Causal Fermion Systems as an Effective Collapse Theory

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Brief summary

- causal fermion systems can describe non-smooth spacetime structures;
  in particular, spacetimes involving fluctuating fields
- causal action principle describes nonlinear dynamics

Main message of this talk:

- gives rise to a effective collapse model,
- has similarities with CSL model.

- Has been worked out in detail in the non-relavitivistic limit with Johannes Kleiner and Claudio Paganini
  - "Causal fermion systems as an effective collapse theory," arXiv:2405.19254 [math-ph]
- Collapse theory derived from first principles.

Effective description by nonlocal Dirac equation

- Consider causal fermion system in Minkowski space:
  - Thus Minkowski space, spacetime points $(t, \vec{x})$
  - $\psi_1(t, \vec{x}), \ldots, \psi_f(t, \vec{x})$ family of spinorial wave functions
- causal action principle describes the interaction of all these wave functions
- the linearized interaction can be described effectively by a nonlocal Dirac equation
  - F.F., "Solving the linearized field equations of the causal action principle in Minkowski space," arXiv:2304.00965 [math-ph], to appear in *Adv. Theor. Math. Phys.* (2024)
- There are nonlinear corrections.

Effective description by nonlocal Dirac equation

- Begin in one-particle description (Fock spaces later).
- Describe the dynamics of the causal action principle in terms of a nonlocal Dirac equation

$$\big(i\partial\!\!\!/ + \mathcal{B} - m \big) \psi = 0$$

$$\big(\mathcal{B}\psi\big)(x) = \int_M \mathcal{B}(x,y)\, \psi(y)\, d^4y$$

$$\mathcal{B}(x,y) = \sum_{a=1}^N \gamma_j\, A^j_a\Big(\frac{x+y}{2}\Big)\, L_a(y-x)$$

Linearized fields in Minkowski space

$$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j \, A^j_a\Big(\frac{x+y}{2}\Big) \, L_a(y-x)$$

- The kernels $L_a(y-x)$ are nonlocal on the scale ℓ_{min} with

$$\ell_{\text{Planck}} \ll \ell_{\text{min}} \ll \ell_{\text{macro}}$$

(and ℓ_{Planck} denotes the Planck scale)

$$L_a(\xi) = 0 \qquad \text{if } |\xi^0| + |\vec{\xi}| \gtrsim \ell_{\text{min}}$$

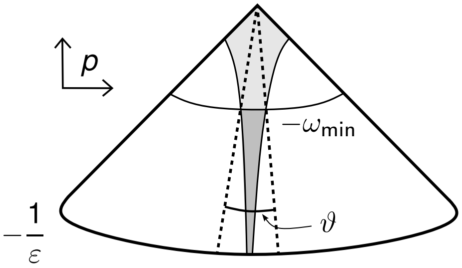
- The number N of fields scales like

$$N \simeq \frac{\ell_{\text{min}}}{\varepsilon}$$

- multitiude of vectorial potentials A^j_a, $a = 1, \ldots, N$, will later be described stochastically
- All potentials satisfy the homogeneous wave equation

$$\Box A^j_a = 0$$

Linearized fields in Minkowski space



$$\vartheta = \frac{1}{\sqrt{\ell_{\text{min}}\, \omega_{\text{min}}}} \qquad \text{with} \qquad \vartheta_{\text{min}} = \sqrt{\frac{\ell_{\text{Planck}}}{\ell_{\text{min}}}}\,.$$

- Different wave functions “feel” different potentials.
- The low-energy wave functions (i.e. $|\omega| \lesssim \ell_{\text{Planck}}^{-1}$) “feel all the potentials at the same time”.

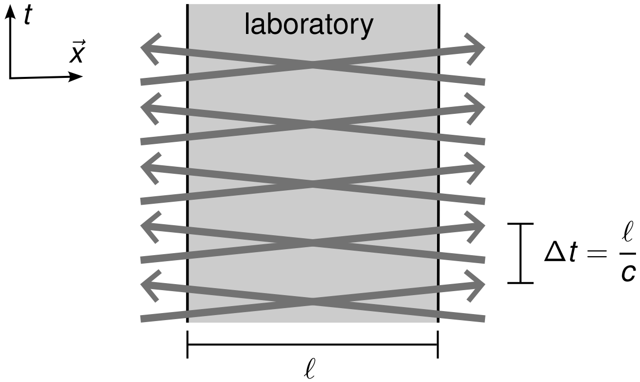
Conserved Scalar Product

- Noether-like theorem: conservation law for scalar product

$$
\begin{aligned}
\langle\psi|\phi\rangle_t := \int \prec\psi \,|\, \gamma^0\, \phi\succ_{(t,\vec{x})}\; d^3x \\
&- i \int_{x^0<t} d^4x \int_{y^0>t} d^4y \;\prec\psi(x)\,|\,\mathcal{B}(x,y)\,\phi(y)\succ_x \\
&+ i \int_{x^0>t} d^4x \int_{y^0<t} d^4y \;\prec\psi(x)\,|\,\mathcal{B}(x,y)\,\phi(y)\succ_x
\end{aligned}
$$

  Has structure of surface layer integral.
- Generalizes probability integral, gives probabilistic interpretation.
- Note: Scalar product depends on stochastic potentials!

