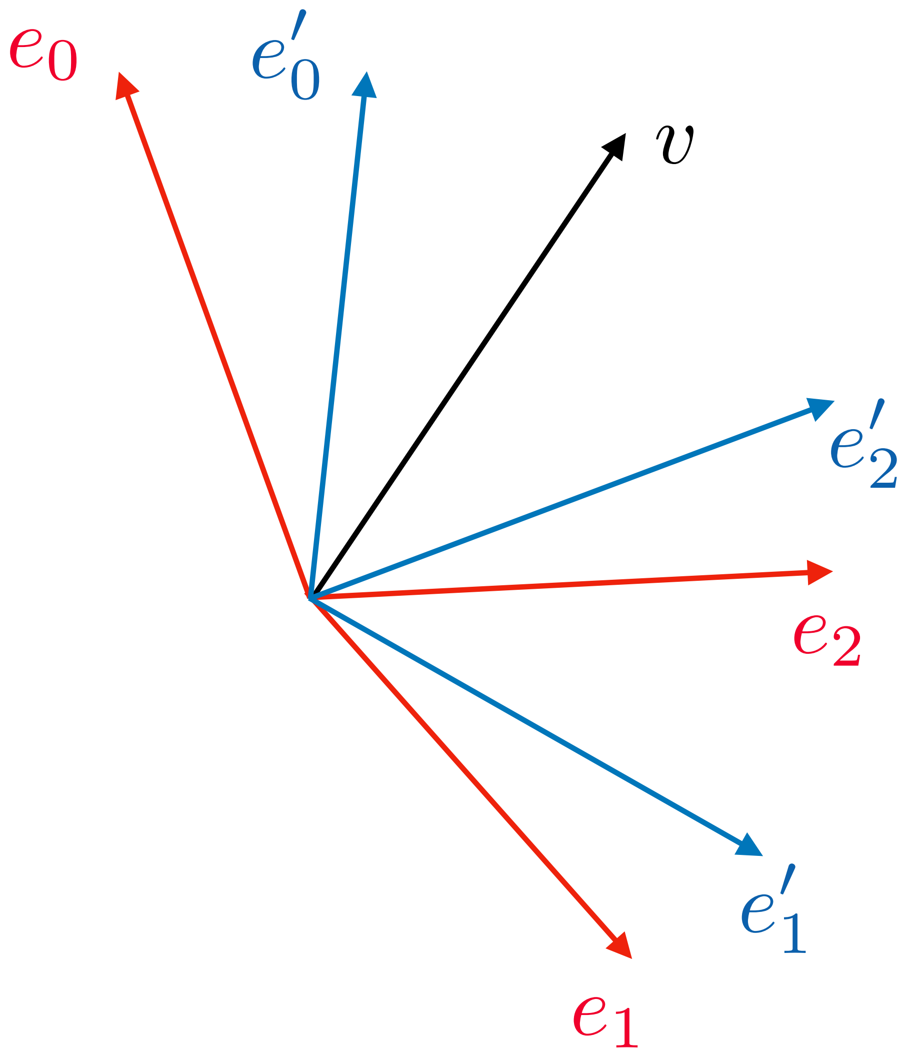


Introduce a second tetrad frame: $e'_{a'}$

$$v_a = \eta_{\mu\nu} e_a^\mu v^\nu = e'^{\,a'}_{\mu} e'_{a'\nu} e_a^\mu v^\nu = \Lambda_a{}^{a'} v_{a'}$$

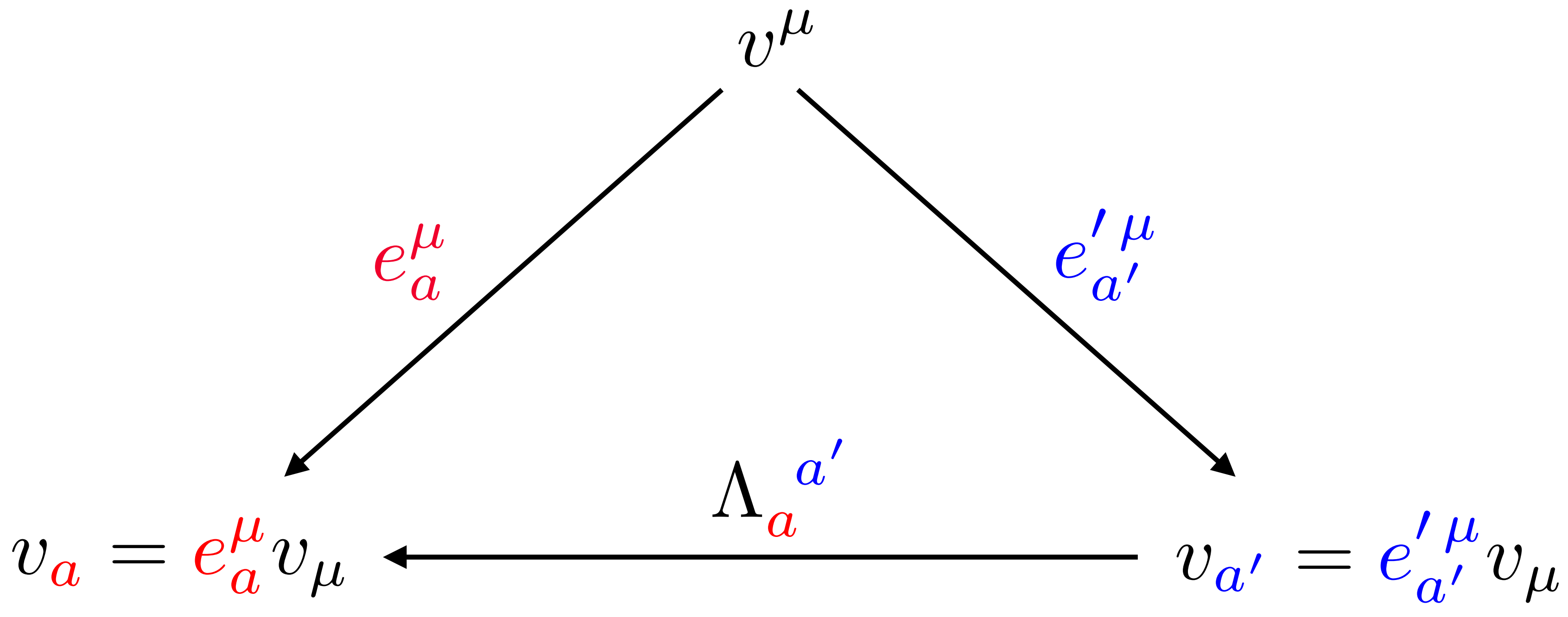
relational observable rel. to e

relational observable rel. to e'

"symmetry-induced" RF transformations

$$\Lambda_a{}^{a'} = e'^{\,a'}_{\mu} e_a^\mu$$

(relational obs. describing 1st rel. to 2nd frame)



change of rel. obs. associated with same kinematical subsystem quantity

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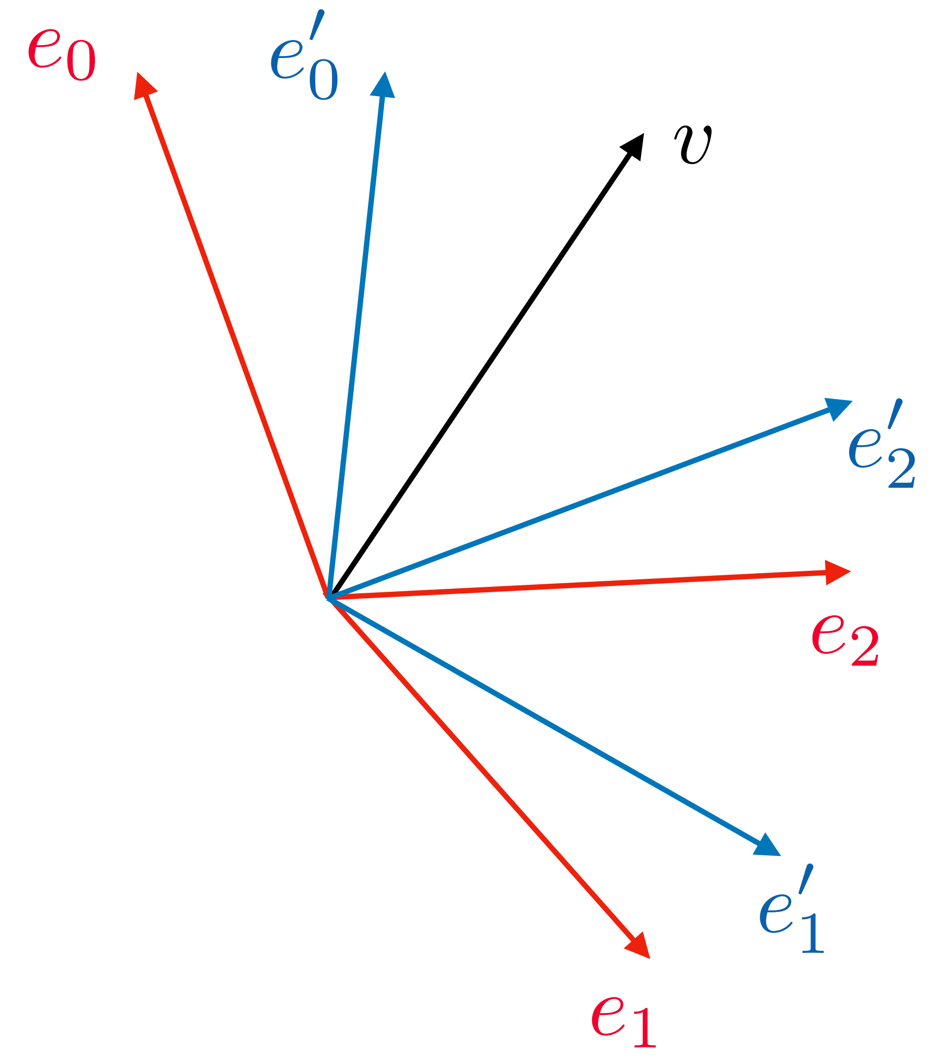

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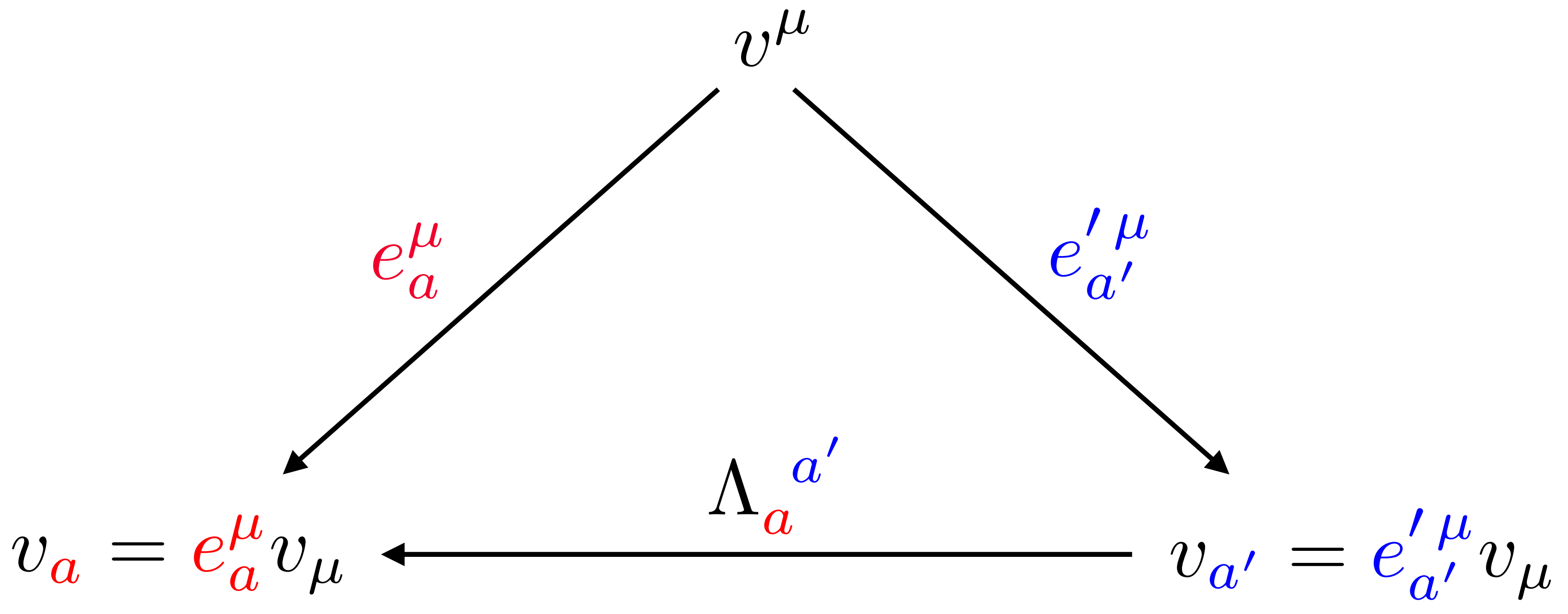
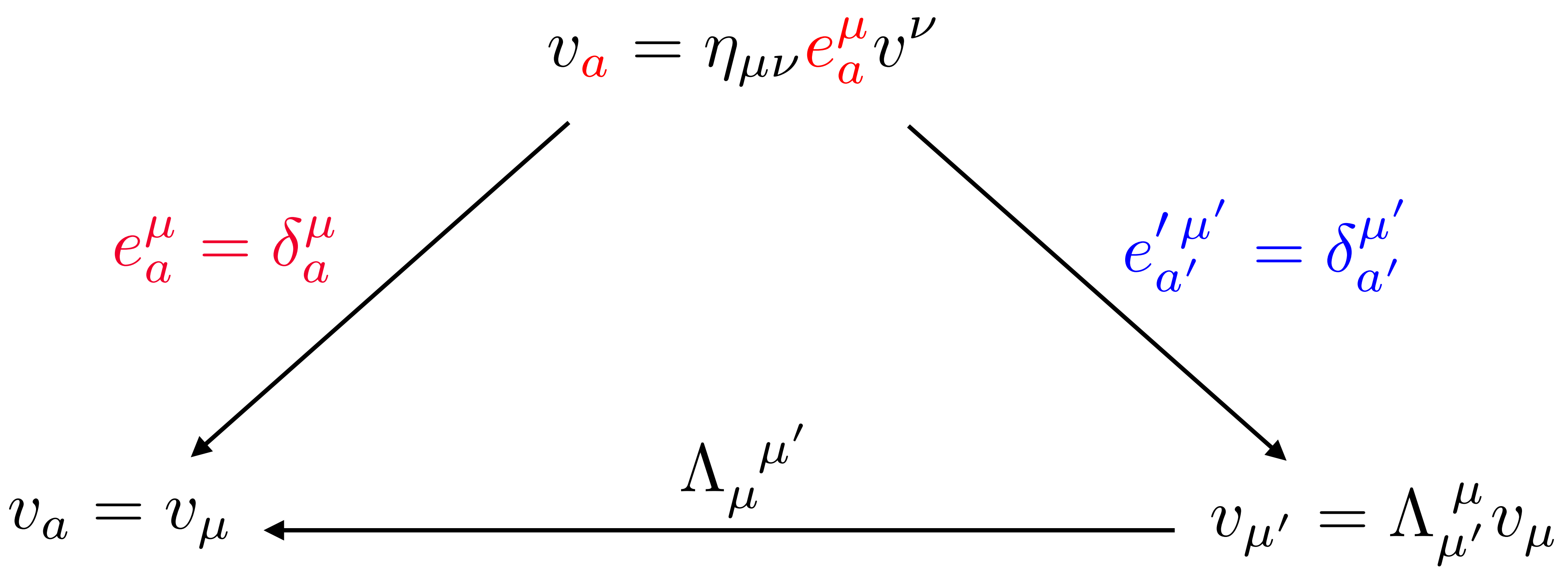


change of rel. obs. associated with same kinematical subsystem quantity

"gauge-induced" RF transformations

$$\Lambda_\mu^{\ \mu'}$$

(coordinate change via gauge fixings)



change of coordinate description of same relational observable

SUBSYSTEM RELATIVITY $\Rightarrow$ RELATIVITY OF SIMULTANEITY

[Höhn, Kotecha, **FMM** '23]

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Relational observables describing v rel. to 1st frame: $v_a \quad , \quad v_a v^a = v_\mu v^\mu$

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Distinct gauge-invariant sets of observables,
i.e., gauge-inv. notions of subsystems,
with non-trivial overlap (functions of $v_\mu v^\mu$)

internal relational observables

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Different frames decompose the total gauge-inv. "algebra" in different ways into subsystem of interest and "other frame"

