THERMODYNAMICS OF SPACETIME: FROM UNIMODULAR GRAVITY TO QUANTUM GRAVITY PHENOMENOLOGY

Ana Alonso-Serrano
Humboldt-Universität zu Berlin
And
Max-Planck-Institut für Gravitationsphysik
(Albert-Einstein-Institut)

*Work in collaboration with: **Marek Liška** (Charles University of Prague),*
***Luis J. Garay** (UCM) and **Antonio Vicente-Becerril** (Ec Polytech, CSIC)*

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MOTIVATION

- Understand the interface between gravitational dynamics and thermodynamics: One step further
- Classical structure of gravity: GR vs UG
- On the search for quantum gravity…
  - Quantum phenomenological models to get insight into possible effects
- Look at the entropy: Quantum modifications of entropy
  - Same qualitative behavior in all the models
- Could we obtain phenomenological modified equations of motion from it?
  - General phenomenological quantum gravity dynamics
  - Quantum cosmology: Resolution of the singularity?

OUTLINE

- Einstein equations from thermodynamics: Basic concepts and UG vs GR
- Modified entropy
- Quantum phenomenological equations of motion
- Interpretation of the modified dynamics and application to cosmology
- Discussion
- Further perspectives

EINSTEIN EQUATIONS FROM THERMODYNAMICS

- Local-causal horizon

- Gravitational dynamics → equilibrium condition for maximal entropy

$$\boxed{\delta S = 0}$$

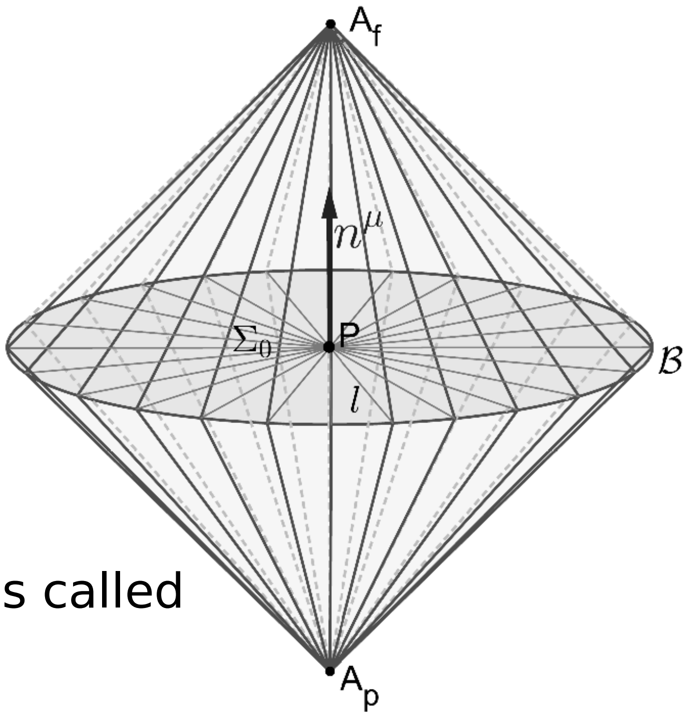
Quantum correlations across horizon + matter-energy crossing it

GEODESIC LOCAL CAUSAL DIAMONDS

- In an arbitrary *P* choose any unit timelike vector n^{μ} and construct Riemann normal coordinates (RNC)

  - Metric expansion around *P*

$$g_{\mu\nu}(x) = \eta_{\mu\nu} - \frac{1}{3} R_{\mu\alpha\nu\beta}(P)\, x^{\alpha} x^{\beta} + O\left(x^3\right)$$



- Area of the 2-sphere

$$\mathcal{A} = 4\pi l^2 - \frac{4\pi}{9} l^4 G_{00}(P) + O\left(l^5\right)$$

- Spacetime region causally determined by this ball is called geodesic local causal diamond (GLCD)

- Conformal Killing vector → conformal Killing horizon

Quantum correlations across horizon + matter-energy crossing it

↓

- Entanglement entropy: $S = \eta \mathcal{A}$ → Bekenstein entropy $S_{BH} = \frac{k_B \mathcal{A}}{4 l_P^2}$

- Entropy of matter: in terms of entanglement for GLCD explicitly evaluated for small perturbations from vacuum

↳ Clausius entropy: Equal to leading order for conformal fields (without gravitational dynamics)

[A.A-S, M. Liska, PRD 102 (2020) 10, 104056]

Key (an minimal) assumptions:
- Einstein equivalence principle
- Entropy of horizon = Bekenstein entropy

[A.A-S, M. Liska, arxiv 2305.11756 [gr-qc]]

Einstein equations are derived

[T. Jacobson, PRL 116 (2016) 20, 201101]

- Different methods: Clausius entropy and entanglement entropy

[A.A-S, M. Liska, PRD 102 (2020) 10, 104056]

