Fermiogenesis

How to determine its occurrence in Causal Fermion Systems.

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Based on:
Felix Finster[‡], Maximilian Jokel[‡], C.F.P.
"A mechanism of baryogenesis for Causal Fermion Systems".
Classical and Quantum Gravity, 2022, 39. Jg., Nr. 16, S. 165005.

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My Goal

My goals for this talk are:

- Explain what Baryogenesis is and why it matters.
- Give you an intuitive understanding why Causal Fermion Systems allows for a new perspective on the problem.
- Explain the role of the regularizing vector field and its motivation.
- Convince you that the effect is consistent with observations in the sense that the effect is absent in an exact FRLW spacetime.

Outline

1. Basics
2. Sakharov's Criteria
3. Deriving the Modified Dynamics
4. The Rate of Fermiogenesis
5. Summary & Conclusion

What is Baryogenesis

- There is more matter than anti-matter in the universe.
- The asymmetries in the standard model are insufficient to explain this asymmetry dynamically.
- Most attempts rely on extensions of the Standard Model. This requires extra particles that have not yet been observed.
- We derived an entirely new mechanism based on the theory of Causal Fermion Systems.
  $\Rightarrow$ Does not require additional particles (besides the non-interacting right-handed neutrino).
  $\Rightarrow$ Allows for zero-temperature fermionic dark matter.
- Our mechanism creates the asymmetry entirely in the fermionic sector of the Standard Model $\Rightarrow$ Fermiogenesis.

Our Result in Technical Terms

In CFS the rate of fermiogenesis can be derived as the spectra flux of a suitable defined spatial operator $A(t)$ on a foliation of Cauchy surfaces.

$$B(t) = -\mathrm{tr}\big(E_{-m}(t)\,\dot{A}(t)\big)\,, \tag{1}$$

- Fermiogenesis as a sea level shift of the Dirac sea.
- $A(t)$ encodes what we refer to as the locally rigid regularization.
- This locally rigid dynamics derived from $A(t)$ is a small deviation from Dirac dynamics.
- For Dirac dynamics we proved that $B(t) = 0$ in a suitable sense.
- In spatially homogeneous spacetimes, e.g. an FRWL universe, the locally rigid dynamics agrees with the Dirac dynamics.

Classical Spacetimes

Let $(\mathscr{M}, g)$ be a Lorentzian spin manifold of dimension four.

- time-oriented, smooth, globally hyperbolic.
- We use the signature convention $(+,-,-,-)$.

This implies:

- There is a global time function t.
- Its level sets give a global smooth foliation by Cauchy surfaces denoted by $(\mathscr{N}_t)_{t\in\mathbb{R}}$.

What is a Causal Fermion System

A CFS consists of a Hilbert space $(\mathcal{H}, \langle .|. \rangle_{\mathcal{H}})$, a particular subset of the bounded self-adjoint linear operators $\mathcal{F}$ and the universal measure ρ.

- $\mathcal{F}$ is a Banach manifold.
- Only those measures represent physical systems which minimize the Causal Action Principle.
- The so called *local correlation map* allows us to map (an approximation of) spacetime plus matter into a CFS

$$
\begin{aligned}
F_\varepsilon[g_{\mu\nu}, A_\mu, \dots] : \mathscr{M} &\mapsto \mathcal{F} \qquad \subset \mathrm{L}(\mathcal{H}) \\
x &\mapsto F\varepsilon[g_{\mu\nu}, A_\mu, \dots](x).
\end{aligned}
$$

- The universal measure ρ can be written as the pushforward measure under the local correlation map of the measure on the spacetime.
- To obtain the classical theories we have to take the continuum limit where $\varepsilon \searrow 0$.
- No metric in CFS $\Rightarrow$ compatible with non-Riemannian measures.

Sakharov's Third Criterion

3)Interactions must be out of thermal equilibrium.

Sakharov's Second Criterion

2) C-symmetry and CP-symmetry must be violated.