The Non-Relativistic Limit



Assume potentials are Gaussian and Markovian,

$$\ll A^j_a(x) \gg = 0$$
$$\ll A^j_a(x)\, A^k_b(x) \gg = \delta(x^0 - y^0)\, \delta_{ab}\, C^{jk}(\vec{y} - \vec{x})$$

The non-relativistic limit

- However, the nonlocality of the potential must be taken into account. (Otherwise, no collapse occurs.)

$$\big(i\partial\!\!\!/ + \mathcal{B} - m\big)\psi = 0$$

Hamiltonian formulation:

$$i\partial_t\psi = \big(H_0 + V\big)\psi$$
$$H_0 = -i\gamma^0\vec{\gamma}\vec{\nabla}$$
$$(V\psi)(t) = \int_{-\infty}^{\infty} V(t,t')\,\psi(t')\,dt'$$
$$(V(t,t')\psi)(\vec{x}) = \int_{\mathbb{R}^3} \Big(-\gamma^0\,\mathcal{B}\big((t,\vec{x}),(t',\vec{y})\big)\Big)\,\psi(\vec{y})\,d^3y$$

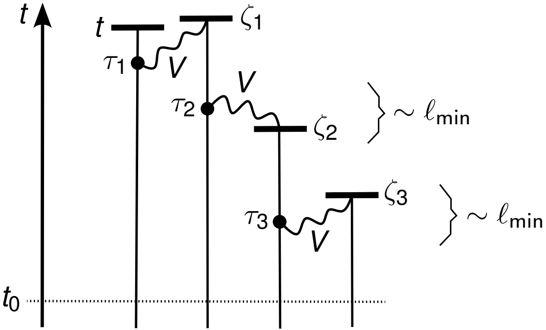
The non-local Dyson series

$$i\partial_t \psi = \big(H_0 + V\big)\psi$$

Can be solved with nonlocal Dyson series

$$\begin{aligned}
\psi(t) &= \psi(t_0) + \int_{t_0}^t \dot{\psi}(\tau)\, d\tau = \psi(t_0) - i \int_{t_0}^t (V\psi)(\tau)\, d\tau \\
&= \cdots = \qquad \text{(apply iteratively)} \\
&= \psi(t_0) + \int_{t_0}^t d\tau \int_{-\infty}^{\infty} d\zeta\, \big(-iV(\tau,\zeta)\big)\, \psi(t_0) \\
&\quad + \int_{t_0}^t d\tau_1 \int_{-\infty}^{\infty} d\zeta_1\, \big(-iV(\tau_1,\zeta_1)\big) \\
&\qquad \times \int_{t_0}^{\zeta_1} d\tau_2 \int_{-\infty}^{\infty} d\zeta_2\, \big(-iV(\tau_2,\zeta_2)\big)\, \psi(t_0) \\
&\quad + \cdots
\end{aligned}$$

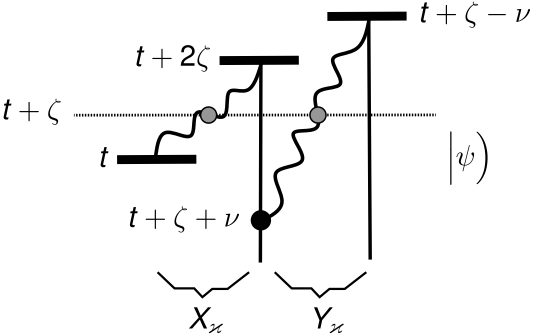
The non-local Dyson series



Gaussian pairings

$$\mathcal{B}(x,y) = \sum_{a=1}^{N} \gamma_j \, A^j_a\Big(\frac{x+y}{2}\Big) \, L_a(y-x)$$

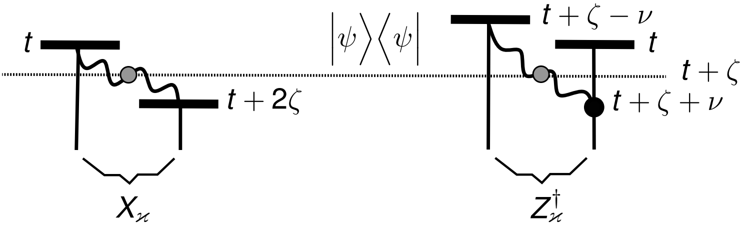
$$\ll A^j_a(x)\, A^k_b(x) \gg \; = \delta(x^0 - y^0)\, \delta_{ab}\, C^{jk}(\vec{y} - \vec{x})$$



