Verlinde's "Emergent gravity and the dark Universe"

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March 3, 2023

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1. References

[1] E. Verlinde, *Emergent Gravity and the Dark Universe*, arXiv:1611.02269 [hep-th].

2. Preliminaries

2.1.The missing mass problem

- There is not enough mass in the observed Universe to account for the observed gravitational pull [Zwicky, Rubin]

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- Two alternatives:
- dark matter;
- Newton–Einstein theory incorrect in yet unexplored areas of the parameter space

2.2. The Tully-Fisher law

- Speed of outermost stars orbiting the centre of a galaxy of mass M [Tully-Fisher]

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$$\frac{GM}{R^2} < K' a_0, \qquad a_0 = cH_0 \simeq 10^{-10} m s^{-2}$$

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- $$\Rightarrow v = (GMa_0)^{1/4}$$

- Total acceleration g_T is a certain function of baryonic acceleration g_B [Tully-Fisher]

$$g_T = f(g_B) = \begin{cases} g_B & \text{for} \quad g_B \gg a_0 \\ \sqrt{g_B a_0} & \text{for} \quad g_B \ll a_0 \end{cases}$$

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- Two regimes:
  $a \gg a_0$, standard Newtonian regime;
  $a \ll a_0$ MOND regime.

2.3. Newtonian gravity in terms of surface densities

- Given ϕ, define the surface mass density

$$\mathbf{\Sigma} = \frac{d-2}{d-3}\frac{\mathbf{g}}{8\pi G}, \qquad \mathbf{g} = -\nabla\phi$$

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- Gravitational self–energy $U_{\rm grav}$ of a mass distribution

$$U_{\rm grav} = \frac{1}{2}\int_{\mathcal{V}} dV\, \mathbf{g}\cdot\mathbf{\Sigma}$$

2.4. Thermodynamics of spacetime

- Area law for the entropy of a horizon [Bekenstein-Hawking]

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- The holographic principle ['t Hooft, Susskind]: the degrees of freedom describing a volume of space are encoded in the surface bounding that volume

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- Gravity describes the change in entanglemente entropy caused by matter
- Entropic/emergent gravity

2.5. de Sitter spacetime

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- Mass in dS space

$$M = -\frac{1}{8\pi G}\int_{\mathcal{S}_\infty} dA\,\phi\nabla\cdot\mathbf{n}$$

n is the outward normal to $\mathcal{S}_\infty$

Plausibility argument: mass in Minkowski space

$$\nabla^2\phi = 8\pi G\rho \Longrightarrow$$

$$M = \int_{\mathcal{V}} dV\,\rho = \frac{1}{8\pi G}\int_{\mathcal{V}} dV\,\nabla^2\phi$$

$$= \frac{1}{8\pi G}\int_{\mathcal{S}} dA\,\mathbf{n}\cdot\nabla\phi = -\frac{1}{8\pi G}\int_{\mathcal{S}} dA\,\phi\nabla\cdot\mathbf{n}$$

3. Entropy as a criterion for a phase transition

3.1. Entropy of dS space

- Empty dS space

$$S_{\rm DE}(r) = \frac{r}{L}\frac{A(r)}{4G\hbar}, \qquad A(r) = \Omega_{d-2} r^{d-2}$$

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- Reduces the horizon size

3.2. The missing mass problem in entropic terms

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- Multiply with r/L, use $a_0 = 1/L$

$$\frac{2\pi Mr}{\hbar} < \frac{r}{L}\frac{A(r)}{4G\hbar}, \quad ie \quad |S_{\mathrm{M}}(r)| < S_{\mathrm{DE}}(r)$$

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- high surface mass density, strong gravitational acceleration, standard gravity; spacetime is stiff
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- $|S_{\rm M}(r)| < S_{\rm DE}(r)$:
- low surface mass density, low gravitational acceleration, MOND; spacetime becomes elastic

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- Entanglement entropy follows area law [BH]
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- When $r \sim L$, GR breaks down and MOND sets in
- Volume law for entanglement entropy overtakes the area law [Verlinde]

4. Basics in elasticity theory

4.1. Definitions

- A displacement vector field u_i within an elastic medium defines a strain tensor

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- Hooke's law: stresses and strains are linearly related

$$\sigma_{ij} = \lambda \varepsilon_{kk}\delta_{ij} + 2\mu \varepsilon_{ij}, \quad \textit{Lame moduli } \lambda, \mu$$

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- Let **B** be the resultant force per unit volume

$$\mathbf{F} = \int_{\mathcal{V}} dV\,\mathbf{B}$$

4.2. Relation with Newtonian gravity

- Assume

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- Elastic energy

$$U_{\mathrm{elas}} = \frac{1}{2} \int dV \, \varepsilon_{ij} \sigma_{ij}$$

5. The elastic phase of emergent gravity

5.1. Gravitational parameters as elastic moduli

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- Elastic moduli of the dark matter medium

$$\mu = \frac{a_0^2}{16\pi G}, \qquad \lambda = -\frac{a_0^2}{8\pi G}$$

5.2. Elasticity analogues of Newtonian gravity

gravity	elasticity	correspondence
Newtonian potential ϕ	displacement field u_i	$u_i = \phi n_i / a_0$
acceleration g_i	strain tensor ε_{ij}	$\varepsilon_{ij} n_j = -g_i / a_0$
surface mass density Σ_i	stress tensor σ_{ij}	$\sigma_{ij} n_j = \Sigma_i a_0$
volumetric mass density ρ	body force B_i	$B_i = -\rho a_0 n_i$
point mass m	point force f_i	$f_i = -m a_0 n_i$

Identification made on the surface $\mathcal{S}$ perpendicular to n_i

5.3. A derivation of the Tully-Fisher law

- In d-dim dS spacetime

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$$\varepsilon_{ij} n_j = \varepsilon n_i, \qquad \Sigma_D = \frac{a_0}{8\pi G} \varepsilon$$

- Estimate of the amount of apparent dark matter in good agreement with observations [arXiv:2206.11685]

6. Disputed questions

6.1. Composition law for accelerations

- First claim [Verlinde]

$$g_T(r) = g_B(r) + g_D(r)$$

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- Suggestive of some $U(1)_{\rm grav}$ invariance

$$\mathbf{g}_X = -\nabla\phi_X, \qquad X = T, B, D$$

- Newtonian energy densities $(\nabla\phi_X)^2$ are additive

$$(\nabla\phi_T)^2 = (\nabla\phi_B)^2 + (\nabla\phi_D)^2$$

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- Theory of elasticity: oscillating plate [Courant-Hilbert]

$$\nabla^4 f + f_{tt} = 0$$

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- Perhaps our knowledge of gravity is incomplete
- Perhaps spacetime is not irreducible but derived
- Verlinde postulates an emergent spacetime composed of qubits,
- spacetime emerges from their entanglement.
- Gravity is the force describing the change in entanglement due to matter

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- MOND exhibits slow relaxation, aging and memory effects.
- Dark matter is only apparent: no new particles
- Instead: phase transition between GR (stiff phase) and MOND (elastic phase)