Minkowski Space as a Causal Fermion System

- The local correlation map allows us to write the Hilbert space $\mathcal{H}$ in terms of so called *physical wave functions* in the space time $\mathscr{M}$.
- When we consider Minkowski space as a minimizer of the Causal Action Principle in the continuum limit, the Hilbert space $\mathcal{H}$ of the CFS corresponds to the negative frequency solutions of the Dirac equation.
  The fundamental description of Minkowski in terms of CFS is no longer *C* nor *CP* invariant.
- The Standard Model of particle physics, including the QFT limit, has been derived in perturbation theory around Minkowski space.
- The same goes for General Relativity.
- Perturbing Minkowski by adding an extra state of positive energy to the Hilbert space leads to Minkowski space containing a Dirac field.

Regularization in Minkowski Space

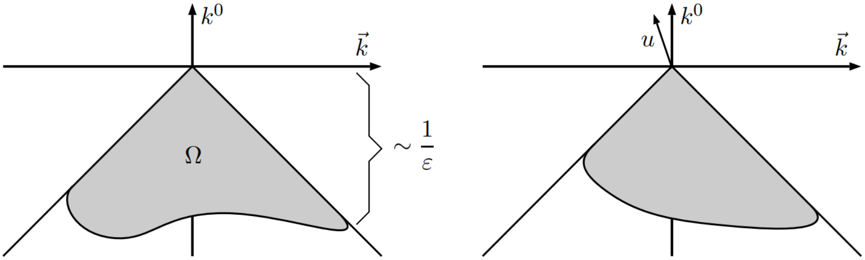


Figure: Only regularizations that can be represented as a cutoff at a certain energy scale with respect to an observer can be minimizers of the Causal Action Principle in Minkowski space[1].

[1] M. Jokel, Second Variations of the Causal Action for Regularized Dirac Sea Configurations, Dissertation, Universität Regensburg

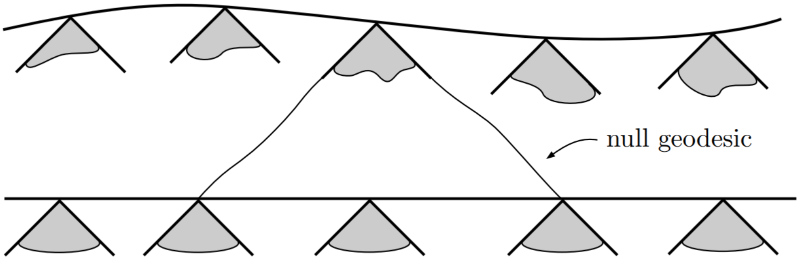
Sakharov's First Criterion

1) Baryon number violation.

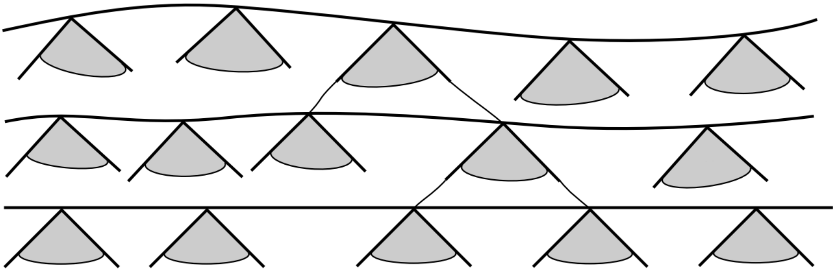
Dirac vs. Modified Dynamics

- With Dirac's classical argument particles and anti particles can only be created in pairs $\rightarrow$ no fermiogenesis.
- Dirac Dynamics is known to describe experiments on earth to a high degree of accuracy.
- Derive a modification of Dirac dynamics which differs on cosmological scales.
- Show that this modified dynamics does lead to fermiogenesis.

Regularization under Dirac Dynamics



Locally Rigid Regularization



Conservation Law

There exists a conservation law for so called *surface layer integrals* in CFS. It can be shown to be equal to the usual flux integral for the physical wave functions.

$$\langle\psi|\phi\rangle_\rho^t = \int_{\mathcal{N}_t} \prec\tilde{\psi}\,|\,\gamma(\nu)\,\tilde{\phi}\succ(x)\,d\mu_{\mathcal{N}_t}(x) + \O\left(\frac{\ell_{\text{lab}}}{\ell_{\text{cosmo}}}\right).$$

Denoting the scalar product on the RHS with $(.|.)_t$ at time t and the Hilbert space by $\mathcal{H}_t \simeq L^2(\mathbb{R}^3, \mathbb{C}^4)$, we can construct an isometric embedding,

$$\Psi^\varepsilon : \mathcal{H} \hookrightarrow \mathcal{H}_t \qquad \text{isometry for all } t\,.$$

What do we want to achive

- The Dirac dynamics should be modified only slightly.
- The regularization should be locally rigid with a regularizing vector field $u(x)$.
- Current conservation should hold. I.e. the evolution should be unitary with regards to the scalar product in the conservation law.

