Relativistic weak quantum gravity, and its significance for the standard model of particle physics

Tejinder P. Singh

Tata Institute of Fundamental Research
Mumbai

University of Regensburg
Online Seminar
January 21, 2022

Reference: *Quantum theory without classical time: a route to quantum gravity and unification,* arXiv: 2110.02062 (Review)

Motivation

- There must exist a formulation of quantum field theory which does not depend on classical time.
- Gravitation, as well as quantum theory, are emergent phenomena.
- How do fermions curve space-time? : a non-commutative spinor space-time.
- Unification of standard model with gravity required at all energy scales, not just at Planck energy scales.

Summary

- There must exist a reformulation of quantum field theory without classical time.

- Such a formulation has been developed as a matrix-valued Lagrangian dynamics.

- Elementary particles live in the non-commutative octonionic space.

- This space possibly determines the standard model, properties of elementary particles, and dimensionless constants of the standard model, all at low energies.

- The Left-Right symmetric extension of the standard model unifies it with pre-gravitation.

- We predict that the neutrino is a Majorana particle.

- We predict three right-handed sterile neutrinos, a dark photon, and two spin-one Lorentz bosons.

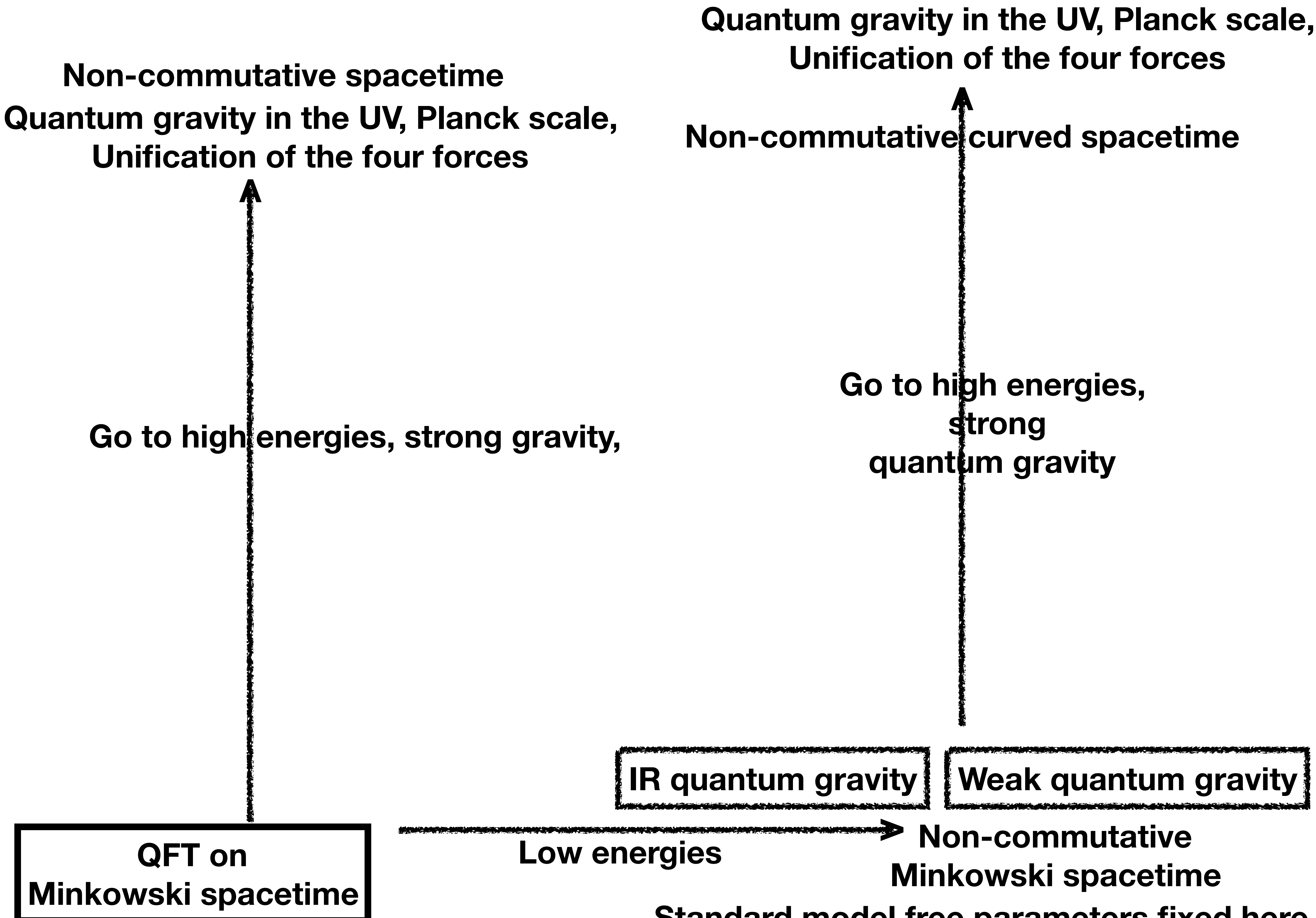
- Prior to measurement, a quantum system obeys L-R symmetry.