Unimodular gravity

- Unimodular gravity: Restricting full diffeomorphism invariance of GR

$$g = -1 \longrightarrow \sqrt{-g} = \omega_0$$

The simplest action $$S = \int_\Omega \left[\frac{c^4}{16\pi G}\left(R - 2\bar{\Lambda}\right) + \mathcal{L}_{\mathrm{matter}}\right]\omega_0 \mathrm{d}^4x$$

when varying it with respect to the metric

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{4}Tg_{\mu\nu}\right)$$

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$$R_{\mu\nu} - \frac{1}{4} R g_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right)$$

- Transverse diff invariance does not suffice to ensure ${T_\mu}^\nu{}_{;\nu} = 0$

  → Λ appears as an integration constant

- UG is fully equivalent to GR

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- Transverse diff invariance does not suffice to ensure $T_\mu{}^\nu{}_{;\nu} = 0$

$\Rightarrow$ Λ appears as an integration constant

- UG is fully equivalent to GR

- Weyl transverse gravity: transverse diffeomorphisms + Weyl invariance

- Why obtaining Weyl transverse gravity from thermodynamics of spacetime?
  - Local energy conservation must be assumed separately
  - Λ appears as an integration constant
  - Only conformally invariant part couples to gravity → conformal invariance
  - GR and UG same classical dynamics

[A.A-S, M. Liska, PRD 102 (2020) 10, 104056]

MODIFIED ENTANGLEMENT ENTROPY

- Leading order quantum gravity modifications to Bekenstein entropy

$$S_{BH,q} = \frac{k_B \mathcal{A}}{4 l_P^2} + \mathcal{C} k_B \ln\left(\frac{\mathcal{A}}{\mathcal{A}_0}\right) + O\left(\frac{k_B l_P^2}{\mathcal{A}}\right)$$

- Predicted by different theories of quantum gravity and phenomenological approaches

- Calculations entanglement entropy → local causal horizons

QUANTUM PHENOMENOLOGICAL EQUATIONS OF MOTION

EQUATIONS OF MOTION FROM ENTANGLEMENT EQUILIBRIUM

[T. Jacobson, PRL 116 (2016) 20, 201101]

- Maximal vacuum entanglement hypothesis (MVEH):

  "When the geometry and quantum fields are simultaneously varied from maximal symmetry, the entanglement entropy in a small geodesic ball is maximal at fixed volume"

- Consider the GLCD carried out in a maximally symmetric spacetime (MSS), and we simultaneously vary geometry and state of quantum fields:

$$\delta S_e + \delta S_m = 0$$

  - On one side: δS_e with the logarithmic correction

$$S_{e,q} = \eta \mathcal{A} + k_B \mathcal{C} \ln \frac{\mathcal{A}}{\mathcal{A}_0} + O\left(\frac{k_B l_P^2}{\mathcal{A}}\right)$$

$$\Big\downarrow \; \delta \mathcal{A}|_V$$

$$\delta S_{e,q} = S_{e,q} - S_{e,q}^{MSS} = \eta \delta \mathcal{A} + k_B \mathcal{C} \frac{\delta \mathcal{A}}{\mathcal{A}_{MSS}} - k_B \frac{\mathcal{C}}{2} \left(\frac{\delta \mathcal{A}}{\mathcal{A}_{MSS}}\right)^2 + O\left((\delta \mathcal{A})^3\right)$$

- On the other side: δS_m

  - Conformal Killing vector → Conformal Killing horizon with T_{Unruh}

  - Quantum fields in the thermal state → Minkowski vacuum (EEP)

  - Variation of entropy of quantum fields (Temperatures cancel out)

$$\delta S_m = \frac{2\pi k_B c}{\hbar}\frac{4\pi}{15}l^4\left(\delta\langle T_{00}\rangle + \delta X\right) - 4\psi l^2 C^2 \frac{2\pi k_B l_P^2}{\hbar c^3}\frac{4\pi}{15}l^4\left(\delta\langle T_{00}\rangle + \delta X\right) + O\left(l^5\right)$$

➢ Considering the MVEH: $\delta S_{e,q} + \delta S_m = 0$

Requirement of recovering Einstein equations for $C \to 0$ fixes $\eta = k_B/4l_P^2$

⇒ { Due to EEP ➡ every spacetime point *P*
GLCD in *P* ➡ any arbitrary unit timelike vector n^μ }

- We obtain:

$$S_{\mu\nu}(P)\,n^\mu n^\nu + \frac{\mathcal{C}l_{Pl}^2}{30\pi} S_{\alpha\beta}(P)\,n^\alpha n^\beta S_{\mu\nu}(P)\,n^\mu n^\nu - \Phi(P) = \frac{8\pi G}{c^4}\delta\langle T_{\mu\nu}(P)\rangle(P)\,n^\mu n^\nu$$

with: $S_{\mu\nu} = R_{\mu\nu} - Rg_{\mu\nu}/4$

➡ We differentiate respect to n^μ getting a system of conditions → Φ

Finally we obtain:

$$S_{\mu\nu} - \frac{Cl_{Pl}^2}{30\pi} S_{\mu\lambda}S^{\lambda}{}_{\nu} - \frac{8\pi G}{c^4}\delta\langle T_{\mu\nu}\rangle = -^{(0)}\Phi g_{\mu\nu}$$

