A microscopic derivation of a non-relativistic collapse model

2.2.24

In causal fermion system approach

- the dynamics is described by causal action principle, non-linear equations
- spacetime non-smooth
  small-scale fluctuation
  best described stochastically
- conserved current
  $\langle \psi | \psi \rangle_S^t$ independent of time
  gives probabilistic interpretation of ψ

April 23 linearized fields in Minkowski space

vacuum: $(i\partial\!\!\!/ - m)\,\psi = 0$

nonlocal potential $(\mathscr{B}\psi)(x) = \int \mathscr{B}(x,y)\,\psi(y)\,d^4y$

$$(i\partial\!\!\!/ - \mathscr{B} - m)\,\psi = 0$$

$\mathscr{B}$ is composed of many homogeneous potentials

$$\mathscr{B}(x,y) = \sum_a A_a\left(\frac{x+y}{2}\right) L_a(y-x)$$

$$\Box A_a = 0$$

$$L_a(\xi) = 0 \quad \text{if} \quad |\xi^0| \gg \ell_{\min}$$

$$\ell_{\text{Planck}} \ll \ell_{\min} \ll \frac{1}{m}$$

A_a real-valued $\qquad \overline{L_a(\xi)} = L_a(-\xi)$

$$\mathcal{D}(x,y)^* = \mathcal{D}(y,x) \quad \text{symmetric}$$

$$\langle \psi | \phi \rangle_S^t = (\psi|\phi)_{L^2(\mathbb{R}^3)}$$
$$+ \frac{i}{2}\left(\int_{x^0>t} d^4x \int_{y^0<t} d^4y - \int_{x^0<t} d^4x \int_{y^0>t} d^4y \right)$$
$$\langle \psi(x) | \mathcal{D}(x,y)\, \phi(y) \rangle$$

- The wave functions generate bosonic fields

$$\Box A_a = e^2 J_a$$
$$J_a = \int d^4\xi \, \langle \psi(z - \tfrac{\xi}{2}) | \gamma^\delta L_a(\xi)\, \psi(z + \tfrac{\xi}{2}) \rangle$$

- Then in total

$$A_a = A_a^{\text{stoch}} + A_a^J$$
$$\begin{cases} \Box A_a^{\text{stoch}} = 0 & \text{stochastic} \\ \Box A_a = e^2 J & \text{nonlinear} \end{cases}$$

- Non-relativistic limit

$$c \longrightarrow \infty$$

$$(i\partial\!\!\!/ + \mathcal{B} - m)\psi = 0$$

$$i\partial_t \psi = \underset{\uparrow}{H_0 \psi} \underbrace{- i\gamma^0 \mathcal{B} \psi}_{V}$$