- Only classical systems live in 4D classical spacetime. Quantum systems live in an octonionic space.

- Unification of standard model with gravitation is required at all energy scales, not j
  just at the Planck energy scale.

Quantum superposition and gravitation

- A quantum object such as an electron has action of the order Planck's constant h
- It obeys position superposition; hence the produced gravitation is not classical.
- This is an example of low-energy [hence weak] quantum gravity.
- Only when the associated energy scale h/T is Planck energy, does this become Planck scale quantum gravity.
- Hence, we need a pre-quantum, pre-spacetime theory at all energy scales.


Non-commutative spacetime
Quantum gravity in the UV, Planck scale,
Unification of the four forces
Go to high energies, strong gravity,
QFT on
Minkowski spacetime
Low energies
Quantum gravity in the UV, Planck scale,
Unification of the four forces
Non-commutative curved spacetime
Go to high energies,
strong
quantum gravity
IR quantum gravity
Weak quantum gravity
Non-commutative
Minkowski spacetime
Standard model free parameters fixed here

Methodology

- Raise classical dynamical variables to the status of matrices/operators, but do not impose quantum commutation relations [Adler's Trace Dynamics, 1996]

- First we do this for non-gravitational degrees of freedom:

$$S = \int dt\ \mathcal{L}(q, \dot{q}) \to \int dt\ Tr[\mathcal{L}(\hat{q}, \dot{\hat{q}})]$$

- This matrix-valued Lagrangian dynamics has a novel conserved charge:

$$\tilde{C} = \sum_i [q_i, p_i]$$

- Assume this dynamics to hold at some time resolution T

- Coarse-graining over much larger time scales leads to the emergence of quantum theory.

How to include gravitation in this matrix dynamics?

- What gravitational degrees of freedom are to be raised to the status of matrices / operators?

- Answer: the eigenvalues of the Dirac operator on a curved space-time manifold:

$$Tr[L_P^2 D^2] \sim L_P^{-2} \int d^4x \, \sqrt{g} \, R + \mathcal{O}(L_P^0) = \sum_i \lambda_i^2$$

- An atom of space-time:

$$\lambda_i \to \hat{\lambda}_i \equiv \dot{q}_{Bi} \qquad\qquad \lambda_i^2 \to \int d\tau \; Tr[\dot{q}_{Bi}^2]$$

- An atom of space-time-matter:

$$\frac{S}{\hbar} = \int \frac{d\tau}{t_{Pl}} \, Tr\left\{ \frac{L_P^2}{L^2} \left[\dot{q}_B^\dagger + \frac{L_P^2}{L^2} \beta_1 \dot{q}_F^\dagger \right] \times \left[\dot{q}_B + \frac{L_P^2}{L^2} \beta_2 \dot{q}_F \right] \right\}$$