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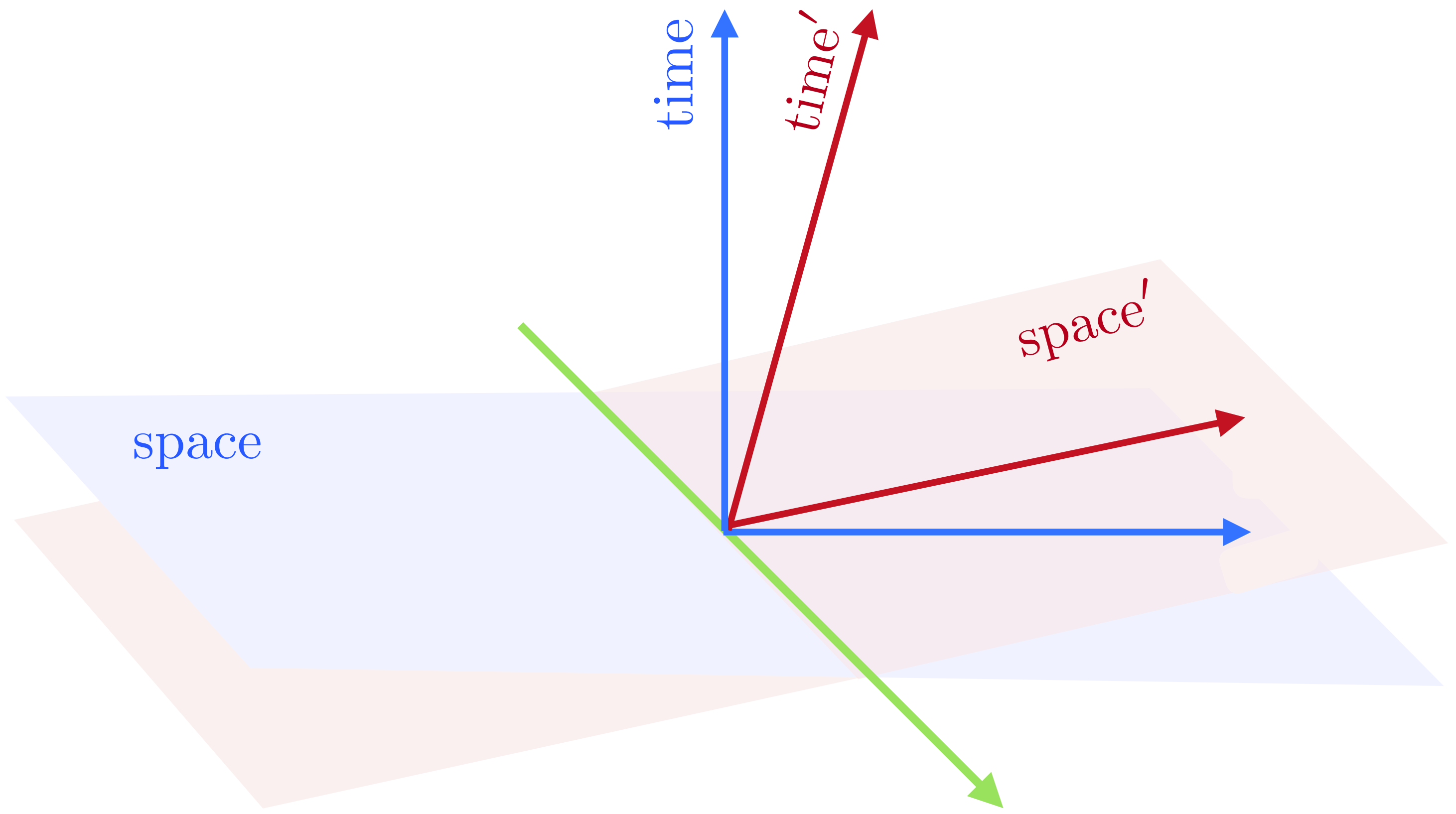
Note: $\Lambda_a^{\ a'}$ would not give a non-trivial transformation if the observables describing the subsystem of interest rel. to the two frames were coincident



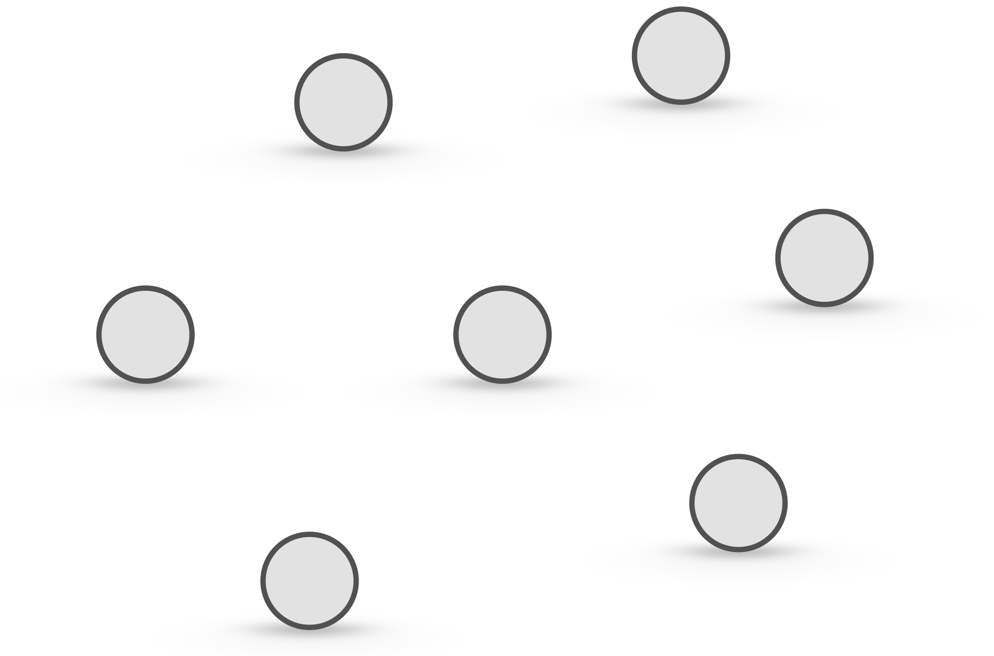
Unless $\Lambda_a^{\ a'}$ is only a spatial rotation, v_a and $v_{a'}$ decompose into space and time components in distinct ways

Part II

Quantum reference frames & relational subsystems

EXTERNAL/KINEMATICAL VS. INTERNAL/PHYSICAL SUBSYSTEMS

(Total) system of particles with translation invariance:



Kinematical factorisation into subsystems (e.g. absolute positions/relative to external frame)

$\nparallel$

Physical/external-frame-indep. factorisation into subsystems (e.g. translation invariant relative distances)

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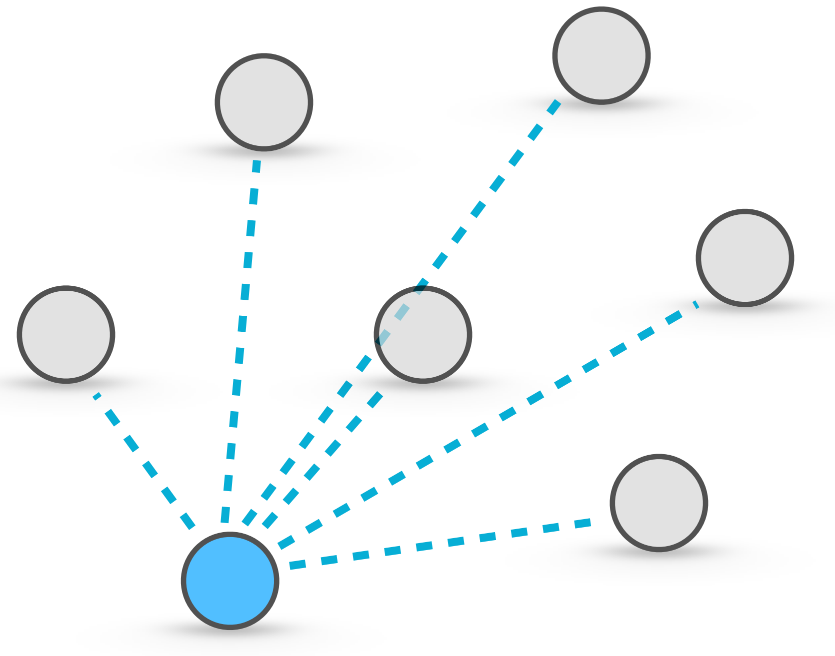

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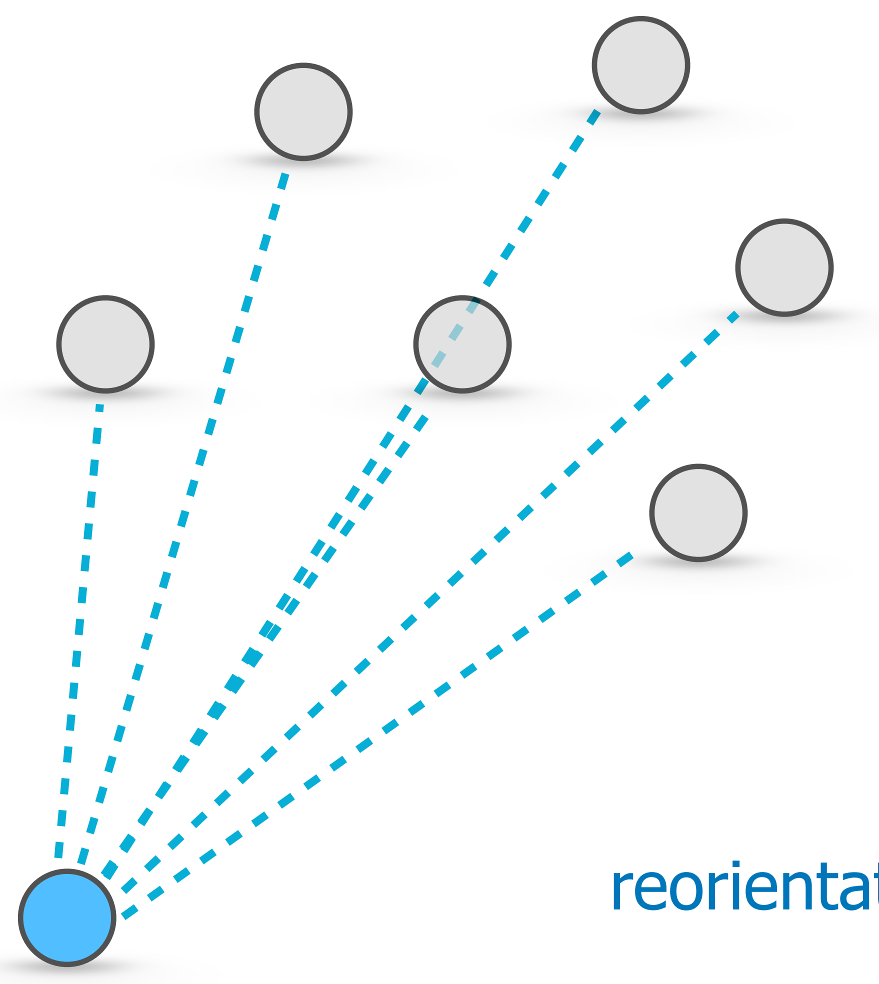


Kinematical factorisation into subsystems (e.g. absolute positions/relative to external frame)

$\neq$

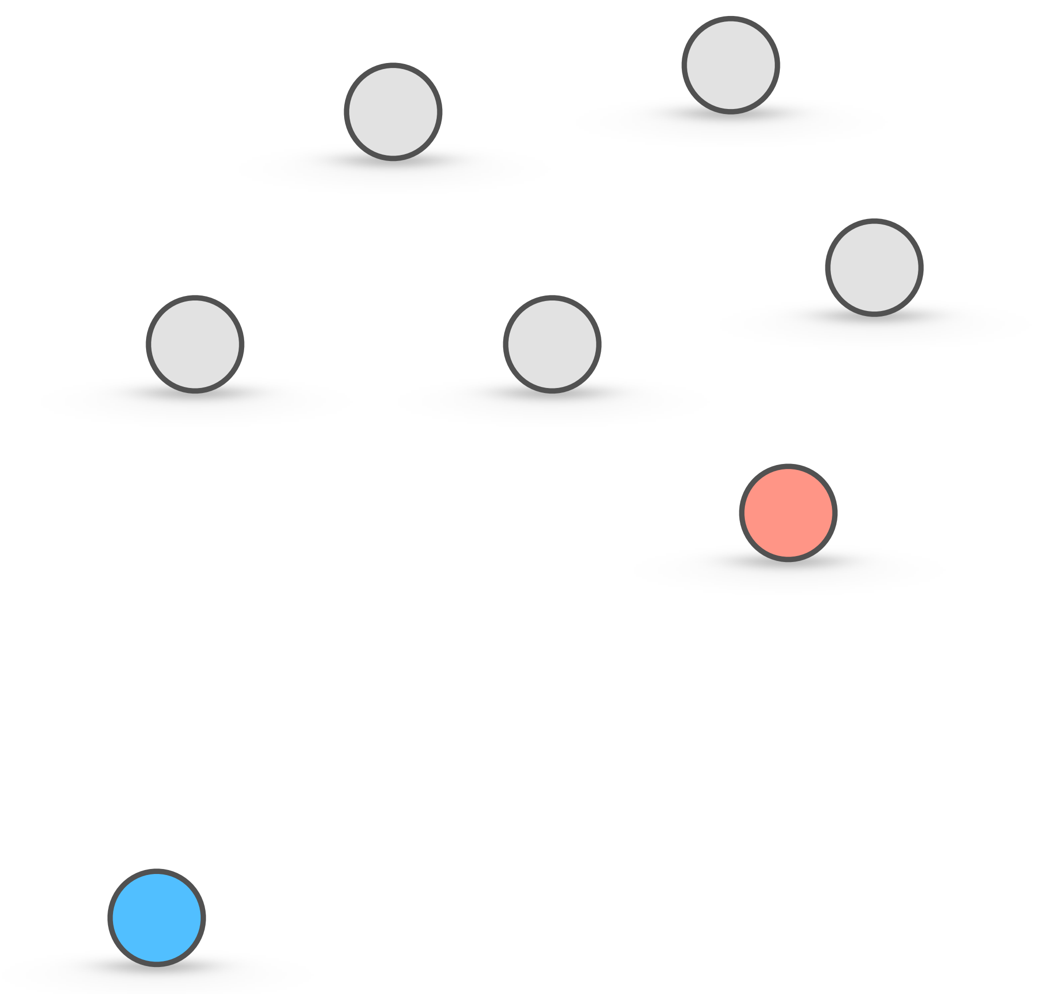
Physical/external-frame-indep. factorisation into subsystems (e.g. translation invariant relative distances)

reorientation (change of state) of internal frame

→ description of remaining particles relative to external frame unchanged
(absolute positions of the other particles unchanged)

→ description relative to internal frames changed
(relations between the frame and the other particles are changed)

EXTERNAL/KINEMATICAL VS. INTERNAL/PHYSICAL SUBSYSTEMS



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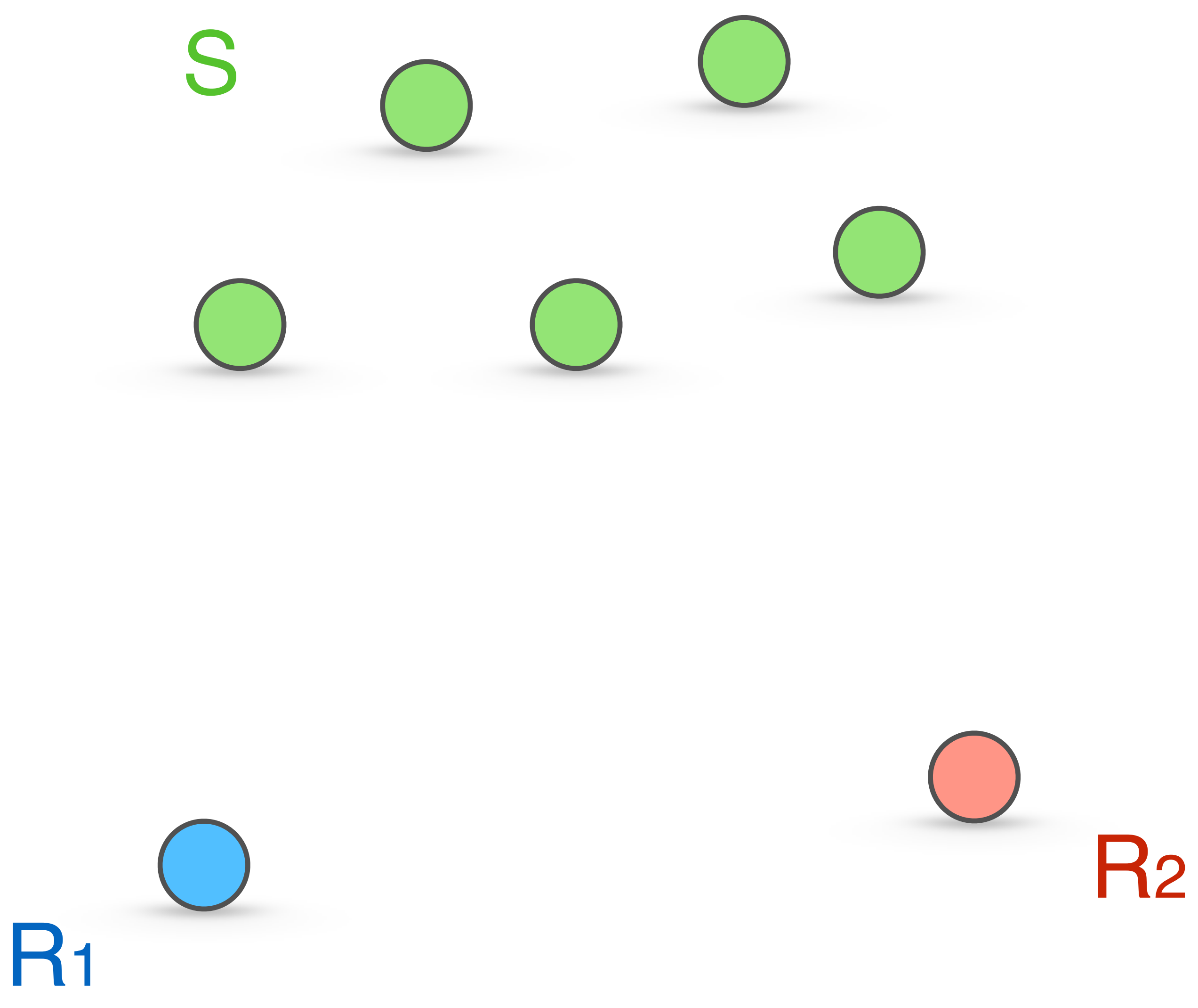