Hamiltian formulation in interaction picture

$$i\partial_t \psi = V\psi$$

$$(V\psi)(t) = \int V(t,t')\,\psi(t')\,dt'$$

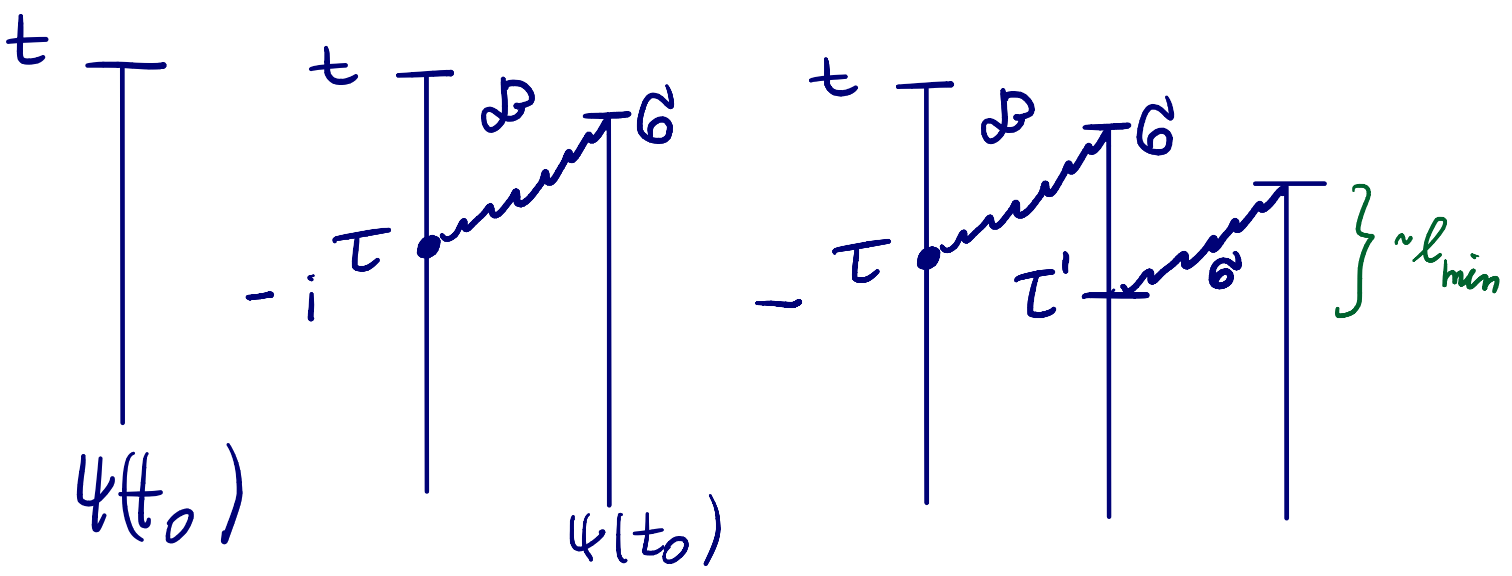
nonlocal in time
on scale t_h

$$V(t,t') \,:\, \mathcal{H}_{t'} \to \mathcal{H}_t\,,\; (V(t,t')\psi)(\vec{x}) = \int_{\mathbb{R}^3} \Big(-\gamma^0\,\mathcal{B}\big((t,\vec{x}),(t',\vec{y})\big)\Big)\,\psi(\vec{y})\,d^3y\,.$$

Dyson series

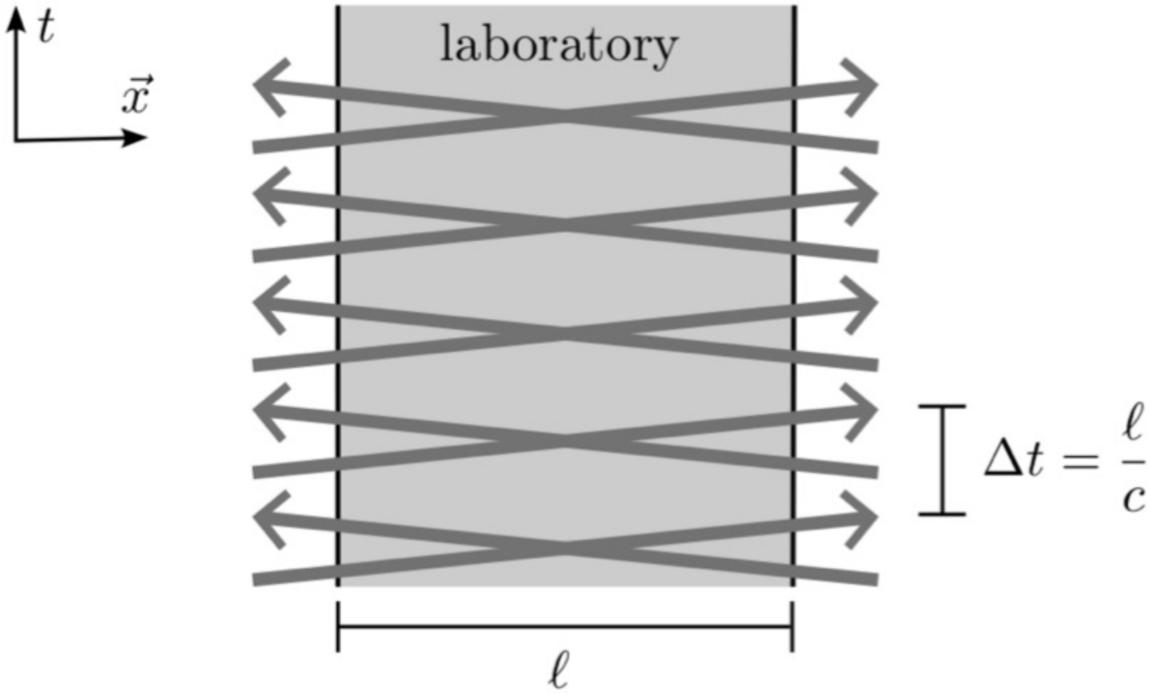
$$\begin{aligned}
\psi(t) &= \psi(t_0) + \int_{t_0}^t \dot{\psi}(\tau)\,d\tau \\
&= \int_{t_0}^t \big(-iV\psi\big)(\tau)\,d\tau = \int_{t_0}^t d\tau \int_{-\infty}^{\infty} d\sigma\,\big(-iV(\tau,\sigma)\big)\,\psi(\sigma) = \cdots = \\
&= \psi(t_0) + \int_{t_0}^t d\tau \int_{-\infty}^{\infty} d\sigma\,\big(-iV(\tau,\sigma)\big)\,\psi(t_0) \\
&\quad + \int_{t_0}^t d\tau_1 \int_{-\infty}^{\infty} d\sigma_1\,\big(-iV(\tau_1,\sigma_1)\big)\int_{t_0}^{\sigma_1} d\tau_2 \int_{-\infty}^{\infty} d\sigma_2\,\big(-iV(\tau_2,\sigma_2)\big)\,\psi(t_0) \\
&\quad + \cdots\,.
\end{aligned} \tag{4.}$$