We apply the following strategy

- Find a suitable spatial operator A which encodes the regularizing vector field $u(x)$.
- Modify the dynamics by continuous adiabatic projections onto a spectral subspace of A.

Fermionic Projector

Let $\mathcal{H}_{\mathscr{M}}$ be a Hilbert space on $\mathscr{M}$ with scalar product $(\cdot|\cdot)_{\mathscr{M}}$.

- The fermionic projector P projects to the subspace of $\mathcal{H}_{\mathscr{M}}$ corresponding to the physical wave function representation of the Hilbert space underlying the CFS.
- It encodes the dynamics of the ensemble of all physical wave functions. ($\approx$ This can be thought of as a sort of dual description of the dynamics of the measure.)
- Can be represented as an integral kernel

$$(\psi|P\phi)_{\mathscr{M}} = \int_{\mathscr{M}\times\mathscr{M}} \psi(x)P(x,y)\phi(y)d\mu(y)d\mu(x) \tag{2}$$

- $P(x,y)$ contains the equivalent information to $F(x)$ and $F(y)$.
- The Lagrangian in the Causal Action Principle can be computed in spacetime using $P(x,y)$.

Fermionic Projector with a Hard Cutoff Regularization

In Minkowski project to the negative frequency subspace of solutions to the Dirac equation

$$P^{\varepsilon}_{\text{vac}}(x,y) = \int \frac{d^4k}{(2\pi)^4}\, \hat{P}^{\varepsilon}_{\text{vac}}(k)\, e^{-ik(y-x)} \tag{3}$$

$$\hat{P}^{\varepsilon}_{\text{vac}}(k) = (k_j\gamma^j + m)\, \delta\big(k^2 - m^2\big)\, \Theta\big(-k^0\big)\, \Theta\big(1 + \varepsilon k^0\big)\,. \tag{4}$$

In considering P as an operator on the induced Hilbert space on a $t = const$ hypersurface it corresponds to the spectral projection operator

$$E_{I_0}(t) := \chi_{I_0}\big(A(t)\big) \tag{5}$$

where $A = \varepsilon H$ is a multiple of the Dirac Hamiltonian

$$H := -i\gamma^0\gamma^\alpha\partial_\alpha + \gamma^0 m\,. \tag{6}$$

and I_0 the interval $I_0 := \big(-1, 0\big)$.

Fermionic Projector with a Regularizing Vector Field

In curved spacetimes Fourier transform in Gaussian coordinates

$$P^\varepsilon(x,y) \approx \int \frac{d^4k}{(2\pi)^4}\, \hat{P}^\varepsilon(k)\, e^{-ik(y-x)}\,.$$

Locally the kernel $\hat{P}^\varepsilon$ should be of the form the form

$$\hat{P}^\varepsilon(k) = \hat{P}^{\text{vac}}\big(k, u(x)\big) \qquad \text{with} \tag{7}$$

$$\hat{P}^{\text{vac}}(k, u(x)) = \big(k_j\gamma^j + m\big)\, \delta\big(k^2 - m^2\big)\, \Theta\big(-k^0\big)\, \Theta\big(1 + ku\big)\,, \tag{8}$$

where u is a future-directed, timelike vector.
In Minkowski this can be realized by projecting to the spectrum of

$$A(u) = u^0\, H + i \sum_{\alpha=1}^{3} u^\alpha\, \frac{\partial}{\partial x^\alpha}\,. \tag{9}$$

where $-i\partial_\alpha$ are the momentum operators.

Spectral Projection Operator in Curved Spacetime

Defining A in *curved spacetime*, we want it to satisfy:

(a) Locally in a Gaussian coordinate system, it has the same form as in Minkowski space.

(b) It is symmetric with respect to the scalar product on the Cauchy surface.