⬇ Taking the trace

$$S_{\mu\nu} - \frac{Cl_{Pl}^2}{30\pi} S_{\mu\lambda}S^{\lambda}{}_{\nu} + \frac{Cl_{Pl}^2}{120\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \frac{8\pi G}{c^4}\left(\delta\langle T_{\mu\nu}\rangle - \frac{1}{4}\delta\langle T\rangle g_{\mu\nu}\right)$$

(Classically Einstein equations recovered by imposing: $T_{\mu}{}^{\nu}{}_{;\nu} = 0$ → Λ)

[A.A-S, M. Liska, JHEP 12 (2020) 196]

EQUATIONS OF MOTION FROM CLAUSIUS ENTROPY

- Clausius entropy of matter crossing arbitrary bifurcate null surfaces in a curved spacetime

[V. Baccetti, M. Visser, CQG 31(2014) 035009]

Derivation of Einstein equations → equivalence of entropies

[A.A-S, M. Liska, PRD 102 (2020) 10, 104056]

Quantum modifications → new construction of Clausius entropy

- The entropy balance equation yields to a similar expression than previous procedure, and finally we obtain

$$\boxed{S_{\mu\nu} - \frac{\mathcal{C}l_P^2}{18\pi} S_{\mu\lambda} S^{\lambda}{}_{\nu} + \frac{\mathcal{C}l_P^2}{72\pi}\left(R_{\kappa\lambda}R^{\kappa\lambda} - \frac{1}{4}R^2\right) g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{4}T g_{\mu\nu}\right)}$$

[A.A-S, M. Liska, JHEP 12 (2020) 196]

INTERPRETATION OF THE EQUATIONS

$$S_{\mu\nu} - D l_P^2 S_{\mu\lambda} S^{\lambda}{}_{\nu} + \frac{D l_P^2}{4} \left(R_{\kappa\lambda} R^{\kappa\lambda} - \frac{1}{4} R^2 \right) g_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{4} T g_{\mu\nu} \right)$$

- Equivalence of the derivations → Equivalence of entropies?
- Traceless equations → Unimodular Gravity
- Nonlinear in second derivatives
- General equations
- Further understanding of the equations
  - Thermodynamics for Weyl transverse gravity

    [A.A-S, L.J. Garay, M. Liska, PRD 106 (2022) 6, 064024 and CQG 40 (2023) 2, 025012]
  - Extra terms due to higher-order corrections?

SKETCH OF A COSMOLOGICAL MODEL

[A.A-S, M. Liska, A. Vicente-Becerril, PLB 839 (2023) 137827]

- Consider a homogeneous, isotropic cosmological model (FLRW model)

$$ds^2 = -c^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right)$$

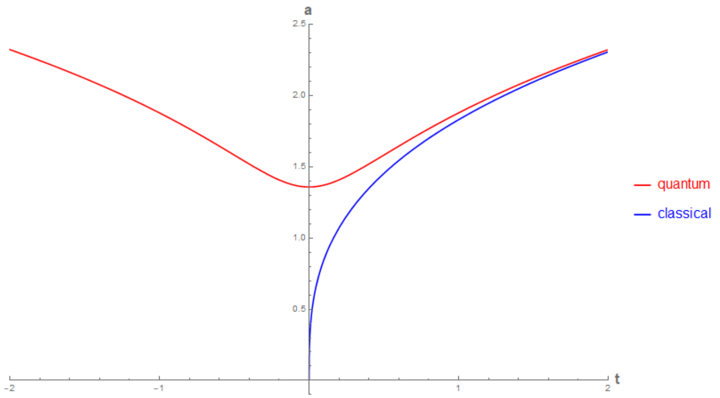
Matter content: Perfect fluid

Raychaudhuri equation: $\dot{H} - \frac{k}{a^2} - \frac{D l_{\mathrm{P}}^2}{c^2}\left(\dot{H} - \frac{k}{a^2}\right)^2 = -4\pi G\left(\rho + \frac{p}{c^2}\right)$

Energy conservation

Perturbative Friedmann equation (k=0) $H^2 = \frac{8\pi G\rho}{3}\left(1 - \frac{2\pi D\rho}{\rho_P}\right) + \tilde{\Lambda}$

- *D>0* Avoid singularity (similar to the bounce in LQC)



DISCUSSION

- Understanding thermodynamics of spacetime
  - Assumptions, principles and implications
  - Equivalence of entropies
  - Unimodular Gravity

- Analyzing general low-energy quantum gravity effects
  - General quantum phenomenological dynamics from logarithmic corrections to entropy
  - Effects in a specific models: Cosmological model

FUTURE PERSPECTIVES

- Full structure of phenomenological equations
- Could we find the action which implies the equations of motion?

- Study of explicit solutions
- Constraints on parameter *D*

- Equivalence of entropies
- Structure of gravity: Weyl transverse gravity

Thank you very much for your attention!

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