Graphically



$+ \cdots$

The non-relativistic limit



" $t_{mi} \longrightarrow 0$ "

Moreover, specify A_a stochastically

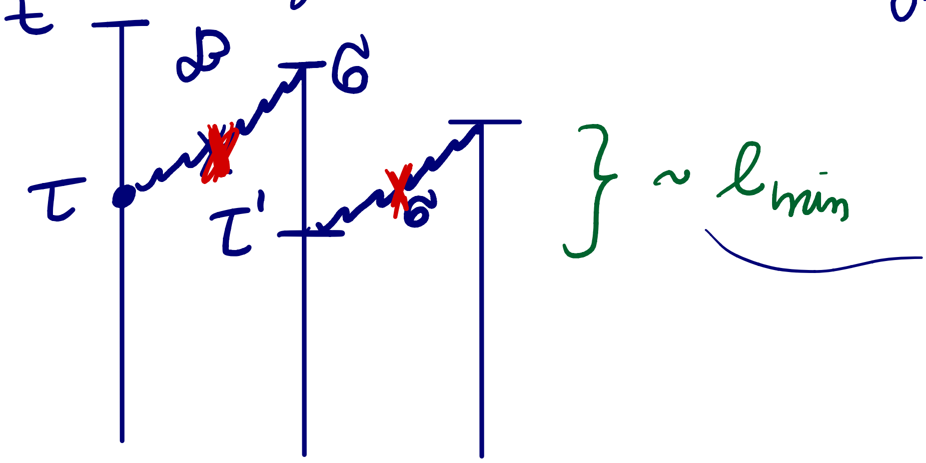
assume Gaussian and Markovian

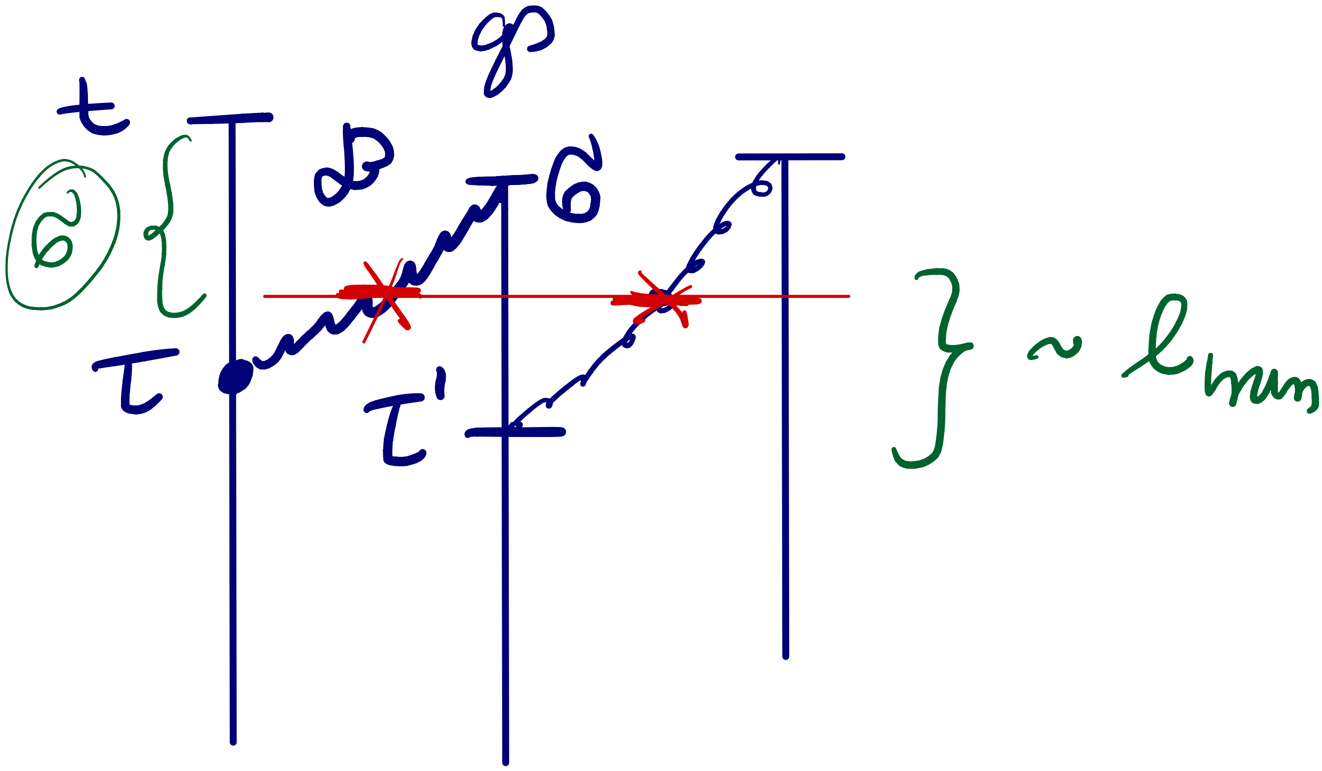
$$\langle\langle A_a \rangle\rangle = 0 \qquad \langle\langle A_a(t,\vec{x})\, A_b(t',\vec{y}) \rangle\rangle$$

$$= \delta_{ab}\, \delta(t-t')\, K(\vec{x},\vec{y})$$

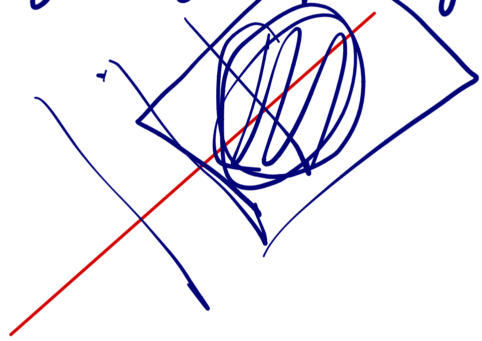
Gaussian pairings

$$\mathcal{B}(x,y) = \sum_a A_a\left(\frac{x+y}{2}\right) L_a(y-x)$$





In this way one gets ~~pseudo~~ spatial operators



$L_a(\xi) = 0$

if $|\xi^0| + |\vec{\xi}| \gg \ell_{min}$

$\langle \psi | \phi \rangle^{\rho}_{t}$ conserved in time

$$= (\psi | \phi)_{L^2(\mathbb{R}^3), t} + (\text{nonlocal contribution involving } \mathcal{D}(x,y)) + (\psi | A_t \phi)$$