This leads to the choice

$$A(t) = \frac{1}{2}\left\{u^0,\ \mathscr{D}_{\mathscr{N}_t}\right\} + \frac{i}{2}\sum_{\alpha=1}^{3}\left\{u^\alpha,\ \nabla_\alpha\right\} .$$

Here $\mathscr{D}_{\mathscr{N}_t}$ denotes the *hypersurface Dirac operator* defined by

$$\mathscr{D}_{\mathscr{N}_t} := -i\gamma(\nu)\sum_{\alpha=1}^{3}\gamma^\alpha\nabla_\alpha \tag{10}$$

and the anti-commutators guarantee that $A(t)$ is symmetric with respect to the scalar product on $\mathscr{N}_t$.

Modified Dynamics from Adiabatic Projections

Setting $I := \big(-1, 1 \big)$ the modified dynamics is given by

$$V^t_{t_0}\, \psi_0 = \lim_{N\to\infty} E_I(t)\, U^t_{t-\Delta t} \cdots E_I\big(t_0+2\Delta t\big)\, U^{t_0+2\Delta t}_{t_0+\Delta t}\, E_I\big(t_0+\Delta t\big)\, U^{t_0+\Delta t}_{t_0}\, \psi(0), \tag{11}$$

where we set $\Delta t := (t-t_0)/N$, and

$$U^{t''}_{t'} : \mathcal{H}_{t'} \to \mathcal{H}_{t''} \tag{12}$$

denotes the unitary Dirac dynamics from time t' to t''.
This time evolution can be written infinitesimally as

$$\frac{d}{dt} V^t_{t_0} = \big(\dot{E}_I(t) - iE_I(t)\, H(t) \big)\, V^t_{t_0}\,, \tag{13}$$

where $H(t)$ is again the Dirac Hamiltonian.

Time Ordered Exponential

The differential equation for V_t^t can be solved with a time-ordered exponential,

$$V_{t_0}^t = \mathrm{Texp}\left(\int_0^t \left(\dot{E}_I(\tau) - iE_I(\tau)\, H(\tau) \right) d\tau \right) , \tag{14}$$

defined via the Dyson series

$$\mathrm{Texp}\left(\int_0^t B(\tau)\, d\tau \right) := \mathbb{1} + \int_0^t B(\tau)\, d\tau + \int_0^t d\tau \int_0^\tau d\tau'\, B(\tau)\, B(\tau') + \cdots .$$

The method of adiabatic projections give rise to a unitary dynamics.

Projection Operator to the Occupied States

We denote the projection operator to the occupied states at time t by

$$\Pi(t) \;:\; \mathcal{H}_t \to \mathcal{H} \subset \mathcal{H}_t$$
$$(\Pi(t)\,\psi)(\mathbf{x}) = -\int_{\mathcal{N}_t} P^\varepsilon\big((t,\mathbf{x}),y\big)\;\gamma(\nu)(y)\;\psi(y)\;d\mu_{\mathcal{N}_t}(y)\,.$$

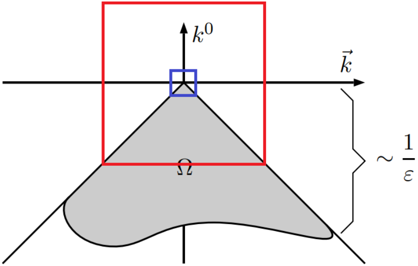
Its time evolution is given either by the Dirac dynamics or by the modified dynamics with adiabatic projections, i.e.

$$\Pi(t) = U^t_{t_0}\,\Pi(t_0)\,\big(U^t_{t_0}\big)^{-1} \qquad \text{(Dirac dynamics)} \tag{15}$$
$$\Pi(t) = V^t_{t_0}\,\Pi(t_0)\,\big(V^t_{t_0}\big)^{-1} \qquad \text{(locally rigid dynamics)} \tag{16}$$

Energy Window

For determining whether fermiogenesis occurs we may restrict attention to an energy scale Λ, which we assume to be in the range

$$m, \ell_{\text{macro}}^{-1} \ll \Lambda \ll \varepsilon^{-1} \,. \tag{17}$$



Detecting Fermiogenesis

We choose an operator η_Λ acting on the spatial Hilbert space $\mathcal{H}_t$ such that

$$\operatorname{tr}\big(\eta_\Lambda \, \Pi(t)\big) \tag{18}$$

tells us about the number of particles in the energy range $(-\Lambda, \Lambda)$.