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Physical/external-frame-indep. factorisation into subsystems (e.g. translation invariant relative distances)

Description of S relative to R_1 invariant under reorientations of R_2, but relative to R_2 it changes since relations between S and R_2 change

$$\mathcal{A}^{\text{phys}}_{S|R_1} \neq \mathcal{A}^{\text{phys}}_{S|R_2}$$

Different internal frames identify distinct relational notions of subsystems

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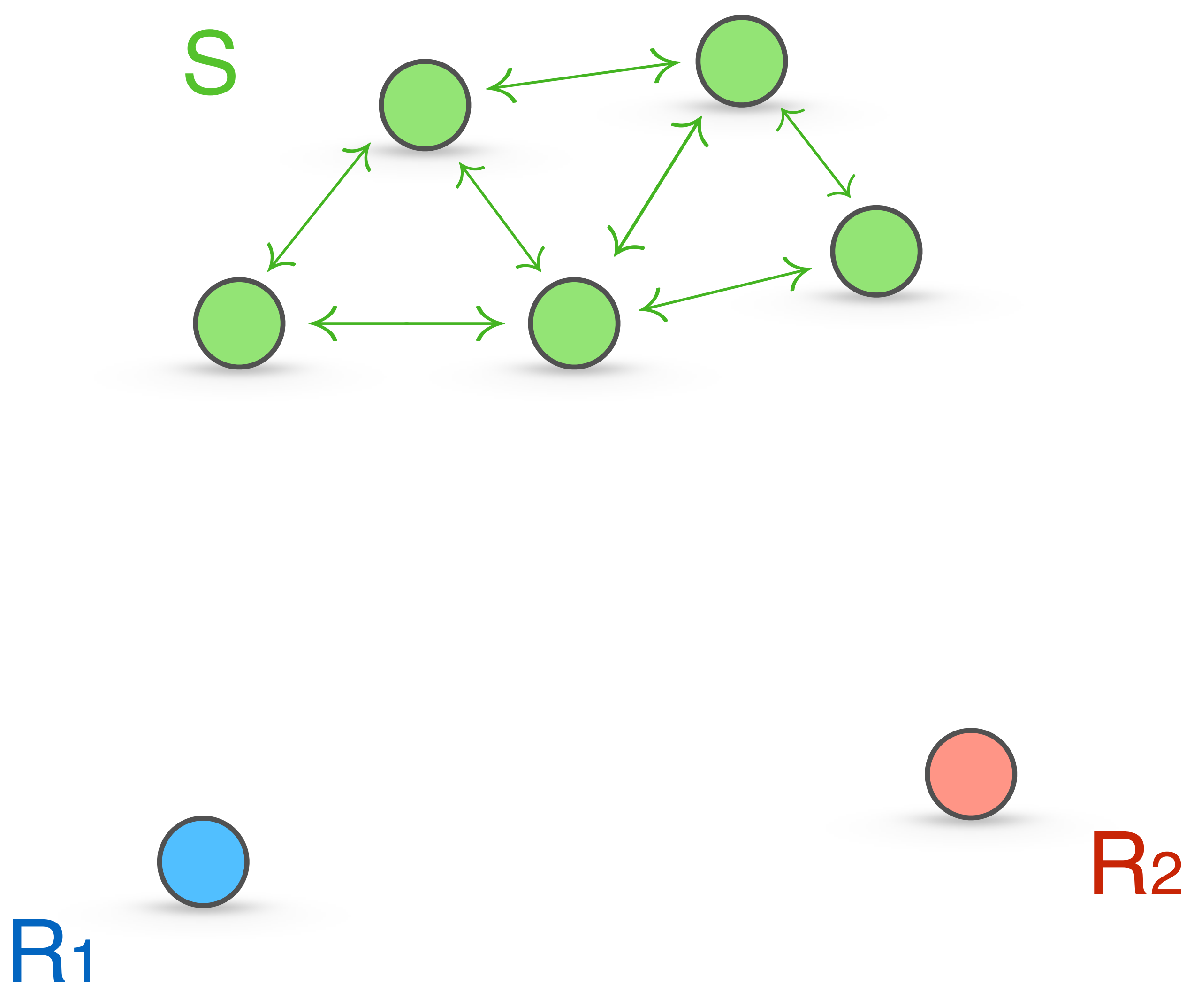

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Different internal frames identify distinct relational notions of subsystems

Internal relations to S are invariant under reorientations of both frames (frame-independent relational observables)

$$\mathcal{A}^{\text{phys}}_{S|R_1} \cap \mathcal{A}^{\text{phys}}_{S|R_2} \neq \emptyset$$

[Höhn, Kotecha, **FMM** '23]

Gauge-inv. properties of subsystems such as correlations, entropies, dynamics (open vs. closed), equilibrium and non-eq. thermodynamics contingent on the internal frame

INTERNAL (QUANTUM) DYNAMICAL REFERENCE FRAMES

[Krumm, Höhn, Müller ’20, ’21;
de la Hamette, Galley, Höhn, Loveridge, Müller ’21]

Setup relative to external (possibly fictitious) frame:

Split total DoFs **into** a **(SUB)SYSTEM OF INTEREST** (subgroup of particle, subregion,…)
and FRAME DoFs (constructed from the complement)

$$\mathcal{H}_{\text{kin}} = \mathcal{H}_R \otimes \mathcal{H}_S$$

frame ↙ ↘ system

space of externally distinguishable states

Unimodular Lie group as gauge transformations (e.g. Galilei, Poincaré, SU(2), reparametrisations,…)

$$U_{RS}(g) = U_R(g) \otimes U_S(g) \quad , \quad g \in G$$

external frame transformations (internal RS relations not affected)

analogue of $\Lambda^\mu_\nu \in SO_+(3,1)$ in SR

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$\mathcal{H}_R$ → frame, $\mathcal{H}_S$ → system

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Dynamical frame R associated with (gauge) group G ⟶ frame configurations associated to group elements

G-frame: subsystem "as non-invariant as possible" under G-action to be used to parametrise the orbits of G

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frame ($\mathcal{H}_R$) — system ($\mathcal{H}_S$)

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G-frame: subsystem "as non-invariant as possible" under G-action to be used to parametrise the orbits of G

frame orientations: system of generalised coherent states (gauge transf. acts transitively from left/gauge cov.)

$$|\varphi(g)\rangle_R \;\to\; U_R(g')|\varphi(g)\rangle_R = |\varphi(g'g)\rangle_R \quad , \quad \int_G \mathrm{d}g\, |\varphi(g)\rangle\langle\varphi(g)|_R = \mathbf{1}_R$$

analogue of $\Lambda^\mu{}_\nu e^\nu_a$

not necessarily orthogonal/perfectly distinguishable orientations

symmetries/frame reorientations (act from right):

$$V_R(g')|\varphi(g)\rangle_R = |\varphi(gg'^{-1})\rangle_R$$

change RS relations (analogue of $\Lambda^a{}_b e^\mu_a$)

commuting group actions

PHYSICAL STATES & RELATIONAL OBSERVABLES

[Krumm, Höhn, Müller '20, '21;
de la Hamette, Galley, Höhn, Loveridge, Müller '21]

Physical states invariant under gauge transformations (external frame transf.):

$$\mathcal{H}_{\text{phys}} = \left\{ |\psi_{\text{phys}}\rangle \quad \text{s.t.} \quad |\psi_{\text{phys}}\rangle = U_R(g) \otimes U_S(g) |\psi_{\text{phys}}\rangle \ , \ g \in G \right\}$$

$$|\psi_{\text{phys}}\rangle = \Pi_{\text{phys}} |\psi_{\text{kin}}\rangle$$

$$\Pi_{\text{phys}} = \int_G \mathrm{d}g \, U_R(g) \otimes U_S(g)$$

group-averaging "projector"

$$\langle \psi_{\text{phys}} | \psi_{\text{phys}} \rangle_{\text{phys}} = \langle \psi_{\text{kin}} | \Pi_{\text{phys}} | \psi_{\text{kin}} \rangle$$

space of relational equivalence classes of states → external-frame indep., internally distinguishable states

[Rovelli '98]

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space of relational equivalence classes of states → external-frame indep., internally distinguishable states

[Rovelli ’98]

Gauge-invariant information encoded in relational observables (e.g. relative distances) obtained via G-twirl:

$$O_{f_S,R}(g) = \int_G \mathrm{d}g'\, U_{RS}(g') \Big(|\varphi(g)\rangle\langle\varphi(g)|_R \otimes f_S \Big) U_{RS}^{\dagger}(g')$$

frame-orientation conditional gauge transf.
(controlled unitary)

(analogue of $v_a = e_a^{\mu} v_{\mu}$)

value of f_S when R is in orientation g

relational obs. in the sense of Rovelli, Dittrich, Thiemann,…

gauge invariance: $[O_{f_S,R}, U_{RS}(g')] = 0$

$O_{\bullet,R}(g) : \mathcal{A}_S \to \mathcal{A}_{\text{phys}}$ *-homomorphism on $\mathcal{H}_{\text{phys}}$

(QUANTUM) FRAME RELATIVITY OF SUBSYSTEMS

[Ahmad Ali, Galley, Höhn, Lock, Smith '21;
de la Hamette, Galley, Höhn, Loveridge, Müller '22;
Höhn, Kotecha, **FMM** '23]

Consider two frames: $\mathcal{H}_{\rm kin} = \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_S$

Change from relational observables of S relative to R1 to those relative to R2 (for the same f_S):

In SR, recall: $v_a = \Lambda_a{}^{a'} v_{a'}$

where $\Lambda_a{}^{a'} = e'^{a'}_{\mu} e^{\mu}_a$

(rel. obs. describing 1st rel. to 2nd frame)

(QUANTUM) FRAME RELATIVITY OF SUBSYSTEMS

[Ahmad Ali, Galley, Höhn, Lock, Smith ’21;
de la Hamette, Galley, Höhn, Loveridge, Müller ’22;
Höhn, Kotecha, **FMM** ’23]

Consider two frames: $\mathcal{H}_{\text{kin}} = \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_S$

Change from relational observables of S relative to R1 to those relative to R2 (for the same f_S):

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where $\Lambda_a{}^{a'} = e'^{a'}_{\mu} e^{\mu}_a$

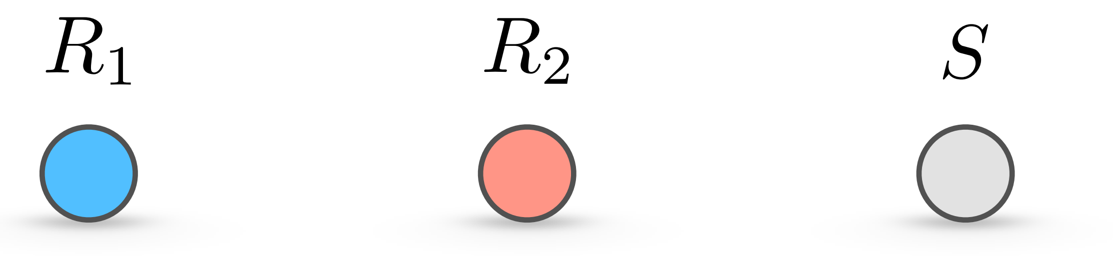
(rel. obs. describing 1st rel. to 2nd frame)

$$O_{f_S,R_1}(g_1) \xrightarrow{\mathcal{V}^{g_1,g_2}_{R_1 \to R_2}} O_{f_S,R_2}(g_2)$$

relation-conditional
frame reorientation

What’s the value of f_S
when R1 is in orientation g_1?

What’s the value of f_S
when R1 is in orientation g_2 ?



$$q_S - q_1 \longrightarrow q_S - q_2$$

$$q_1 \mapsto q_1 + (q_2 - q_1)$$

translation of R1 by relative distance between R2 and R1

(QUANTUM) FRAME RELATIVITY OF SUBSYSTEMS

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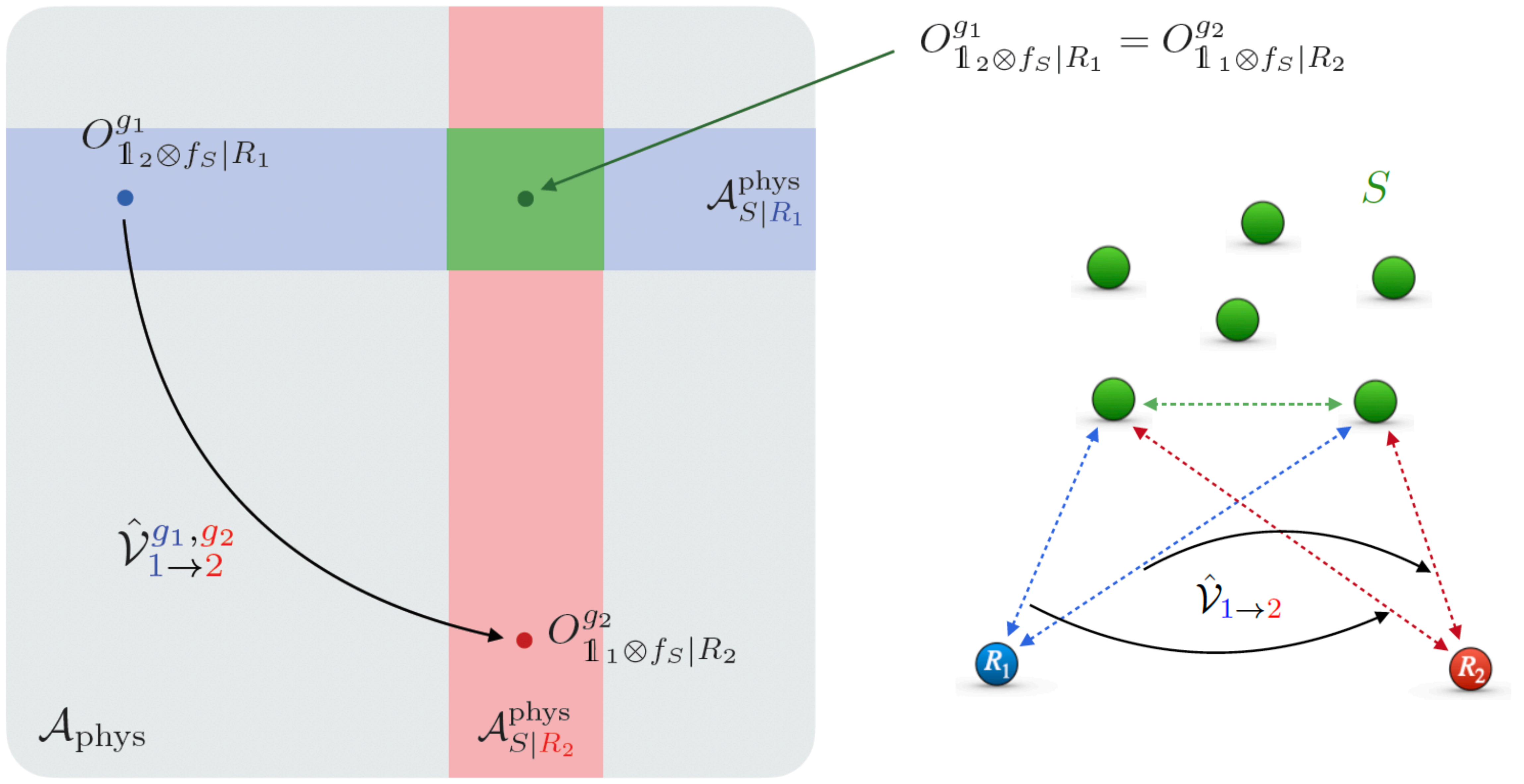
- **S-observables relative to R1 invariant under R2-reorientations, and viceversa**

- **Different gauge-invariant subalgebras describing S relative to R1 and R2**

- $\mathcal{A}^{\rm phys}_{S|R_1} \cap \mathcal{A}^{\rm phys}_{S|R_2}$ **invariant under reorientations of both frames**
  (in particular, relation-conditional ones)

  internal S relations
  (already gauge-inv./indep. of R1 and R2)