not conserved

nonlocal in time

$\langle \psi | \phi \rangle^{t}_{\rho} = (\psi | (1 + A_t) \phi)_t$

conserved

transform to standard scalar product

$$\tilde{\Psi} := \sqrt{1 + A}\, \psi$$

$$\langle \psi | \phi \rangle^{t}_{\rho} = (\tilde{\Psi} | \tilde{\phi})_t$$

$$i \partial_t \tilde{\Psi} = \tilde{V} \tilde{\Psi}$$

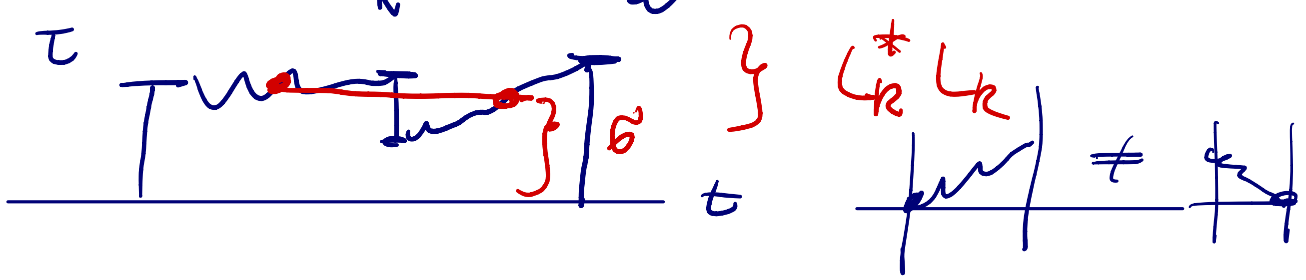
$\langle\!\langle\, |\tilde{\Psi}\rangle \langle \tilde{\Psi}|\, \rangle\!\rangle =: \rho$ statistical operator

$\mathrm{Tr}(\rho) = \langle\!\langle \langle \tilde{\Psi} | \tilde{\Psi} \rangle \rangle\!\rangle$ conserved

work out

$$\frac{d}{dt} \rho = -\frac{1}{2} \sum_k \left[L_k^{+}, [L_k, \rho] \right]$$

$$\sum_k = \sum_a \int d\Omega$$



Reduction of the state vector

Following Bassi - Dürr - Hinrichs (2013)

$$\begin{aligned} \mathrm{d}\psi_t &= \Bigg[-iH\mathrm{d}t + \sum_{k=1}^{n}(\underbrace{L_k - \ell_{k,t}}_{-iV})\,\mathrm{d}W_{k,t} \\ &\quad - \frac{1}{2}\sum_{k=1}^{n}\left(L_k^\dagger L_k - 2\ell_{k,t}L_k + |\ell_{k,t}|^2\right)\mathrm{d}t\Bigg]\psi_t, \quad (1) \end{aligned}$$

$$\ell_{k,t} \equiv \frac{1}{2}\langle\psi_t,(L_k^\dagger + L_k)\psi_t\rangle \quad (2)$$

The resulting Lindblad equation is

$$\frac{d\rho_t}{dt} = -i[H,\rho_t] + \sum_{k=1}^{n}\left(L_k\rho_t L_k^\dagger - \frac{1}{2}L_k^\dagger L_k\rho_t - \frac{1}{2}\rho_t L_k^\dagger L_k\right).$$

$i\partial_t\psi = V\psi$; $\partial_t\psi = -iV$

$L_k^* = -L_k$ corresponds classical potential

$$\frac{d}{dt}\langle\psi|\psi\rangle = \langle -iV\psi|\psi\rangle + \langle\psi|-iV\psi\rangle$$
$$= 0 \quad \text{if} \quad V = V^*$$

$$\frac{d}{dt}\langle\psi|A\psi\rangle = 0 \quad \text{if} \quad V = V^\dagger \text{ and } \underline{[A,V]=0}$$

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general procedure

assume A with $[A,L_a]=0 \quad \forall a$

$$d\langle\psi|A\psi\rangle = \sum_a \langle\psi|(\overline{L_a}A + AL_a^*)\psi\rangle \, dW_a$$

$$\langle\!\langle \langle\psi|A\psi\rangle \rangle\!\rangle = 0$$

$$\langle\langle \underbrace{\langle\psi|A^2\psi\rangle}_{=0} - \underbrace{\langle\psi|A\psi\rangle^2}_{>0} \rangle\rangle < 0$$