Including Yang-Mills fields

$$\frac{S}{\hbar} = \int \frac{d\tau}{t_{Pl}}\, Tr\Big\{ \frac{L_P^2}{L^2} \left[\left(\dot{q}_B^\dagger + i\frac{\alpha}{L}q_B^\dagger\right) + \frac{L_P^2}{L^2}\beta_1 \left(\dot{q}_F^\dagger + i\frac{\alpha}{L}q_F^\dagger\right)\right] \times \left[\left(\dot{q}_B + i\frac{\alpha}{L}q_B^\dagger\right) + \frac{L_P^2}{L^2}\beta_2 \left(\dot{q}_F + i\frac{\alpha}{L}q_F\right)\right] \Big\}$$

which can also be written as

$$\frac{S}{\hbar} = \int \frac{d\tau}{\tau_{Pl}}\, Tr\Big\{ \frac{L_P^2}{L^2}\Big[\left(\dot{q}_B^\dagger + \frac{L_P^2}{L^2}\beta_1\dot{q}_F^\dagger\right) + i\frac{\alpha}{L}\left(q_B^\dagger + \frac{L_P^2}{L^2}\beta_1 q_F^\dagger\right)\Big] \times \Big[\left(\dot{q}_B + \frac{L_P^2}{L^2}\beta_2\dot{q}_F\right) + i\frac{\alpha}{L}\left(q_B + \frac{L_P^2}{L^2}\beta_2 q_F\right)\Big]$$

- Define $q_1^\dagger = q_B^\dagger + \beta_1 q_F^\dagger; \quad q_2 = q_B + \beta_2 q_F$

$$S \equiv \int d\tau\; Tr\mathcal{L} \quad a_0 \equiv L_P^2/L^2 \quad a_1 \equiv \hbar/cL_P$$

$$Tr\mathcal{L} = \frac{1}{2}a_1 a_0\; Tr\left[\dot{q}_1^\dagger \dot{q}_2 - \frac{\alpha^2 c^2}{L^2} q_1^\dagger q_2\right]$$

The free particle Lagrangian

Define: $\dot{\tilde{Q}}_B = \frac{1}{L}(i\alpha q_B + L\dot{q}_B)$; $\dot{\tilde{Q}}_F = \frac{1}{L}(i\alpha q_F + L\dot{q}_F)$;

$$\mathcal{L} = Tr\left[\frac{L_p^2}{L^2}\left(\dot{\tilde{Q}}_B^\dagger + \frac{L_p^2}{L^2}\beta_1\dot{\tilde{Q}}_F^\dagger\right)\left(\dot{\tilde{Q}}_B + \frac{L_p^2}{L^2}\beta_2\dot{\tilde{Q}}_F\right)\right] =$$

$$\mathcal{L} = Tr\left[\frac{L_p^2}{L^2}\left\{\dot{\tilde{Q}}_B^\dagger\dot{\tilde{Q}}_B + \frac{L_p^2}{L^2}\left(\beta_1\dot{\tilde{Q}}_F^\dagger\dot{\tilde{Q}}_B + \dot{\tilde{Q}}_B^\dagger\beta_2\dot{\tilde{Q}}_F\right) + \frac{L_p^4}{L^4}\beta_1\dot{\tilde{Q}}_F^\dagger\beta_2\dot{\tilde{Q}}_F\right\}\right]$$

Define: $\dot{\tilde{Q}}_{1sed}^\dagger = \dot{\tilde{Q}}_B^\dagger + \frac{L_p^2}{L^2}\beta_1\dot{\tilde{Q}}_F^\dagger$; $\dot{\tilde{Q}}_{2sed} = \dot{\tilde{Q}}_B + \frac{L_p^2}{L^2}\beta_2\dot{\tilde{Q}}_F$

$$\mathcal{L} = Tr\left[\frac{L_p^2}{L^2}\dot{\tilde{Q}}_{1sed}^\dagger\;\dot{\tilde{Q}}_{2sed}\right]$$

$$\frac{S}{\hbar} = \frac{1}{2}\int\frac{d\tau}{\tau_{Pl}}\;Tr\left[\frac{L_p^2}{L^2}\dot{\tilde{Q}}_{1sed}^\dagger\;\dot{\tilde{Q}}_{2sed}\right]$$

Atoms of Space-time-matter

What is the noncommutative spacetime?