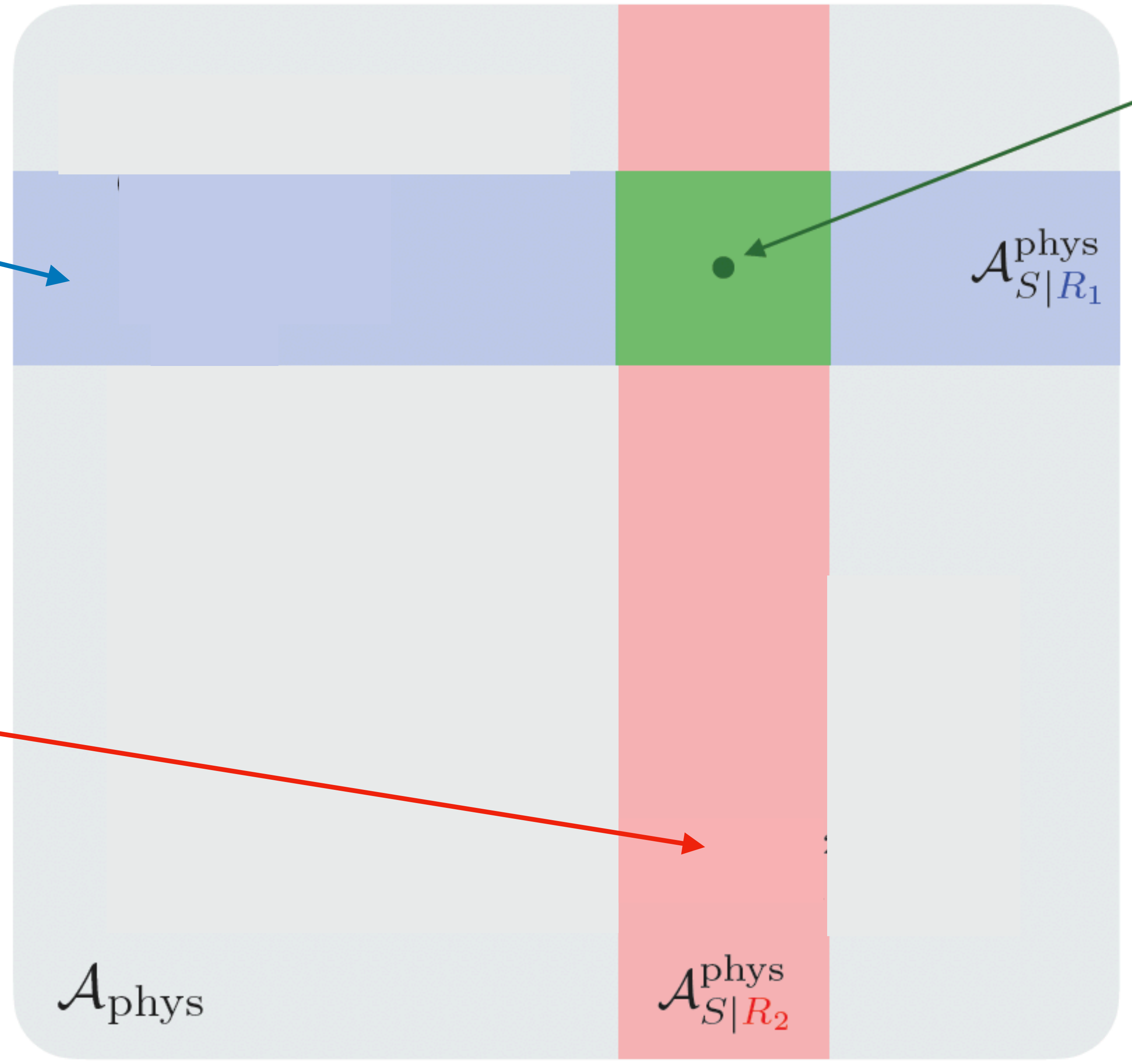


(QUANTUM) FRAME RELATIVITY OF SUBSYSTEMS

[Höhn, Kotecha, **FMM** '23]

Different frames identify different gauge inv. subsystems

Different relational ways to refer to a kinematical subsystem



For finite-systems & ideal frames:

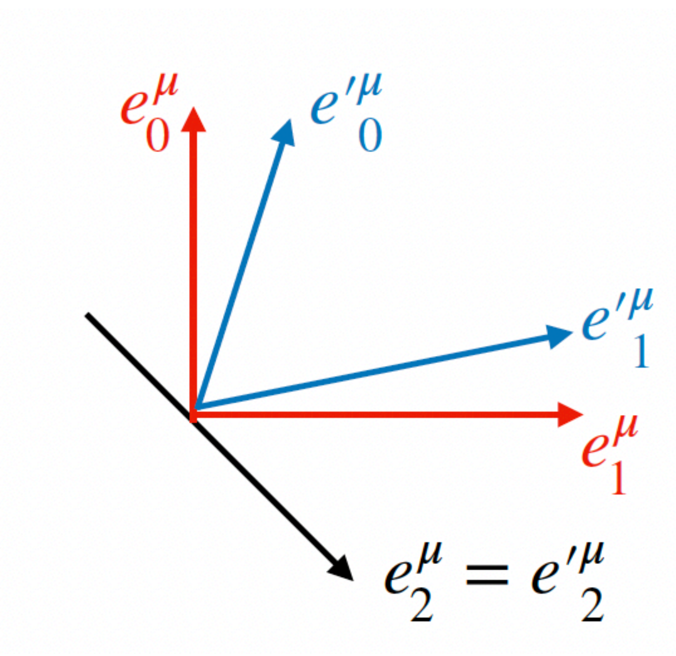
$$\mathcal{A}_{\rm phys} \simeq \mathcal{A}^{\rm phys}_{R_2|R_1} \otimes \mathcal{A}^{\rm phys}_{S|R_1} \simeq \mathcal{A}^{\rm phys}_{R_1|R_2} \otimes \mathcal{A}^{\rm phys}_{S|R_2}$$

but

$$\mathcal{A}^{\rm phys}_{S|R_1} \neq \mathcal{A}^{\rm phys}_{S|R_2}$$

Inequivalent factorisations of total algebra relative to the two frames

(not in general the same as factorisations across kinematical DoFs)

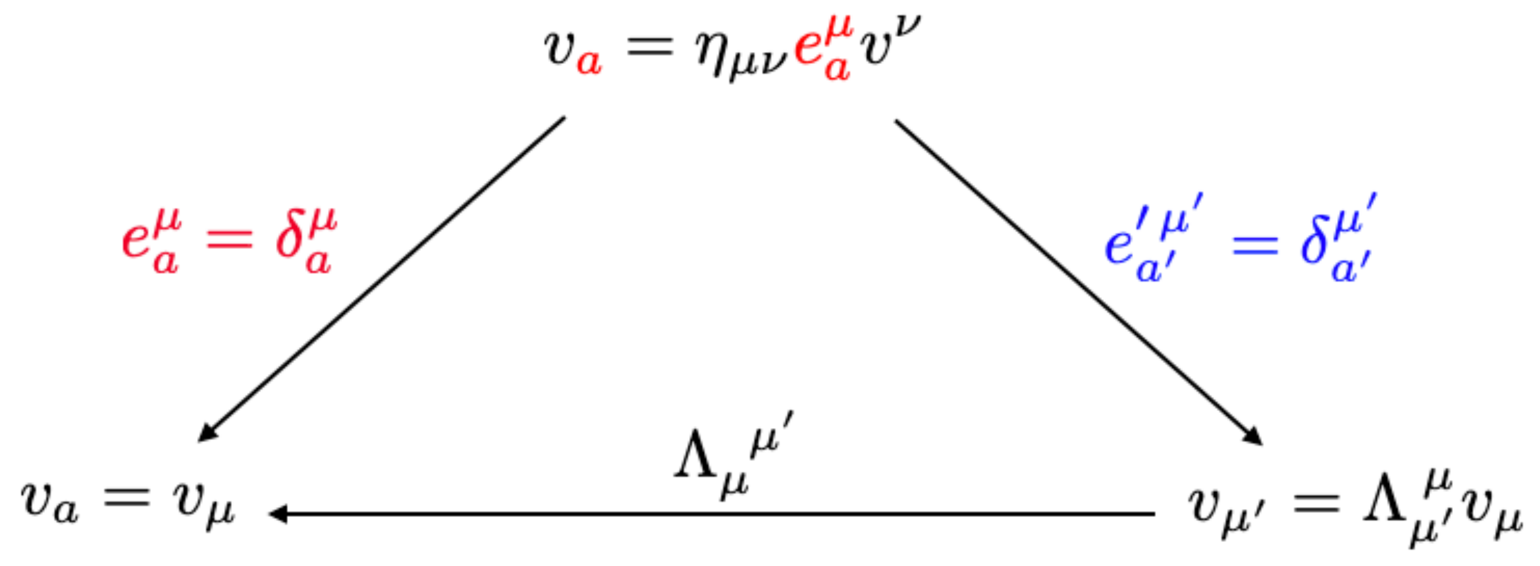


relativity of simultaneity:

different observers decompose space of (relational) length observables in different ways into space and time

JUMPING INTO INTERNAL FRAME PERSPECTIVE

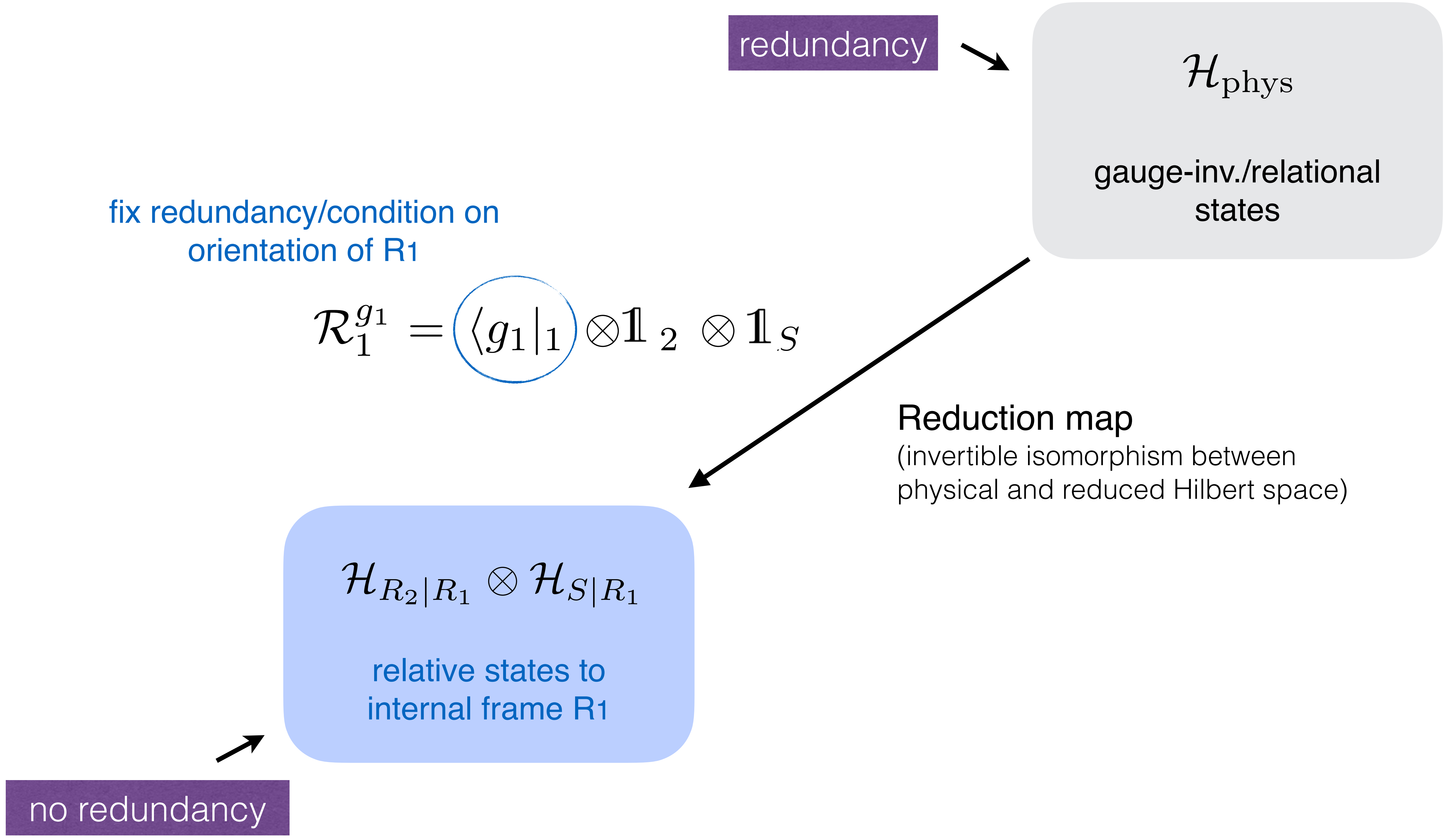
Recall: “Jumping into frame perspective via gauge fixing”



Idea: **identify redundant DoFs with those of the frame**

different gauge choices ⟶ different frame perspectives

Finite Abelian case (frame orientations ⟶ $|g_1\rangle_1$ regular representation, $\mathcal{H}_R = \ell^2(G)$)



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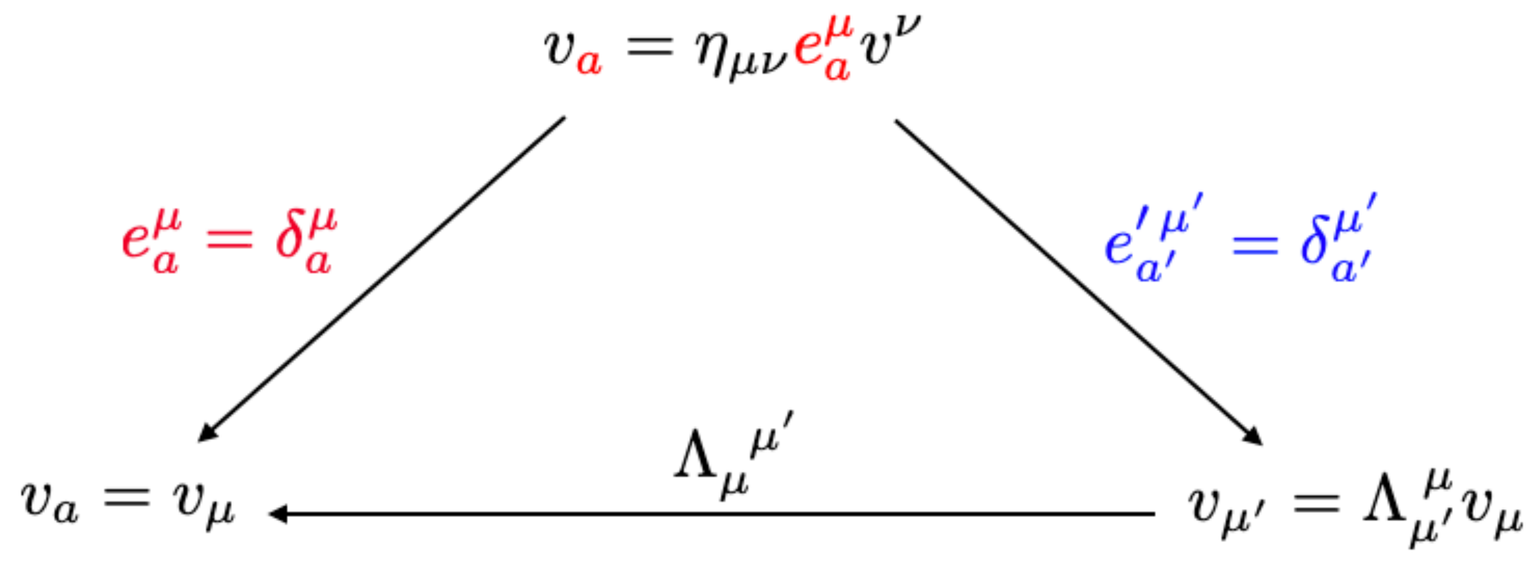


redundancy ↘

$\mathcal{H}_{\text{phys}}$

gauge-inv./relational states

fix redundancy/condition on orientation of R1

$$\mathcal{R}_1^{g_1} = \langle g_1|_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_S$$

Reduction map
(invertible isomorphism between physical and reduced Hilbert space)

$\mathcal{H}_{R_2|R_1} \otimes \mathcal{H}_{S|R_1}$

relative states to internal frame R1

no redundancy ↗

Example: $\mathcal{G} = \mathbb{Z}_n$-transl. inv. (+ mod n) N particles

$$|\psi_{\text{phys}}\rangle = \frac{1}{\sqrt{|\mathcal{G}|}} \sum_{g \in \mathcal{G}} |g, \vec{h}g\rangle$$