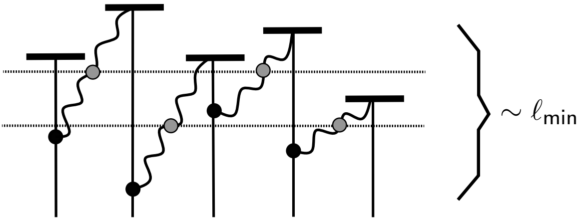
- Nonlocality in time desribed by ζ. Can be treated as additional parameter of the effective collapse model.

Gaussian pairings

- Take into account pairings between bra and ket



- Example in higher order



Main results of analysis

- Statistical operator σ_t has time evolution of Kossakowski-Lindblad form

$$\frac{d\sigma_t}{dt} = -i[H, \sigma_t] - \frac{1}{2} \sum\!\!\!\!\!\!\int_{\varkappa} \big[K_\varkappa, [K_\varkappa, \sigma_t]\big] \, \Big(1 + \mathcal{O}\big(\ell_{\min} \, \|V\|\big)\Big)$$

$$\varkappa := (a, \zeta, \ldots)$$

- There is dynamical state reduction, in agreement with Born's rule.
- Similar to CSL model, but not the same, due to nonlocality in time.

Derivation of Lindblad dynamics

- Note: Standard scalar product

$$(\psi|\phi)_t := \int \prec\!\psi \,|\, \gamma^0\, \phi\!\succ_{(t,\vec{x})}\; d^3x$$

is *not* conserved in time.

- Only the modified scalar product is conserved,

$$\begin{aligned}\langle\psi|\phi\rangle_t = (\psi|\phi)_t \\ &- i \int_{x^0<t} d^4x \int_{y^0>t} d^4y \;\prec\!\psi(x)\,|\,\mathcal{B}(x,y)\,\phi(y)\!\succ_x \\ &+ i \int_{x^0>t} d^4x \int_{y^0<t} d^4y \;\prec\!\psi(x)\,|\,\mathcal{B}(x,y)\,\phi(y)\!\succ_x\end{aligned}$$

.

Derivation of Lindblad dynamics

Transform one into the other:

$$\langle\psi|\phi\rangle_{t_0} = \big(\psi \,|\, (\mathbb{1} + \mathcal{S}_{t_0})\phi\big)_{t_0} \qquad \text{for all } \psi, \phi \in \mathcal{H}_m$$

$$\psi \mapsto \tilde{\psi} := \sqrt{\mathbb{1} + \mathcal{S}_{t_0}}\,\psi$$

$$\langle\psi|\phi\rangle_{t_0} = (\tilde{\psi}|\tilde{\phi})_{t_0}$$

Working with $\tilde{\psi}$, one can use the standard scalar product.

statistical operator $\qquad \sigma_t := \ll |\psi\rangle\langle\psi| \gg \, = \, \ll |\tilde{\psi})(\tilde{\psi}| \gg$

Now compute

$$\frac{d}{dt}\sigma_t = \ll \frac{d}{dt}\Big(|\tilde{\psi})(\tilde{\psi}|\Big) \gg = \cdots$$

to leading order in $\ell_{\min} \|\mathcal{B}\|$.

Reduction of the state vector

Try to use standard assumption:

- Observable $\mathcal{O}$ commutes with Hamiltonian.
- Typical example: Position measurement, use locality of time evolution.

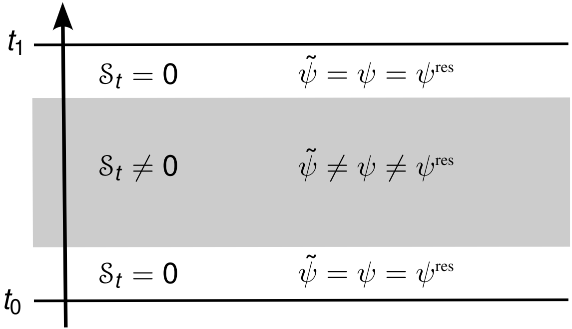
Problem: Operator $\mathcal{S}_t$ is nonlocal! Therefore:

- Work with the original (untilded) wave functions.
- Makes it necessary to also work with the time-dependent scalar product $\langle . | . \rangle_t$.

