Applications of Finsler Geometry: from Zermelo navigation to wildfire spread modeling

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Abstract

The non-symmetric nature of Finsler metrics allows some intrinsically anisotropic physical problems and phenomena to be modeled in a very natural way. A motivating example is Zermelo's navigation problem, which seeks the fastest trajectory between two prescribed points for a moving object in the presence of a current. In the most general case, where the velocity of the object may depend on the space, time and direction, the solution to this problem can be interpreted as a lightlike geodesic of a certain Finsler spacetime.

This can be translated to (anisotropic) waves: lightlike geodesics orthogonal to the initial wavefront are the trajectories that minimize the traveltime and, therefore, make up the wavefront [1, 2]. Some relevant situations include the case when the wave travels between two different media, in which case it gets refracted following a generalized Snell's law [3]. Furthermore, this theoretical framework can be applied to any physical phenomenon that satisfies Huygens' principle (i.e., behaves as a wave in terms of propagation), such as the spread of wildfires [4].

References

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