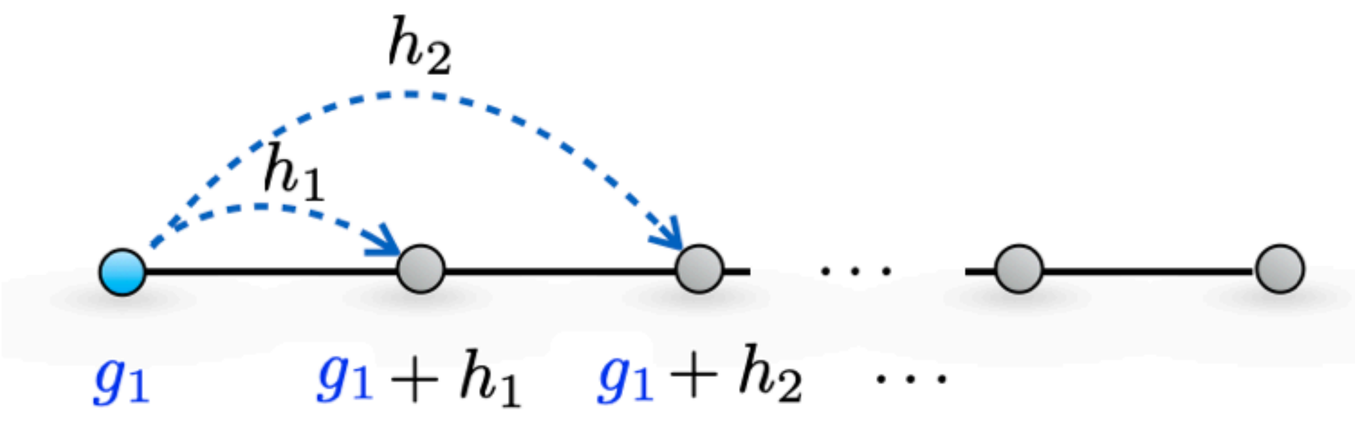
N-1 relative distances

1

↓

$$|\psi(g_1)\rangle_{\bar{1}} = |\vec{h}\, g_1\rangle_{N-1|1}$$

remaining N-1 particles relative to particle 1 in position g_1



JUMPING INTO INTERNAL FRAME PERSPECTIVE

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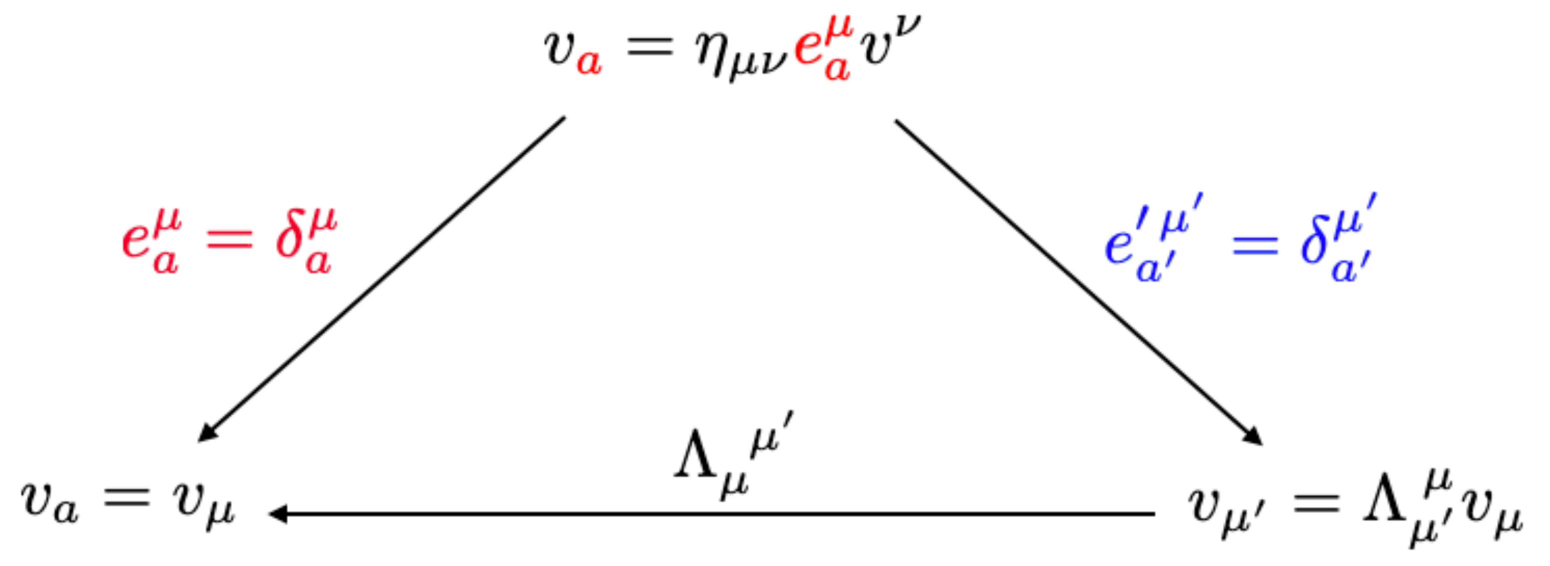

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relative states to internal frame R1

no redundancy

Tensor Product Structure (TPS)

[Zanardi '01; Zanardi, Lidar, Lloyd '03; Cotler, Penington, Ranard '19]

A TPS $\mathcal{T}$ on $\mathcal{H}$ is an equivalence class of isomorphisms (unitaries)

$$\mathbf{T} : \mathcal{H} \to \bigotimes_{\alpha=1}^{n} \mathcal{H}_\alpha$$

such that

$\mathbf{T} \sim \mathbf{T}'$ if $\mathbf{T}' \circ \mathbf{T}^{-1} =$ product of local unitaries $\otimes_\alpha U_\alpha$ and permutations of subsystem factors with equal dim.

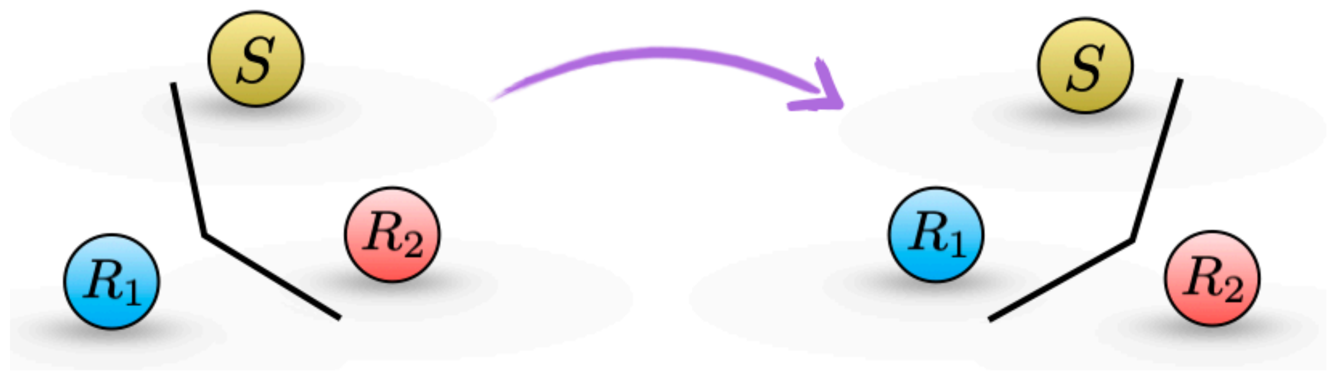
same notion of locality

Jumping into perspective = TPS on $\mathcal{H}_{\text{phys}}$

[Höhn, Kotecha, **FMM** '23]

$$\mathbf{T}_1^{g_1} = \mathcal{R}_1^{g_1} \quad , \quad \mathcal{R}_1^{g} \sim \mathcal{R}_1^{g'}$$

CHANGING THE INTERNAL FRAME PERSPECTIVE



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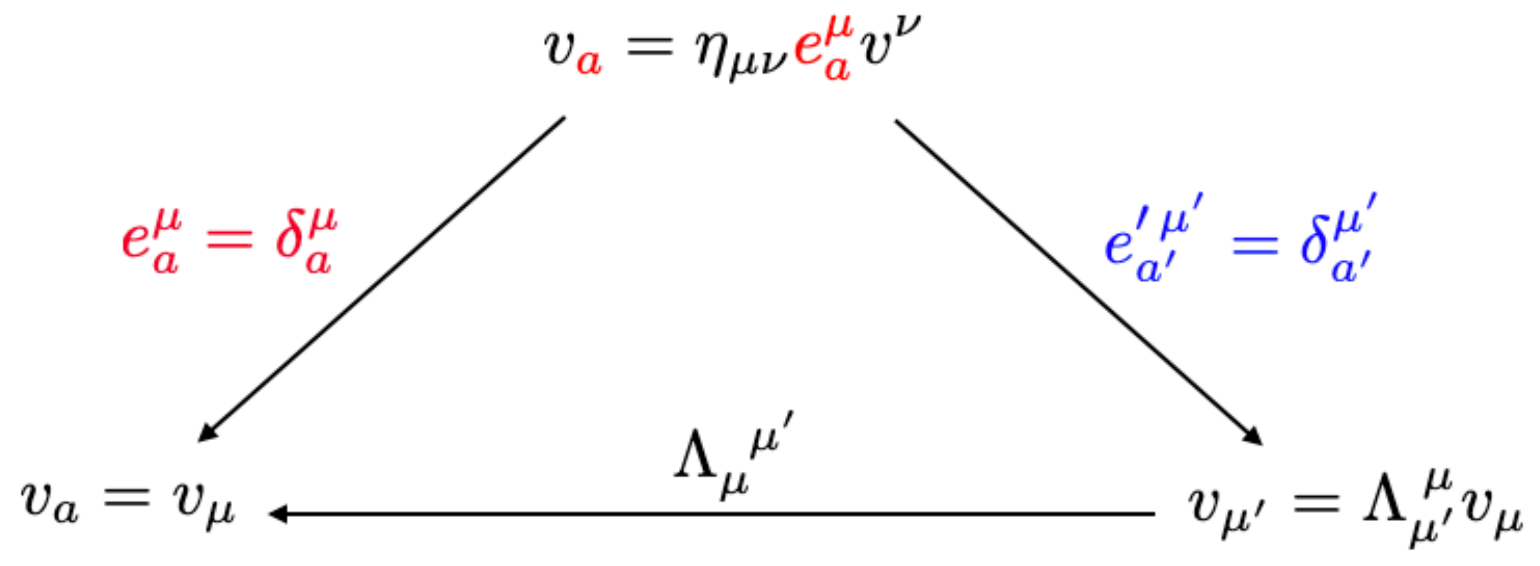


$\mathcal{H}_{\text{phys}}$

perspective-neutral

reduction to R1 perspective

$$\mathcal{R}_1^{g_1} = \langle g_1|_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_S$$

reduction to R2 perspective

$$\mathcal{R}_2^{g_2} = \mathbb{1}_1 \otimes \langle g_2|_2 \otimes \mathbb{1}_S$$

different frame perspectives linked through perspective-neutral stage

$\mathcal{H}_{R_2|R_1} \otimes \mathcal{H}_{S|R_1}$

relative states to internal frame R1

$\mathcal{H}_{R_1|R_2} \otimes \mathcal{H}_{S|R_2}$

relative states to internal frame R2

QRF transformation

$$V_{1\to 2}^{g_1\to g_2} = \mathcal{R}_2^{g_2} \circ (\mathcal{R}_1^{g_1})^{-1}$$

CHANGING THE INTERNAL FRAME PERSPECTIVE

Recall: "Jumping into frame perspective via gauge fixing"

$$v_a = \eta_{\mu\nu} e^{\mu}_{a} v^{\nu}$$

$e^{\mu}_{a} = \delta^{\mu}_{a}$ $\qquad$ $e'^{\mu'}_{a'} = \delta^{\mu'}_{a'}$

$v_a = v_\mu$ $\qquad$ ${\Lambda_\mu}^{\mu'}$ $\qquad$ $v_{\mu'} = {\Lambda^{\mu}}_{\mu'} v_\mu$

$\mathcal{H}_{\rm phys}$

perspective-neutral

reduction to R1 perspective

$$\mathcal{R}_1^{g_1} = \langle g_1|_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_S$$

&

TPS-neutral

reduction to R2 perspective

$$\mathcal{R}_2^{g_2} = \mathbb{1}_1 \otimes \langle g_2|_2 \otimes \mathbb{1}_S$$

$\mathcal{H}_{R_2|R_1} \otimes \mathcal{H}_{S|R_1}$

relative states to internal frame R1

$\mathcal{H}_{R_1|R_2} \otimes \mathcal{H}_{S|R_2}$

relative states to internal frame R2

QRF transformation

$$V_{1\to2}^{g_1\to g_2} = \mathcal{R}_2^{g_2} \circ (\mathcal{R}_1^{g_1})^{-1}$$

non-local unitary → **Inequivalent TPSs**

Hilbert space counterpart of the algebra story we've seen before

PHYSICAL CONSEQUENCES

[Höhn, Kotecha, **FMM** '23]

Gauge-invariant physical properties generically depend on the chosen frame:

- Correlations/entanglement of S with its complement ⟶ gauge-inv. entanglement entropy in general $\mathcal{S}(\rho_{S|R_1}) \neq \mathcal{S}(\rho_{S|R_2})$ for same global physical state

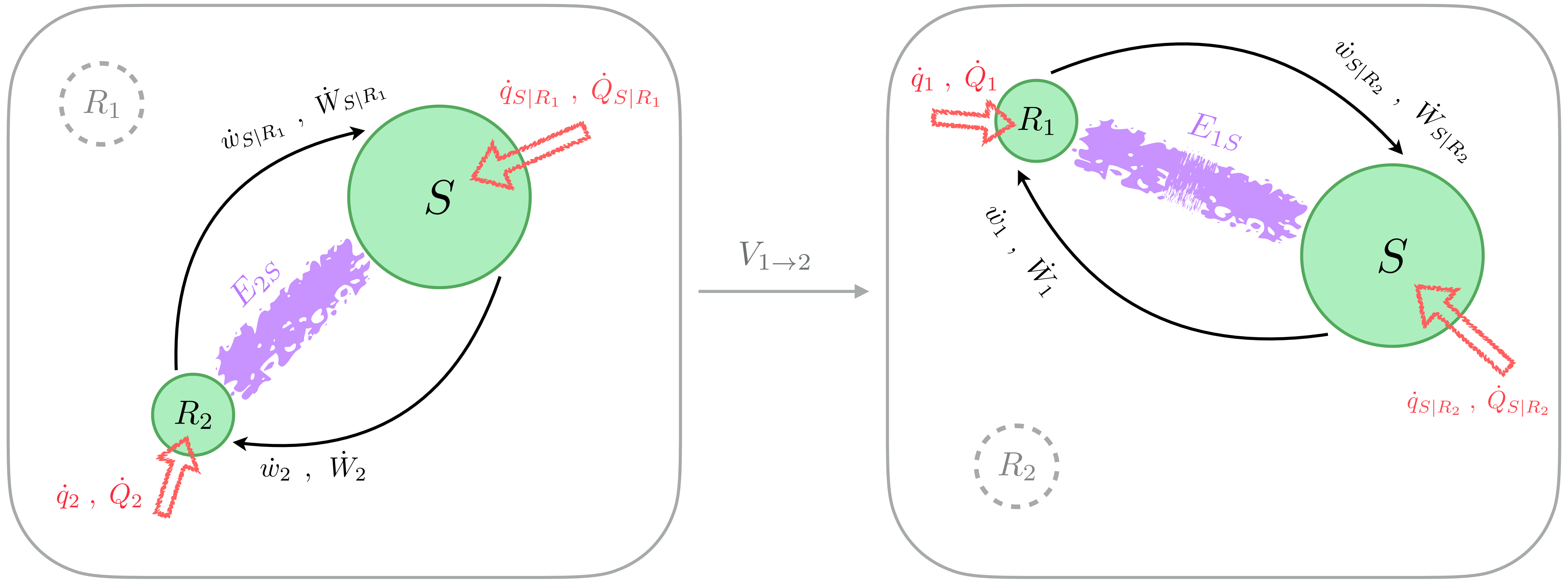
  [see also Giacomini, Castro-Ruiz, Brukner '17; Castro-Ruiz, Oreshkov '21)]

- QRF-relativity of interactions (degree of locality of total Hamiltonian) ⟶ dynamics of S can be isolated/closed relative to R1, but open relative to R2

- QRF-relativity of (quantum) thermodynamics: thermal equilibrium & non-equilibrium processes (heat/work exchanges, entropy production, and entropy flow)

Total system R1R2S in isolation (unitary dynamics)

Subsystems can interact, exchange heat, work, & energy



Part III

Entanglement entropy: relational vs. center construction

RELATIONAL vs CENTER CONSTRUCTION

N+M particles in 1D with translation invariance $G = (\mathbb{R}, +)$ → $P_{\text{tot}}|\psi_{\text{phys}}\rangle = 0$ → $\mathcal{A}_{\text{phys}} = \{q_2 - q_1, \ldots, q_{N+M} - q_1, p_2, \ldots, p_{N+M}\}$

