OPTIMAL REGULARITY AND UHLENBECK COMPACTNESS IN LORENTZIAN GEOMETRY AND BEYOND

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Abstract. We prove that curvature alone controls the derivatives of a connection, including the gravitational metrics of General Relativity, and the Yang-Mills connections of Particle Physics. Specifically, we prove that the regularity of L^p connections can be lifted by coordinate/gauge transformation to one derivative above their L^p bounded Riemann curvature, (i.e., to *optimal regularity*), thereby removing apparent singularities in the underlying geometry. This extends the classical result of Kazdan-DeTurck for Riemannian metric connections. As an application to General Relativity, our optimal regularity result implies that the Lipschitz continuous metrics of shock wave solutions of the Einstein-Euler equations are non-singular, (geodesic curves, locally inertial coordinates and the Newtonian limit all exist in a classical sense). By the extra connection derivative, we extend *Uhlenbeck compactness* from Riemannian to Lorentzian geometry, and from compact to non-compact gauge groups. The proofs are based on our discovery of, and existence theory for, a novel system of non-linear partial differential equations, (the *RT-equations*), non-invariant equations which are *elliptic* independent of metric signature, and which provide a general procedure for constructing coordinate and gauge transformations that regularize spacetime connections.

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