Reduction of the state vector

Consider situation similar to a scattering process and rescale the wave functions,

$$\psi^{\text{res}}(t) := c(t)\, \psi(t) \qquad \text{with} \qquad c(t) := \frac{1}{\sqrt{\ll\!(\psi(t)|\psi(t))\!\gg}}$$



Reduction of the state vector

$$\frac{d}{dt} \ll\!(\psi^{\text{res}}(t)|\psi^{\text{res}}(t))\!\gg = 0$$
$$\frac{d}{dt} \ll\!(\psi^{\text{res}} \,|\, \mathcal{O}\, \psi^{\text{res}})\!\gg = 0$$
$$\frac{d}{dt} \ll\!(\psi^{\text{res}} \,|\, \mathcal{O}^2\, \psi^{\text{res}}) - (\psi^{\text{res}} \,|\, \mathcal{O}\, \psi^{\text{res}})^2\!\gg \leq 0$$

and strictly negative unless ψ^{res} is an eigenstate.

- Shows collapse
- Proves Born rule

Effective description in Fock spaces

- System can be described at any time t by a

$$\text{Quantum state} \qquad \omega^t : \mathscr{A} \to \mathbb{C}\,,$$

where $\mathscr{A}$ is the algebra of observables.

- can be represented on Fock space $\mathcal{F}$ (fermionic and bosonic)

$$\omega^t(A) = \mathrm{Tr}_{\mathcal{F}}\big(\sigma^t A\big) \overset{\text{if pure state}}{=} <\Psi|A|\Psi>$$

- F.F. and Kamran, N., "Fermionic Fock spaces and quantum states for causal fermion systems," arXiv:2101.10793 [math-ph], *Ann. Henri Poincaré* **23** (2022) 1359–1398
- F.F., Kamran, N. and Reintjes, M., "Entangled quantum states of causal fermion systems and unitary group integrals," arXiv:2207.13157 [math-ph], to appear in *Adv. Theor. Math. Phys.* (2024)
- Is ongoing work with C. Dappiaggi, N. Kamran, M. Reintjes.

Effective description in Fock spaces

For our purpose, it suffices to consider Hartree-Fock state

$$\Psi = \psi_1 \wedge \cdots \wedge \psi_q$$

- Dynamics again described by the nonlocal Dirac equation
- Collapse happens as soon as one one-particle wave function collapses.
- Thus also here: Collapse phenomena are predominant for mesoscopic and macroscopic systems

Parameters of the model

- The length scale $\ell_{\min}$. Equivalently, the number N of fields related to $\ell_{\min}$ by

$$N \simeq \frac{\ell_{\min}}{\varepsilon}$$

- The strength of the stochastic field as described by the covariance,

$$\ll A^j_a(x)\, A^k_b(x) \gg \;=\; \delta(x^0 - y^0)\, \delta_{ab}\, C^{jk}(\vec{y} - \vec{x})$$

and the nonlocal kernels $L_a(y - x)$

The nonlinear term

Interestingly, it does not need to be specified. Description on various levels possible:

- Take into account nonlinear coupling,

$$-\Box (A_a)^k = e^2 J_a^k$$

$$J_a^k(z) = \int_M \prec \psi(x) | \gamma^k \, L_a(x,y) \, \psi(y) \succ \big|_{x=z-\xi/2,\; y=z+\xi/2} \, d^4\xi$$

- Take the many-particle perspective:

Hartree-Fock state $\quad \Psi = \psi_1 \wedge \cdots \wedge \psi_q$

In the causal fermion system description, the potential $\mathcal{B}$ is encoded in this family of wave functions. Therefore, Dirac equation for Ψ is nonlinear.

The Nature of the Collapse

- It is not the gravitational field which triggers the collapse.
- Instead, it is a multitude of bosonic fields, specific to the causal action principle
- Remark:
  - This multitude of fields can be described effectively by a second-quantized electromagnetic field.
  - Therefore: collapse is closely related to the electromagnetic interaction in QFT
- But: length scale of nonlocality comes into play. Related to Planck scale. Also gives connection to strength of gravitational field.

Outlook: Next Steps

- Compute the energy spectrum of radiation emitted in collapse.

Outlook: The Relativistic Model

ongoing work also with Simone Murro

- Dirac equation is already relativistic.
- Stochastic background fields break Lorentz invariance.
  - Concept: Stochastic background fields originate from the early universe and/or are generated by the matter on earth and of the surrounding stars and galaxies.
- Replace Markov property by propagation with speed of light. Also gives rise to "smearing in time."

Summary

- Consider causal fermion systems in Minkowski space
- Described by family of fermionic wave functions, encoded in wave evaluation operator Ψ
- Causal action principle gives rise to plethora of fields
- Coupling of these fields to the Dirac equation is nonlocal on a scale $\ell_{\text{min}} \ll m^{-1}$.
- Similar to CSL model, we obtain a stochastic and a nonlinear term.
- But: has a different mathematical structure, due to nonlocality in time and different form of conserved current.
- Further consequences are work in progress ...

www.causal-fermion-system.com

Thank you for your attention!