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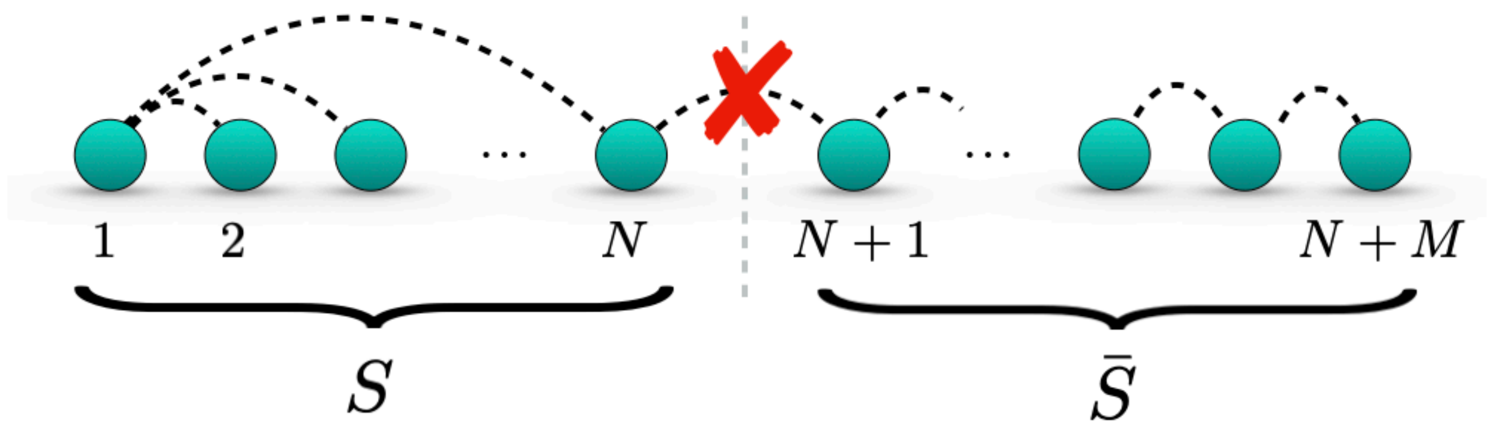
$P_{\text{tot}}|\psi_{\text{phys}}\rangle = 0$

Center construction



[Casini, Huerta, Rosabal '13; Donnelly '11, '14; Van Acoleyen, Bultinck, Haegeman, Marien, Scholz, Verstraete '15,…]

Assign "regional"/internal gauge-inv. algebras to kinematical complements

$$\mathcal{A}_S = \{q_2 - q_1, \ldots, q_N - q_1, p_1, \ldots, p_N\}$$

$$\mathcal{A}_{\bar{S}} = (\mathcal{A}_S)' = \{q_{N+2} - q_{N+1}, \ldots, q_{N+M} - q_{N+M-1}, p_{N+1}, \ldots, p_{N+M}\}$$

Non-trivial center

$$\mathcal{Z}_S = \mathcal{A}_S \cap (\mathcal{A}_S)' \neq \mathbb{C}\mathbf{1}$$

Algebra generated by $\mathcal{A}_S$ and its commutant is a strict subalgebra of $\mathcal{A}_{\text{phys}}$

$$\mathcal{A}_S \vee \mathcal{A}_{\bar{S}} = \bigoplus_z \mathcal{A}_S^z \otimes \mathcal{A}_{\bar{S}}^z \subset \mathcal{A}_{\text{phys}}$$

Entropy associated to local subalgebra (not entanglement entropy)

$$\mathcal{S}_{\text{vN}}(S) = \sum_z p_z \mathcal{S}_{\text{vN}}(\rho_S^z) + H(\{p_z\})$$

← classical Shannon

RELATIONAL vs CENTER CONSTRUCTION

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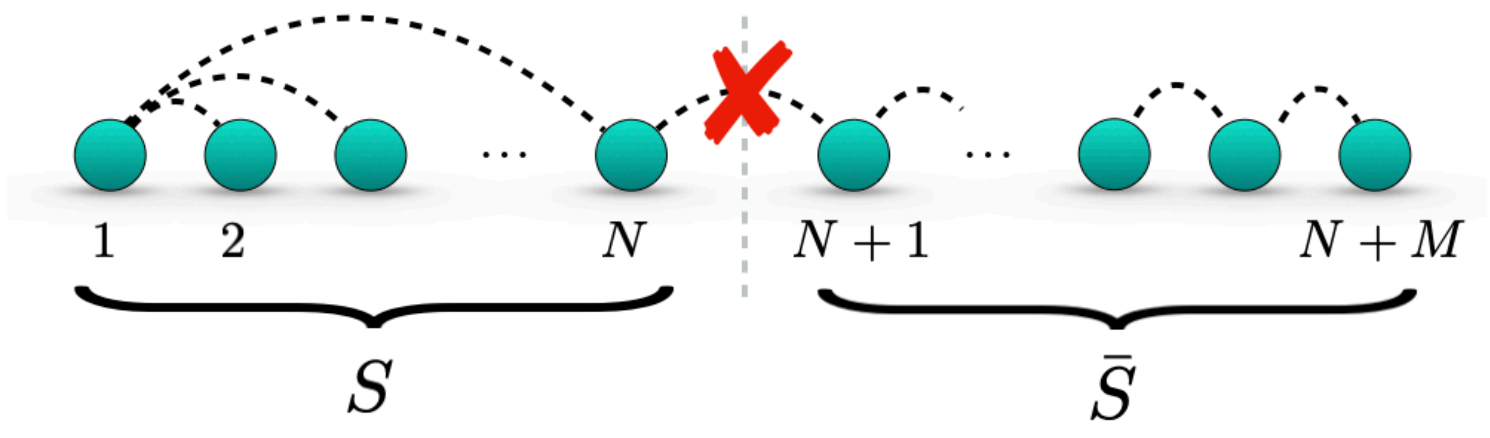
Center construction



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Non-trivial center

$$\mathcal{Z}_S = \mathcal{A}_S \cap (\mathcal{A}_S)' \neq \mathbb{C}\mathbf{1}$$

$$\mathcal{A}_S \ni p_1 + \cdots + p_N = -p_{N+1} - \cdots - p_{N+M} \in \mathcal{A}_{\bar{S}}$$

Algebra generated by $\mathcal{A}_S$ and its commutant is a strict subalgebra of $\mathcal{A}_{\rm phys}$

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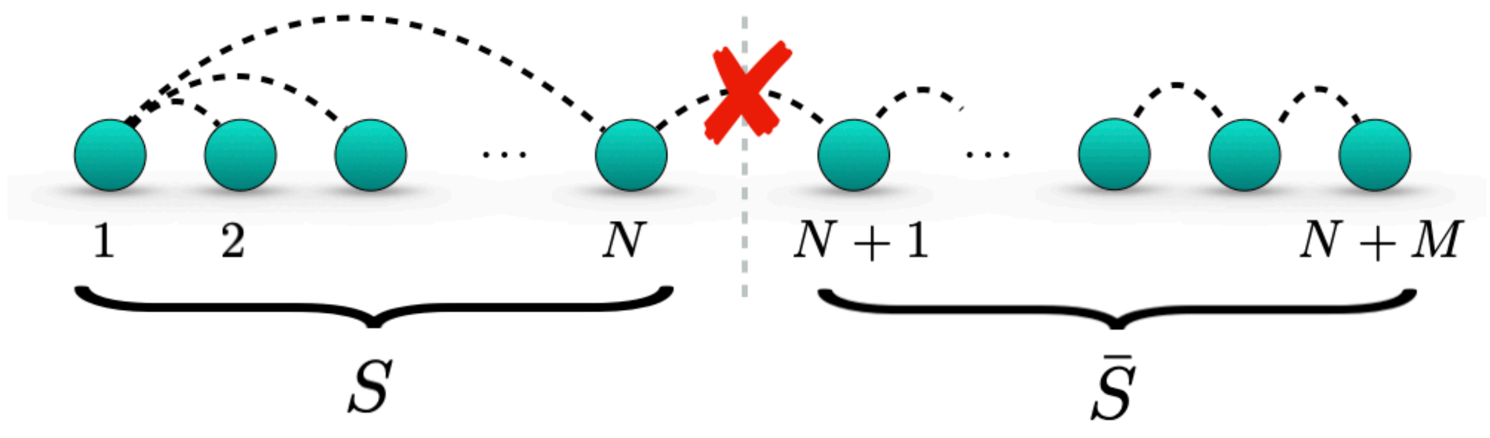
$P_{\rm tot}|\psi_{\rm phys}\rangle = 0$

Center construction



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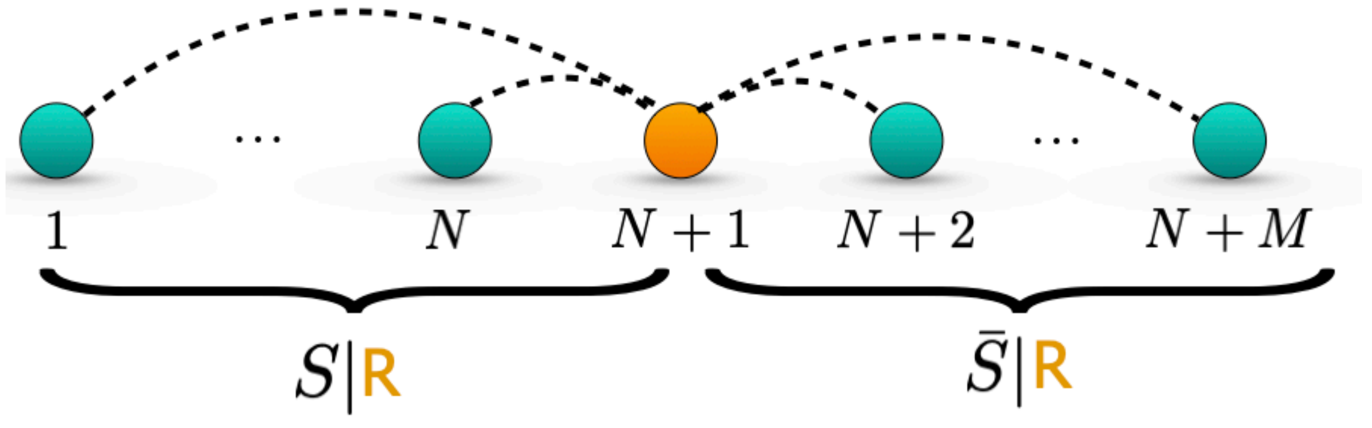
Relational construction



Assign gauge-inv. algebras via complements relative to R

$$\mathcal{A}^{\rm phys}_{S|R} = \{q_1 - q_R, \dots, q_N - q_R, p_1, \dots, p_N\}$$

$$\mathcal{A}^{\rm phys}_{\bar S|R} = \{q_{N+2} - q_R, \dots, q_{N+M} - q_R, p_{N+2}, \dots, p_{N+M}\}$$

No gauge-invariant data missing $\quad \mathcal{A}^{\rm phys}_{S|R} \vee \mathcal{A}^{\rm phys}_{\bar S|R} = \mathcal{A}_{\rm phys}$

Proper entanglement entropy $\quad \mathcal{S}_{\rm vN}(\rho_{S|1}) = -{\rm Tr}(\rho_{S|1} \log \rho_{S|1})$

generically frame dependent

Locality defined relationally (non-local combinations of $S\bar S$ kinematical DoFs)

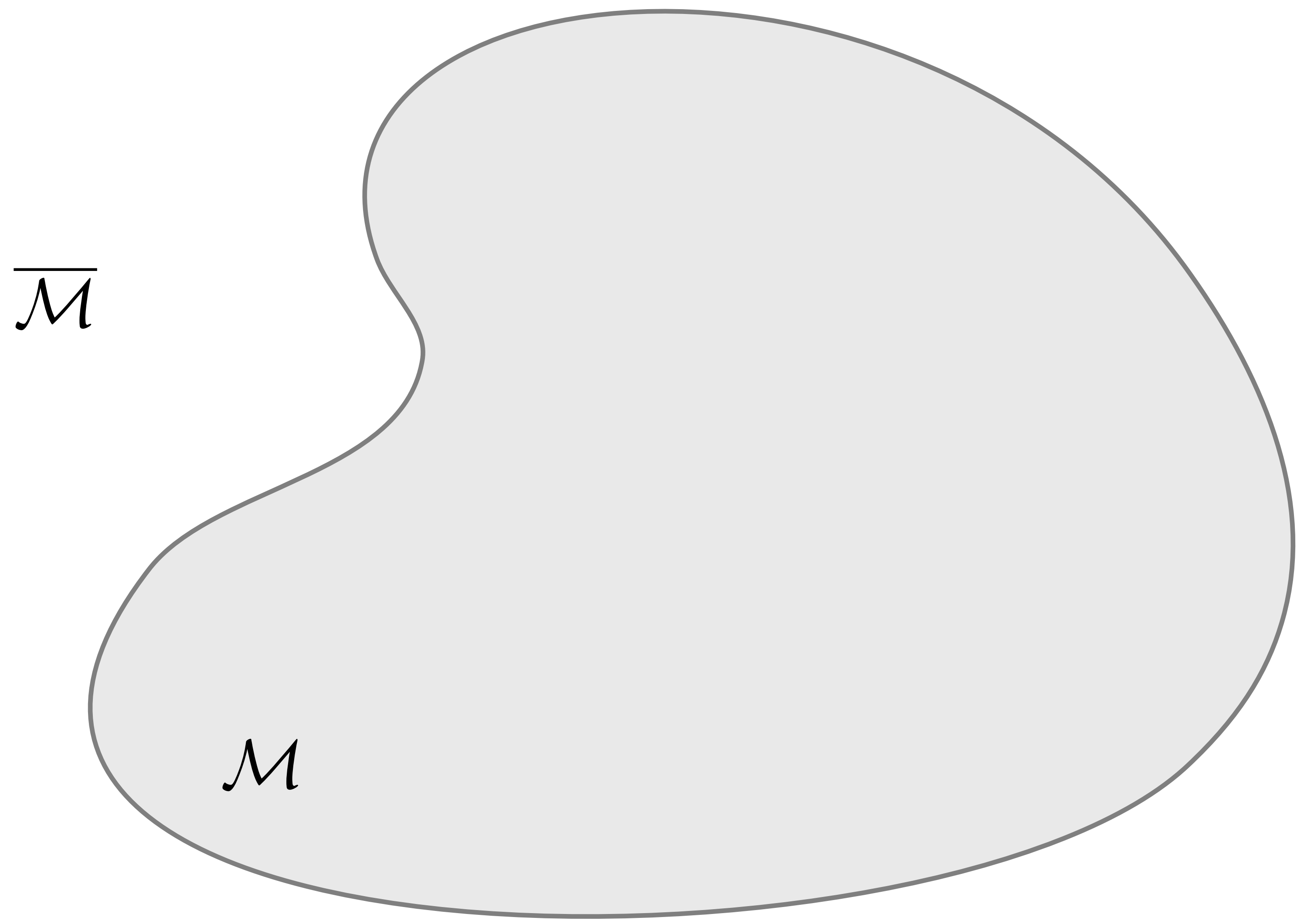
Bigger subalgebras associated with subsystems

"regional"/internal algebras appear as the frame-indep. data $\mathcal{A}^{\rm phys}_{S|R_1} \cap \mathcal{A}^{\rm phys}_{S|R_2} = \mathcal{A}_S$