The rate of fermiogenesis is then,

$$B(t) := \frac{d}{dt} \operatorname{tr}\big(\eta_\Lambda \, \Pi(t)\big) \; . \tag{19}$$

No Fermiogenesis for Dirac Dynamics

Theorem

For the Dirac dynamics,

$$B(t) = \mathcal{O}\left(\frac{1}{\Lambda}\right) , \tag{20}$$

uniformly in ε.

When choosing Λ large, one must keep in mind the condition $\Lambda \ll \varepsilon^{-1}$ must be satisfied.

We could write the error term in a scale invariant way in terms of the energy scale of macroscopic physics

$$\mathcal{O}\left(\frac{\ell_{\text{macro}}}{\Lambda}\right) .$$

Exact Expression for Fermiogenesis.

We finally arrive at the exact formula for Fermiogenesis for the modified dynamics (up to error terms $\varnothing(1/\Lambda)$)

$$B(t) = \frac{d}{dt}\mathrm{tr}\big(\eta_\Lambda\, \Pi(t)\big)$$
$$= 2\,\mathrm{Im} \int_{\mathcal{N}_t} \mathrm{Tr}_{S_x\mathscr{M}} \Big(\big(\eta_\Lambda\, \gamma^0 \big(i\dot{E}_I(t) + [E_I(t),\, H(t)]\big) P\big)(x,x) \Big)\, N(x)\, d\mu_{\mathcal{N}_t}(x)\,.$$

Approximation

Assume that $A(t)$ has a perturbation expansion of the form

$$A(t) = A + \Delta A(t), \qquad \text{with} \qquad \Delta A(t) := \sum_{p=1}^{\infty} \lambda^p \, A^{(p)}(t). \tag{21}$$

Using this perturbation expansion we obtain

$$B(t) = \frac{d}{dt} \mathrm{tr}\big(\eta_\Lambda \, \Pi(t)\big) = \int_{-\frac{1}{\varepsilon}}^{-m} \mathrm{tr}\big(\eta_\Lambda(t) \, \dot{E}_\omega(t)\big) \, d\omega \; = -\eta(-m/\Lambda) \, \mathrm{tr}\big(E_{-m} \, \dot{A}\big) \, .$$

For large Λ, we can set approximately $\eta(-m/\Lambda) \approx \eta(0)$. Normalizing η by $\eta(0) = 1$, we end up with the simple formula

$$B(t) = -\mathrm{tr}\big(E_{-m}(t) \, \dot{A}(t)\big) \, . \tag{22}$$

Conceptual Insights

- Sakharov's third criterion is replaced by the condition that the spacetime must deviate from spatial homogeniety.
- These results suggest that one can think of the physical wave functions that make up the Dirac sea as the gravitational degrees of freedom of the theory. I.e. gravity ”lives” in the backward lightcone.
- The non-Riemannian measures are essential for the effective description to be consistent as they imply that the divergence of the stress-energy tensor does not vanish.
- The last point gives a possible connection to thermodynamic interpretations of General Relativity.
- Fermiogenesis occurs prior to reheating in contrast to usual scenarios.

Open Questions

- Observational implications of matter creation equally across all three generations in a Fermi groundstate.
  $\Rightarrow$ Allows for zero-temperature fermionic dark matter. Degeneracy pressure gives a different equation of state for dark matter.
  $\Rightarrow$ Modification of the expansion history of the universe primarily in the matter dominated era.
- Correct magnitude of the effect in relevant spacetime configurations?
- What happens in black hole spacetimes?
- What are the correct dynamics for the Non-Riemannian-Measure required for the mechanism?

Summary

- Locally rigid regularization generically leads to a small deviation from Dirac dynamics.
- To maintain the conservation laws we are lead to introduce non-Riemannian measures.
- The formalism gives no fermiogenesis for the Dirac dynamic (and accordingly for the spatially homogeneous FRLW universe).
- Fermiogenesis for the locally rigid regularization can be characterized by the spectral flow across the tip of the lower mass shell measured by a suitable spatial operator.
- Using suitable approximations we can obtain a simple formula for the rate of fermiogenesis.

Thank You

Thank you for your attention.

For an introduction to Causal Fermion Systems see the website
causal-fermion-system.com