Title: Exact characterization of entropy and free energy without thermodynamic limit or averaging


#### Abstract

: We usually think of quantities like Helmholtz free energy $F$ and von Neumann entropy $S$ as attaining their physical meaning in the thermodynamic limit: for example, the second law is only true for large numbers of particles or on average. Similarly, information-theoretic quantities like von Neumann entropy only tell us operational facts (such as the achievable transmission rate) in the limit of infinitely long messages. Here, however, I describe a physical, operational interpretation of $F$ and $S$ that holds exactly, without fluctuations, and without averaging or thermodynamic limit, for single, small quantum systems. This interpretation follows from the resource-theoretic approach: the (im)possibility of state transitions in thermodynamics or information theory is exactly characterized by F or S if the (thermodynamic or catalytic) machine is allowed to become correlated or entangled with the work medium, while (as always) exactly preserving its own state. This feature becomes inessential in the thermodynamic limit, but it turns out to be crucial in the nanoscale. Furthermore, I present a no-go theorem ("no broadcasting of coherence and asymmetry") showing that this theorem cannot hold in the presence of genuine quntum coherence between energy levels, and I give a brief outline on further implications for quantum information theory.


## References:

M. P. Müller, Correlating Thermal Machines and the Second Law at the Nanoscale, Phys. Rev. X 8, 041051 (2018). DOI:10.1103/
PhysRevX.8.041051.
M. Lostaglio and M. P. Müller, Coherence and Asymmetry Cannot be Broadcast, Phys. Rev. Lett. 123, 020402 (2019). DOI:10.1103/ PhysRevLett.123.020403; arXiv:1812.08214.
P. Boes, J. Eisert, R. Gallego, M. P. Müller, and H. Wilming, Von Neumann Entropy from Unitarity, Phys. Rev. Lett. 122, 210402 (2019). DOI:10.1103/PhysRevLett.122.210402; arXiv:1807.08773.