- What does the 4D spacetime manifold R^4 get replaced by?
- Appeal to division algebras: $\mathbb{R}, \mathbb{C}, \mathbb{H}, \mathbb{O}$
- Quaternions: $q = a_0 + a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

$$\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = -1 \text{ ; } \hat{i}\hat{j} = -\hat{j}\hat{i} = \hat{k} \text{ ; } \hat{j}\hat{k} = -\hat{k}\hat{j} = \hat{i} \text{ ; } \hat{k}\hat{i} = -\hat{i}\hat{k} = \hat{j}$$

- Octonions:

$$O = a_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3 + a_4\mathbf{e}_4 + a_5\mathbf{e}_5 + a_6\mathbf{e}_6 + a_7\mathbf{e}_7$$

- Sedenions: not a division algebra
- Bosons and fermions live in octonionic spacetime: $SL(2, \mathbb{O}) \sim SO(1, 9)$

$$Q_B = Q_0 + Q_1\mathbf{e}_1 + Q_2\mathbf{e}_2 + Q_3\mathbf{e}_3 + Q_4\mathbf{e}_4 + Q_5\mathbf{e}_5 + Q_6\mathbf{e}_6 + Q_7\mathbf{e}_7$$

Octonionic spacetime reveals the standard model !

FROM DIVISION ALGEBRAS TO

CLIFFORD ALGEBRAS & ELEMENTARY PARTICLES

BIG PICTURE

Three fermion generations including sterile neutrinos Standard Model and pre-Gravitation

LH Fermions		**RH Fermions**
U(1) is electric charge		U(1) is square-root mass
Charge eigenstates		***Mass eigenstates***
LH Majorana Neutrino		RH Majorana Neutrino
	Higgs ⟷	
LH anti-down quark 1/3		RH positron 1/3
LH up quark 2/3		RH up quark 2/3
LH positron 1		RH anti-down quark 1
$SU(3)_c \times SU(2)_W \times U(1)_{em}$		$SU(3)_g \times SU(2)_R \times U(1)_g$

Complex Quaternions and the Clifford algebra Cl(2) (Furey, 2016)

$\mathbb{C}\otimes\mathbb{H}$ $\quad 1,\ i,\ \hat{i},\ \hat{j},\ \hat{k},\ i\hat{i},\ i\hat{j},\ i\hat{k}$ $\quad SL(2,C)$

- Complex quaternions give a faithful representation of Cl(2)

$$\alpha = \frac{1}{2}(-\hat{j}+i\hat{i}); \qquad \alpha^\dagger = \frac{1}{2}(\hat{j}+i\hat{i}); \qquad \alpha^2 = \alpha^{\dagger 2} = 0, \{\alpha,\alpha^\dagger\} = 1$$

These are fermionic ladder operators. $\quad N = \sum_i \alpha_i^\dagger \alpha_i = \alpha^\dagger\alpha$

- Spinors can be defined as minimal left ideals of Clifford algebras.

$(\mathbb{C}\otimes\mathbb{H})\ V$ $\qquad V = \alpha\alpha^\dagger$ Projector/“vacuum” ..Idempotent