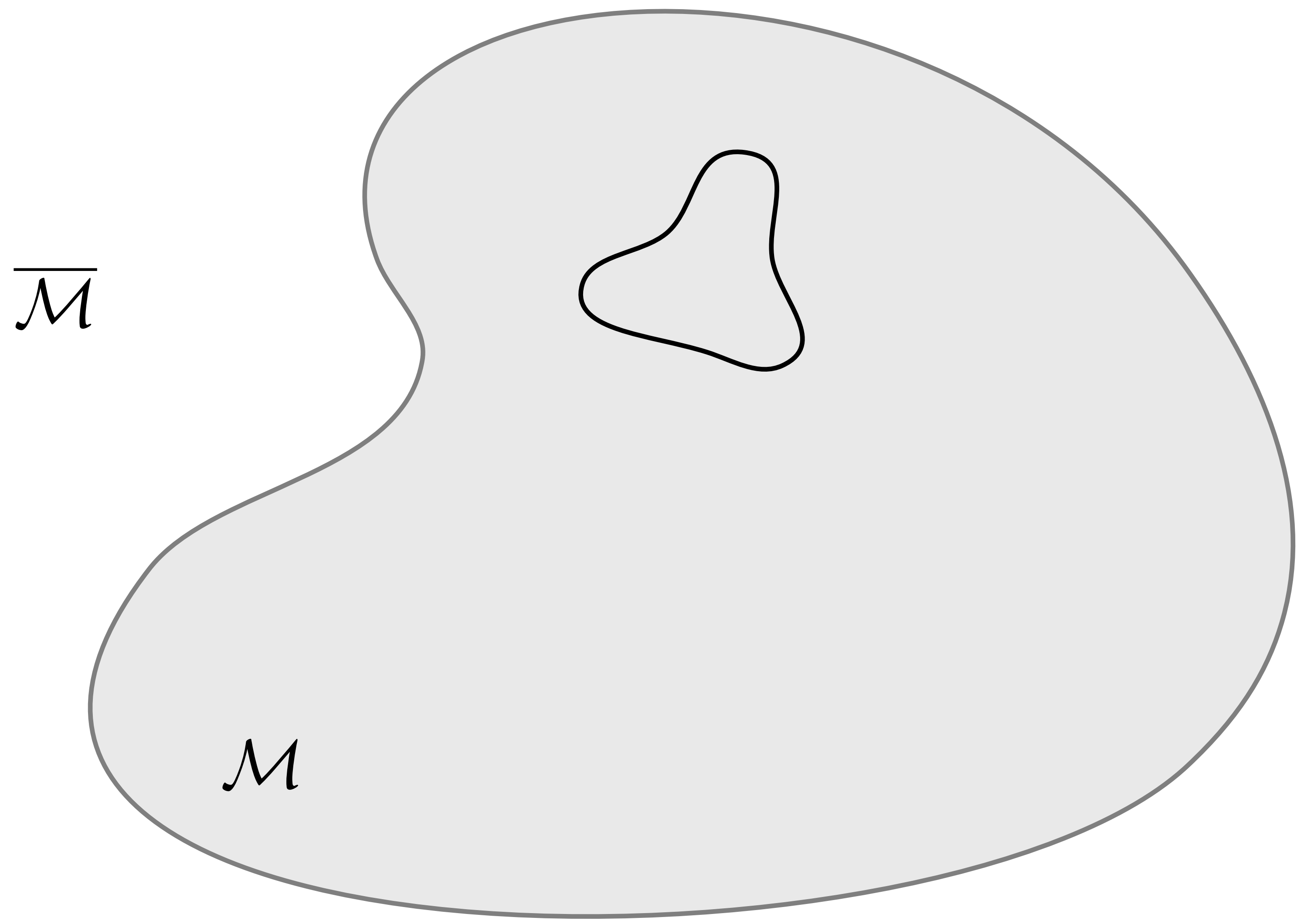
Part IV

Subregion relativity in gauge theories

LOCAL SUBSYSTEMS IN GAUGE THEORIES



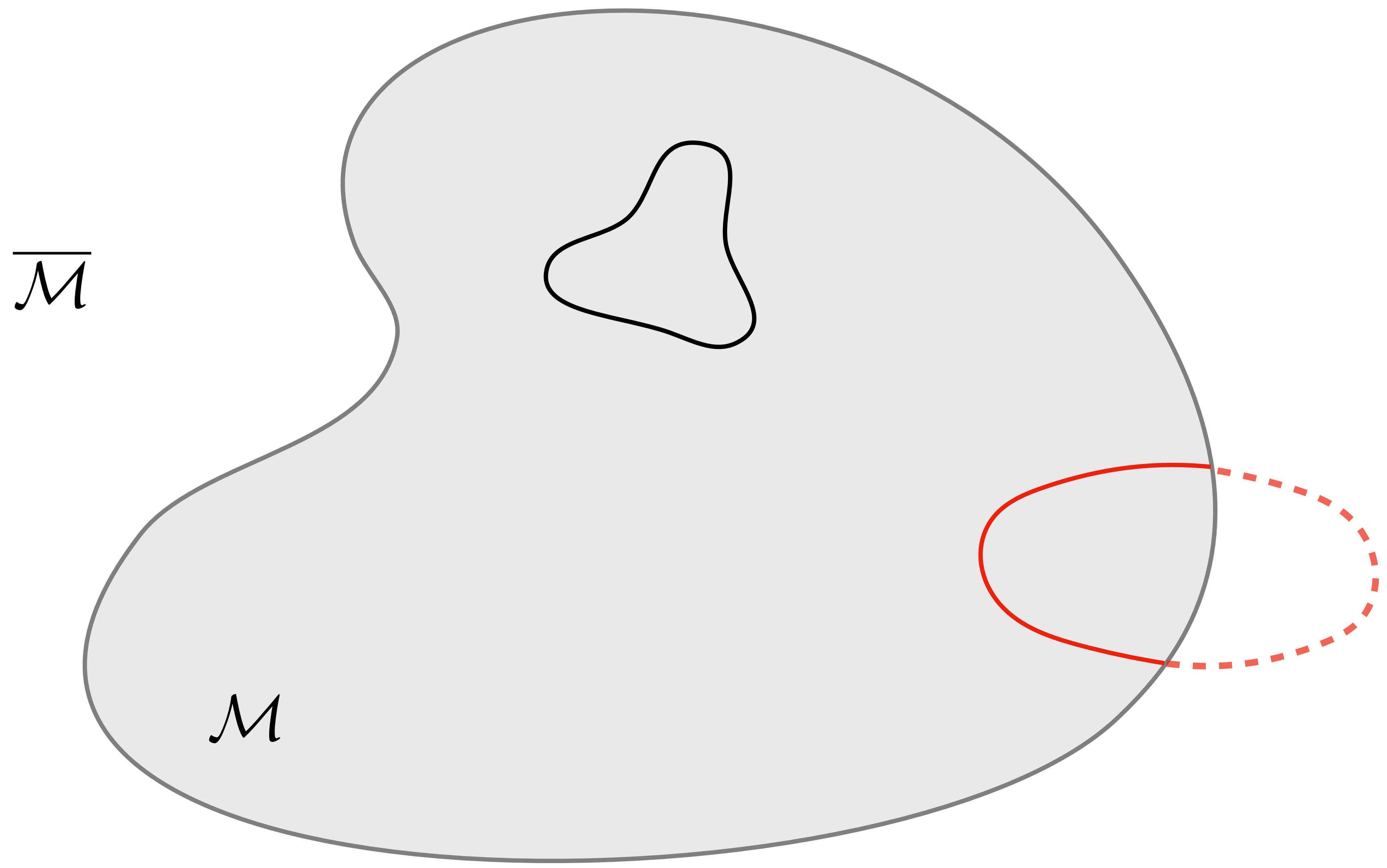
LOCAL SUBSYSTEMS IN GAUGE THEORIES



Kinematical (non gauge-inv.) DoFs different from gauge-invariant DoFs

→ non-local
(e.g. Wilson loops)

LOCAL SUBSYSTEMS IN GAUGE THEORIES



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In presence of (finite) boundaries, subtleties arise for the assignment of gauge-invariant DoFs to a subregion

gauge-invariance of cross-boundary DoFs would be spoiled

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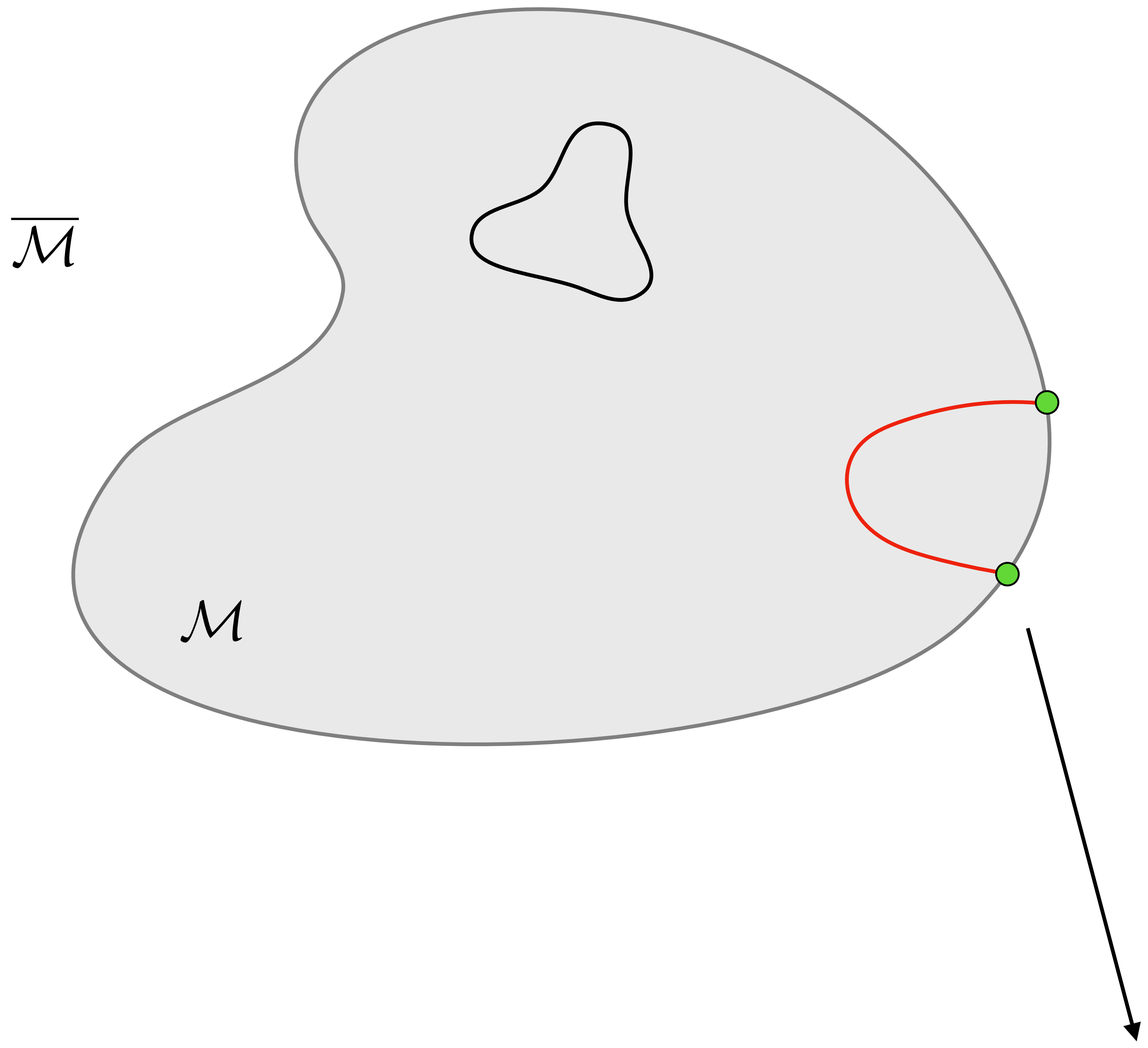

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→ non-local
(e.g. Wilson loops)

In presence of (finite) boundaries, subtleties arise for the assignment of gauge-invariant DoFs to a subregion

gauge-invariance of cross-boundary DoFs would be spoiled

edge modes appear at finite boundaries to restore gauge-invariance

on-shell invariance of subregional presymplectic structure under bulk and boundary gauge transf.

unravel boundary symmetries generated by non-vanishing charges

[Donnelly, Freidel, Francois, Geiller, Gomes, Pranzetti, Riello,
Speranza, Speziale, Wieland, Carrozza, Eccles, Höhn,…]

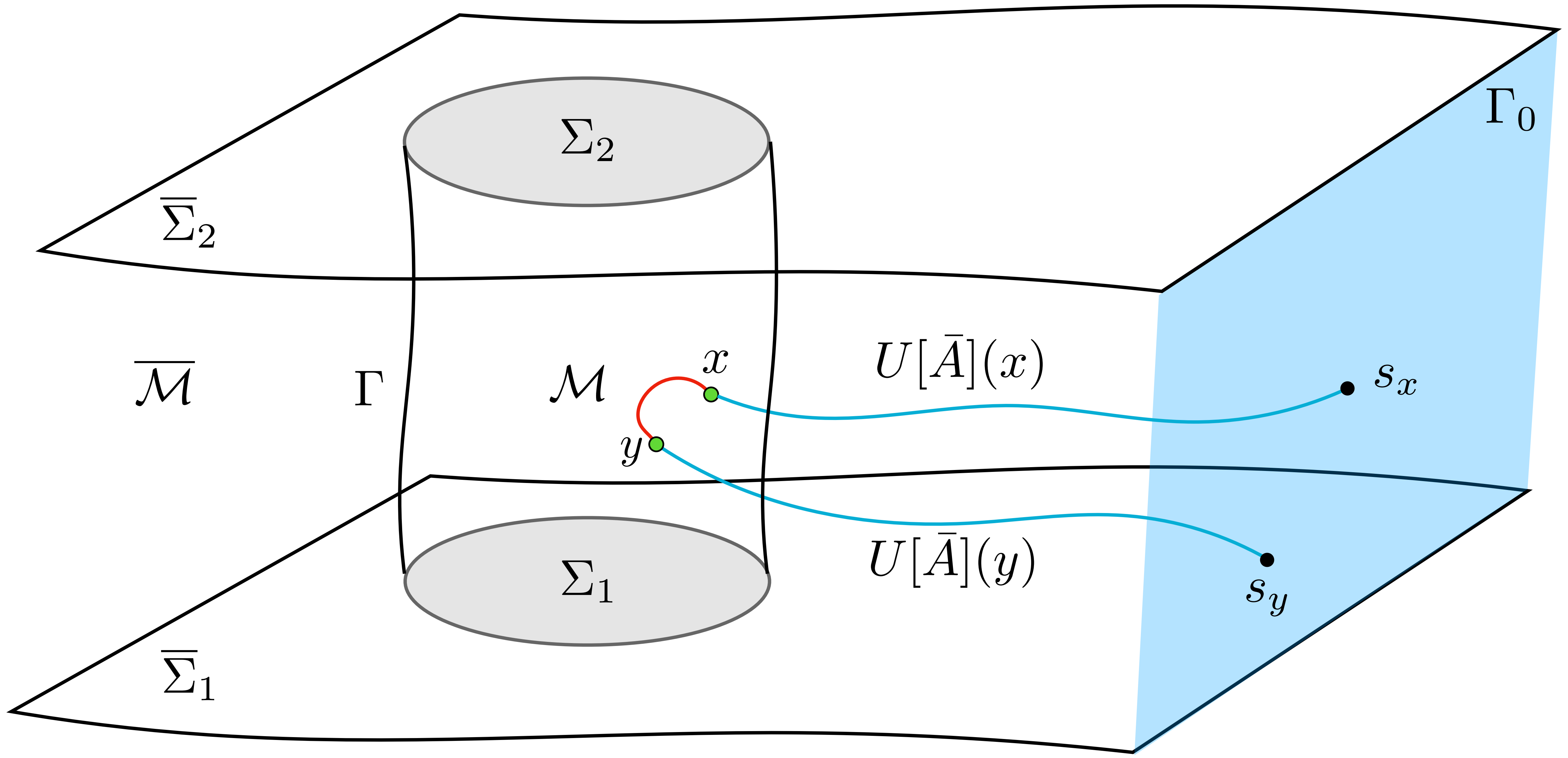
EDGE MODES AS DYNAMICAL FRAMES & DRESSED OBSERVABLES

[Carrozza, Höhn '21; Carrozza, Eccles, Höhn '22]

Edge modes can be understood as group-valued "internalised" external frames via e.g. Wilson lines originating in the complement

→ not new DoFs to be postulated, but understood from the global theory

→ describe how subregion relates to its complement



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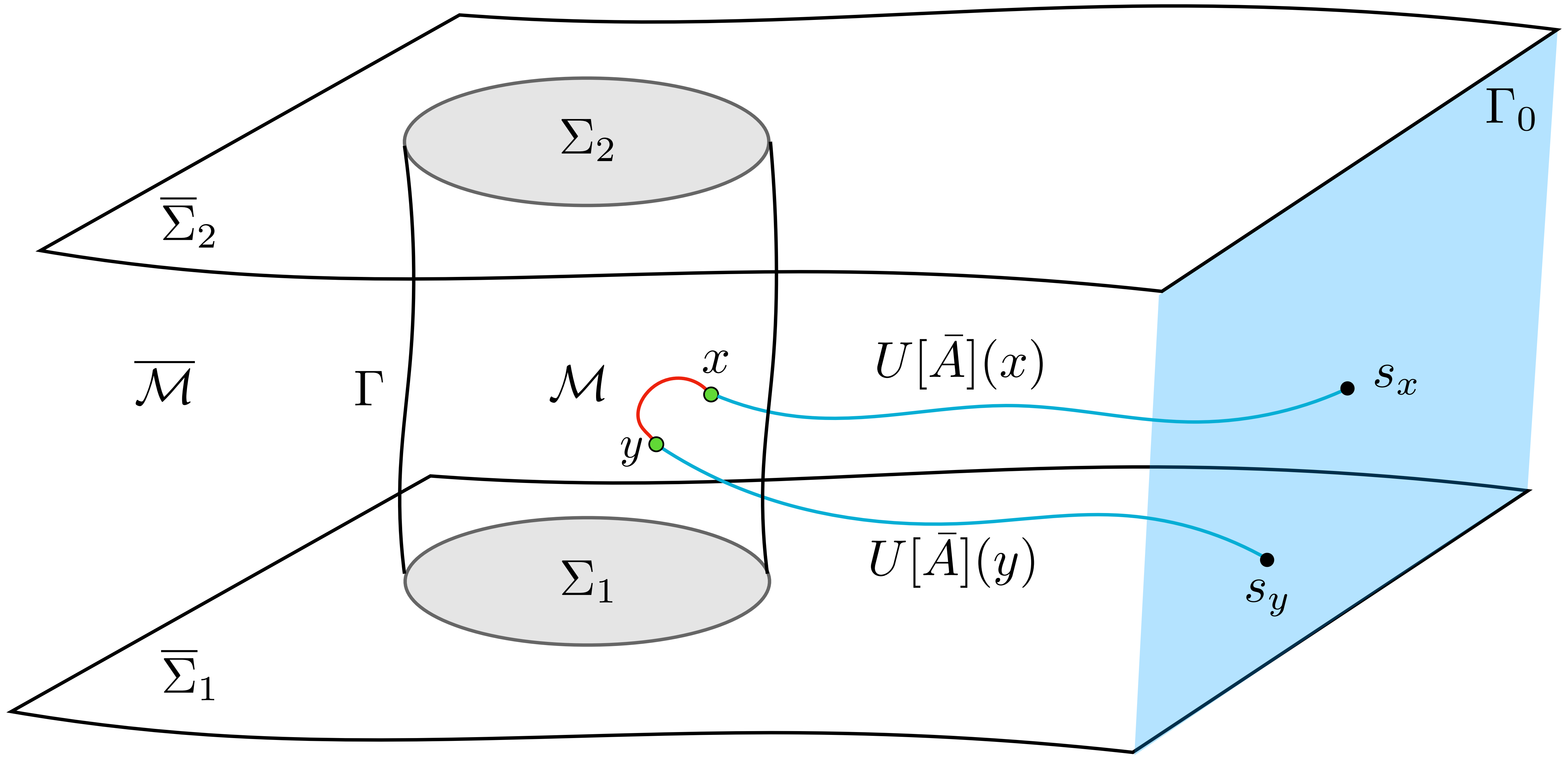
Gauge transformation (from left): $g \triangleright U = g\,U$

