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"Differential Geometry with Algebraifolds"

Abstract: It is well-known that the basic notions of (pseudo-)Riemannian, like tensors, connections and curvature, can be defined algebraically by reference to the algebra of smooth functions on a manifold. In other words, the only non-algebraic concept needed for basic differential geometry is the notion of manifold. Here, I will show how one can do away with manifolds as well by introducing the definition of "algebraifold", which is an algebra whose module of derivations is finitely generated projective. This allows one to state all the same basic definitions of differential geometry. I will also argue that dual bases play the role of charts, and showcase some interesting specimen from the zoo of algebraifolds: besides algebras of smooth functions, we also have algebras of generalized functions containing distributions and algebraic function fields, where the latter formalize the non-rigorous physicist's tendency not to worry about inverting any nonzero function. Another interesting class of examples is given by submersions, for which the algebra of smooth functions on the total space is an algebraifold over the ring of smooth functions on the base space, and therefore our formalism thus automatically contains a fibred differential geometry.