$(\tilde\psi|\tilde\psi)_t$ conserved

$$= \langle\psi|(1+A)\psi\rangle_s^t$$

O Observable $\longrightarrow \psi(t)$ causal time evolution

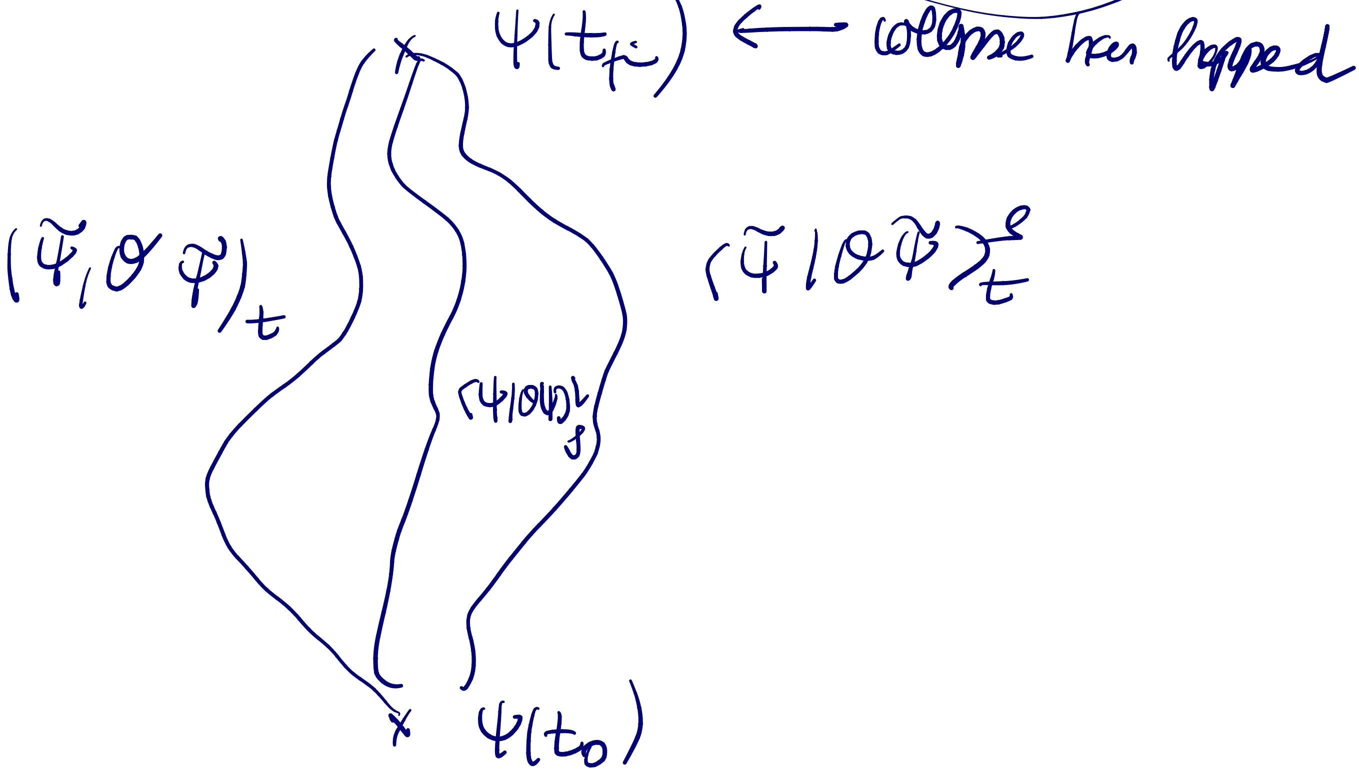
$\downarrow$ $\quad \tilde\psi = \sqrt{1+A_t}\,\psi$ non-local

$$d\psi = -i V\psi$$

assume that $[V, O] = 0$

$$\langle\langle d\langle\psi|O\psi\rangle_s^t \rangle\rangle = 0$$

V is non-symmetric w.r.t. $(\cdot|\cdot)_t$



conclude by

$\left\{ \begin{array}{l} \underline{\text{strength}} \text{ of stochastic field} \\ \text{time} \end{array} \right. \Longrightarrow$ parameters of CSL