Symmetries = (asymptotic) frame reorientations

right action on frame $g \odot U = U\,g^{-1}$



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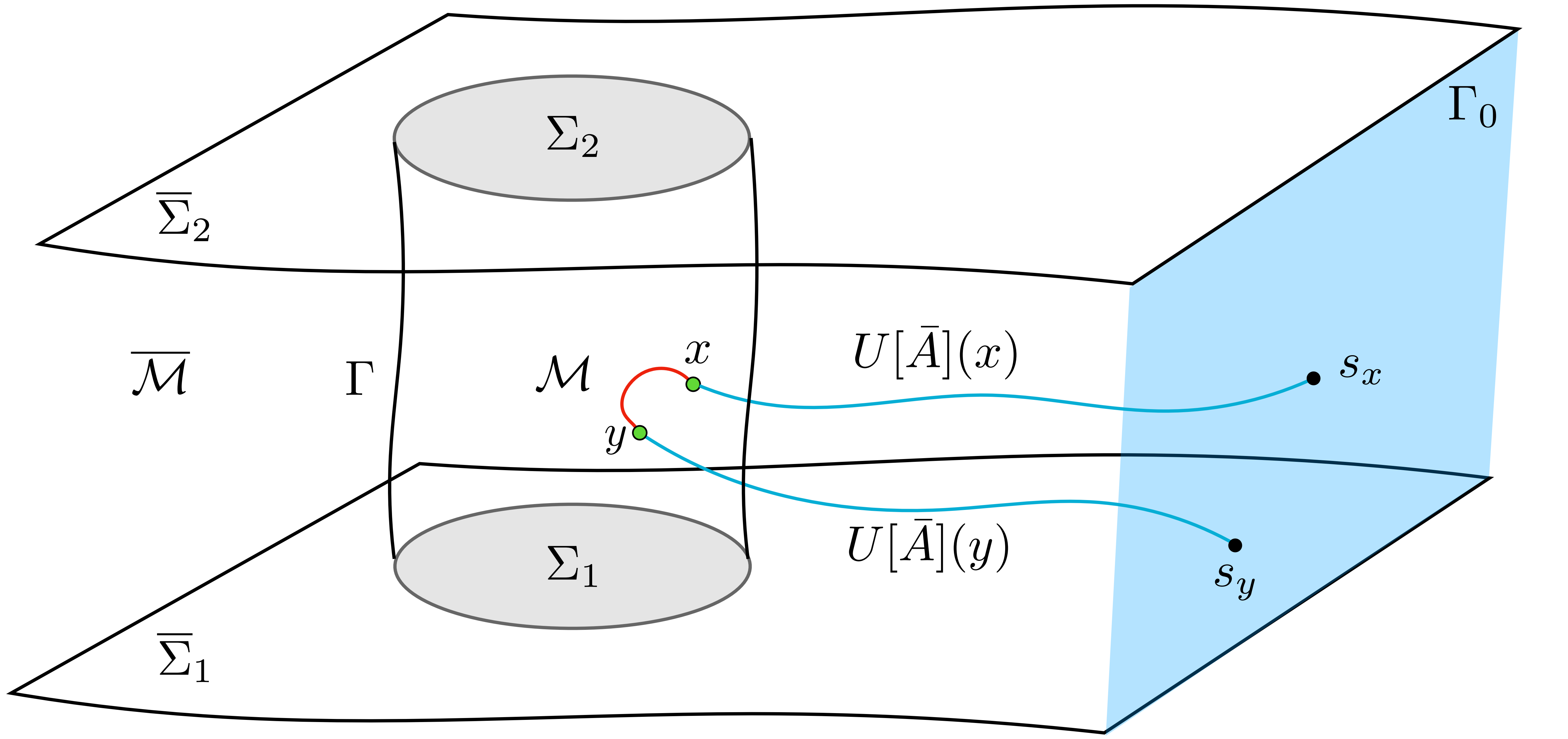


Gauge transformation (from left): $g \triangleright U = g\,U$

Symmetries = (asymptotic) frame reorientations

right action on frame $g \odot U = U\,g^{-1}$

Frame dressed observables = relational observables (non-locally supported on both $\mathcal{M}$ and $\overline{\mathcal{M}}$)

$$O_{f|U}(g) = (U g^{-1})^{-1} \triangleright f$$

frame-orientation conditional gauge transf.

(Non-inv.) functional of fields in $\mathcal{M}$

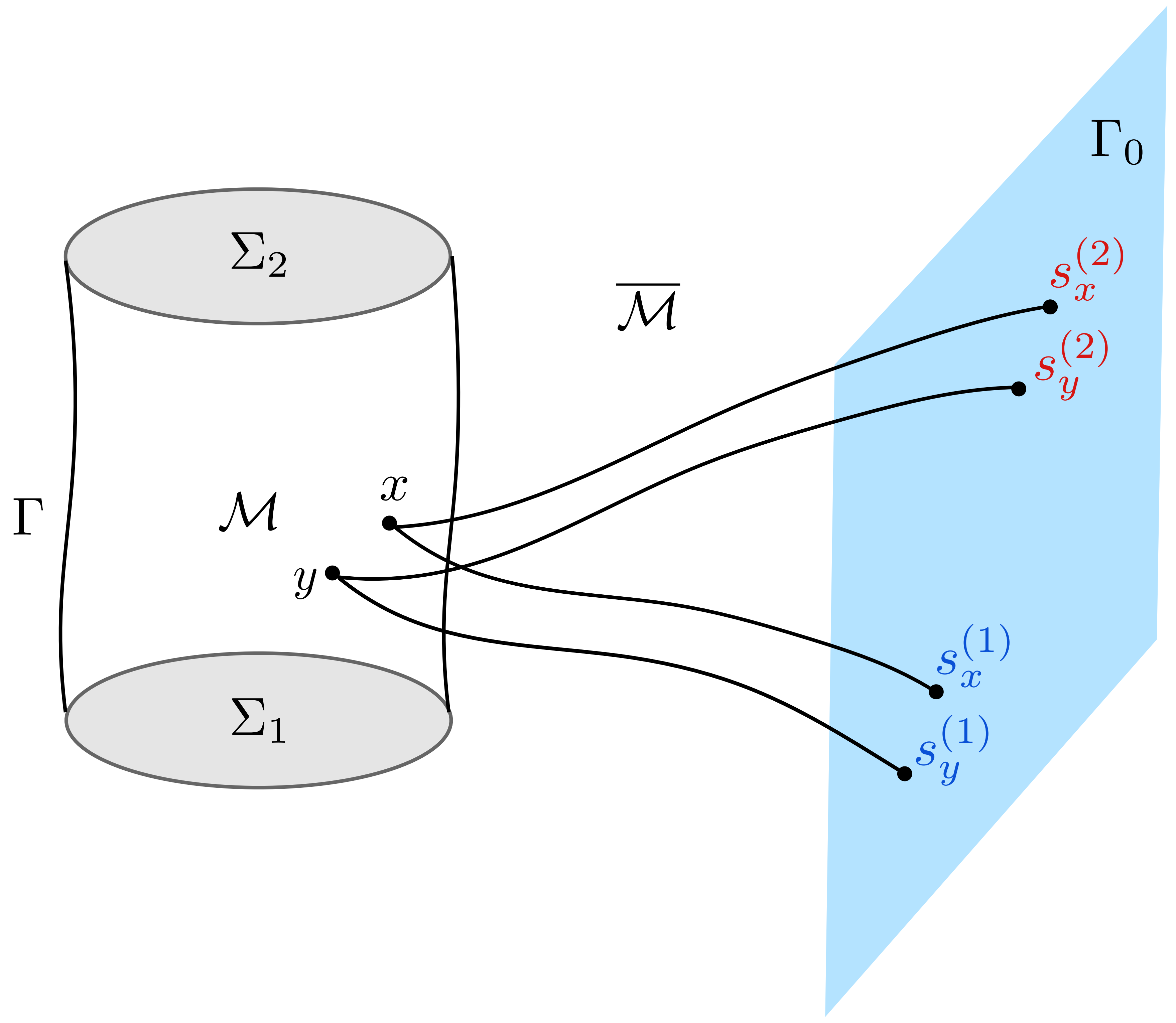
gauge transf. of f compensated by gauge action on U

frame reorientations

$$O_{f|U}(g) \xrightarrow{(\tilde{g}^{-1} g)\odot} O_{f|U}(\tilde{g})$$

SUBSYSTEM RELATIVITY IN GAUGE THEORY

[Carrozza, Höhn, Kirklin, **FMM**, to appear]



Non unique edge mode frame fields (e.g. different systems of Wilson lines)

Change of relational observables relative to frames U1 and U2:

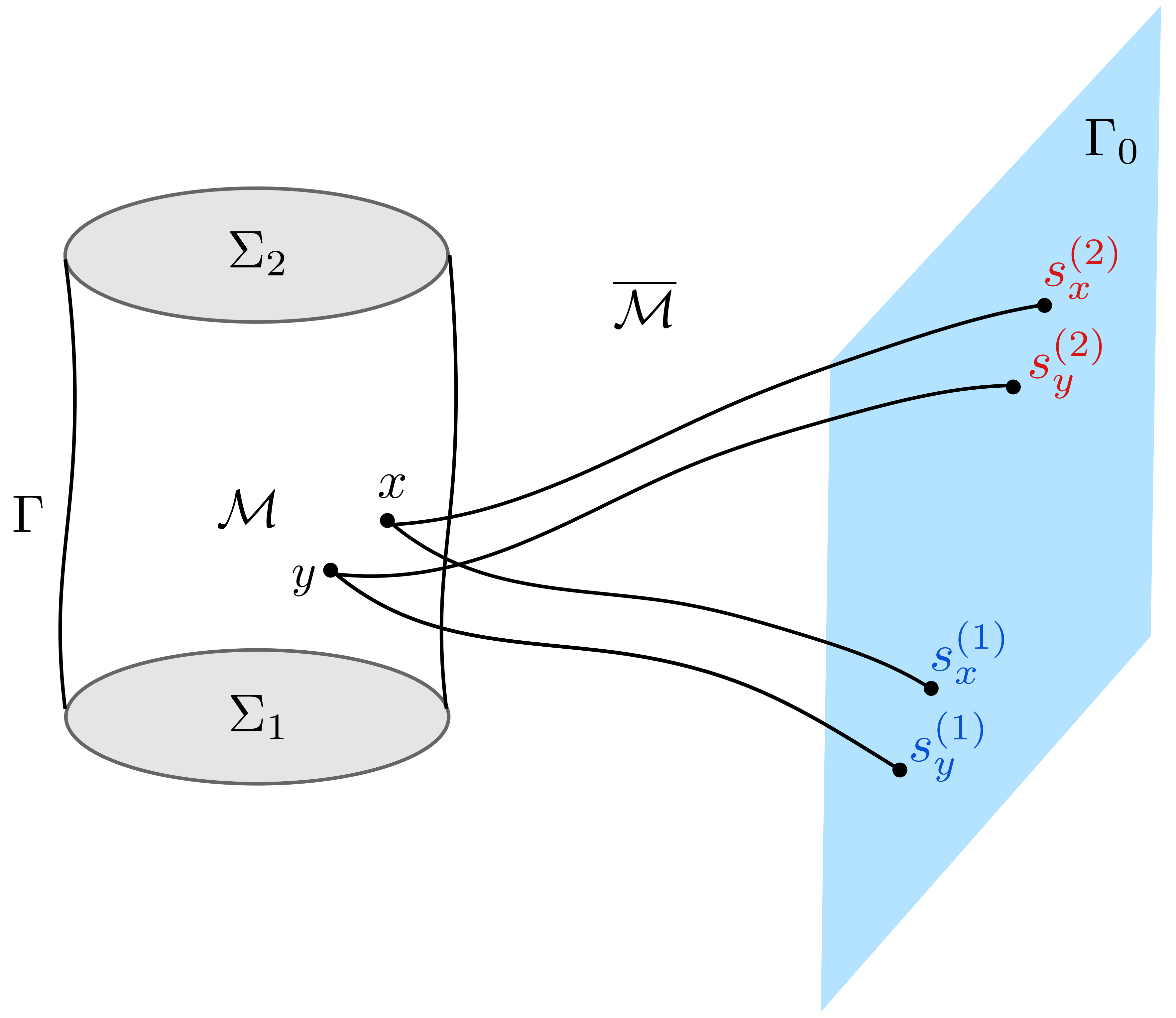
$$O_{f|U_1}(g_1) \xrightarrow[\left(g_1^{-1} g_2\, g_{21}\right)^{-1} \odot]{} O_{f|U_2}(g_2)$$

relation-conditional frame reorientation

$$g_{21} = U_2^{-1} U_1$$

SUBSYSTEM RELATIVITY IN GAUGE THEORY

[Carrozza, Höhn, Kirklin, **FMM**, to appear]



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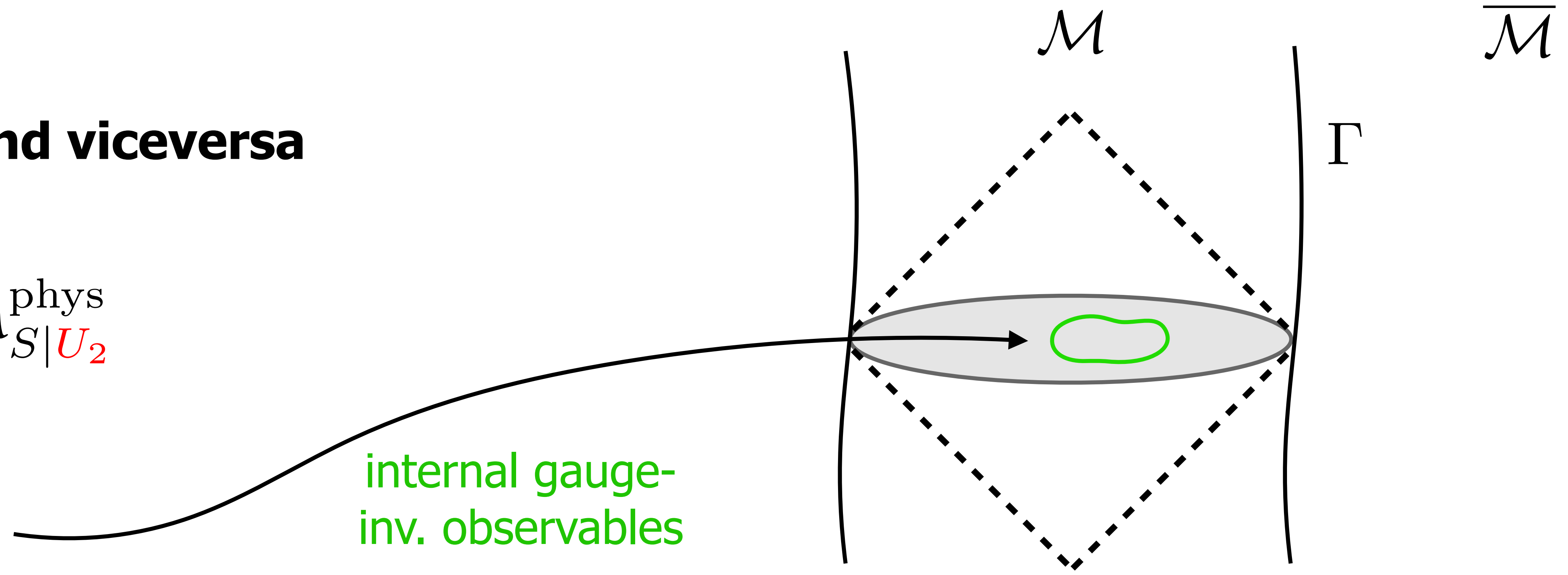
For independent frames:

- **observables relative to U1 invariant under U2-reorientations, and viceversa**
- **different gauge-invariant description of subregion** $\mathcal{A}^{\text{phys}}_{S|U_1} \neq \mathcal{A}^{\text{phys}}_{S|U_2}$
- $\mathcal{A}^{\text{phys}}_{S|U_1} \cap \mathcal{A}^{\text{phys}}_{S|U_2}$ **invariant under reorientations of both frames** (also, relation-conditional ones)



CONCLUSION

- We discussed internal dynamical frames in finite-dim. (quantum) systems and (classical) gauge field theories (edge modes)
- Gauge-invariant/relational notion of subsystems depend on the frame $\longrightarrow$ frame-relativity of (gauge-invariant) properties of subsystems
- Alternative proposal for entanglement entropy in constrained systems and gauge theories

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OUTLOOK

- Subsystem relativity in gauge theory (quantum)
- Gravitational subregions

} [work in progress with S. Carrozza, P. A. Höhn, and J. Kirklin + work in progress with P. A. Höhn, L. Marchetti, J. De Vuyst]

→ minisuperspaces,… [with F. Sartini and P. A. Höhn]

- Quantum gravity:
  - → Relational subsystems in spin networks, entanglement & quant. therm. in LQG
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THANK YOU FOR YOUR ATTENTION !