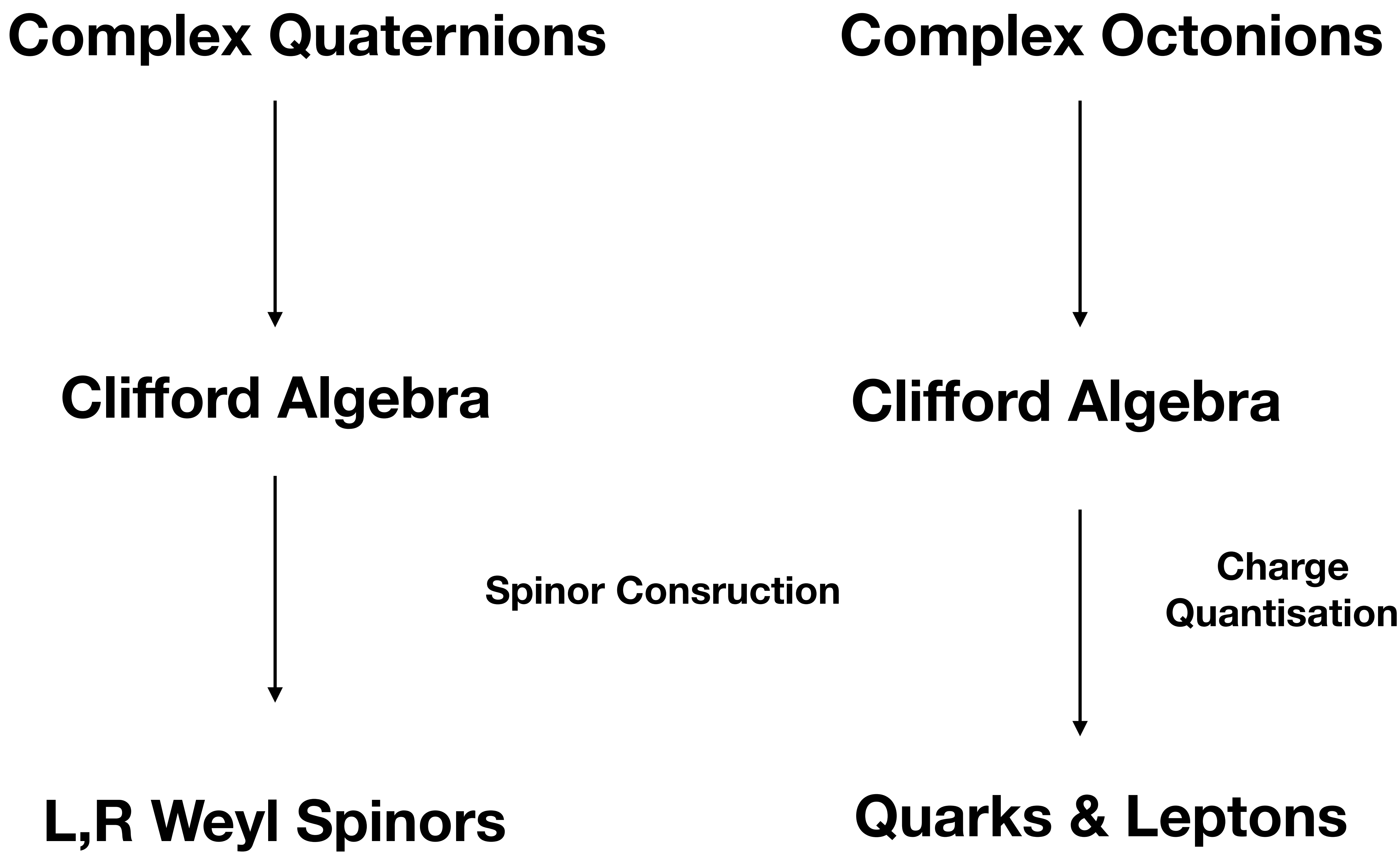
- Two (complex) dim space: $\quad V \qquad \alpha^\dagger V$

$$\psi_L = \psi_L^\uparrow V + \psi_L^\downarrow \alpha^\dagger V \qquad \psi_R = \psi_R^\downarrow V^* + \psi_R^\uparrow \alpha V^*$$

■ Using the complex quaternions one can describe:

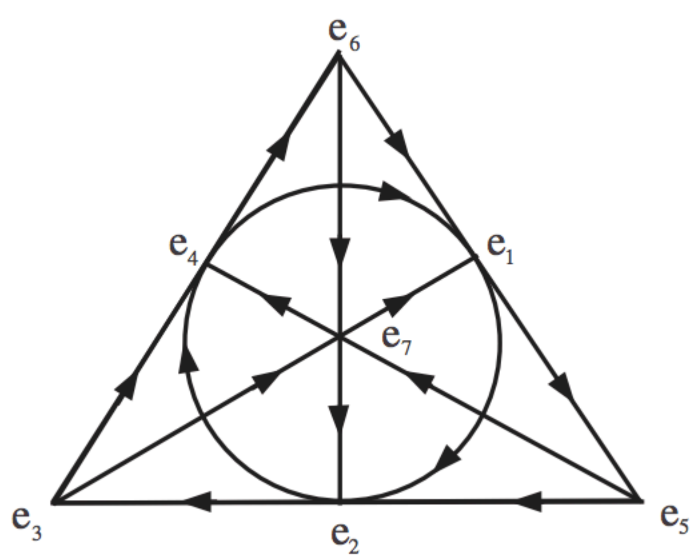
- Left and Right handed Weyl spinors
- Dirac spinors and Majorana spinors
- Contra-variant and covariant four vectors
- Scalars
- Field strength tensors

These are all the Lorentz representations of the standard model.


Complex Quaternions
Clifford Algebra
L,R Weyl Spinors
Spinor Consruction
Complex Octonions
Clifford Algebra
Charge Quantisation
Quarks & Leptons

Complex Octonions and the Clifford algebra Cl(6) (Furey, 2016)

$\mathbb{C} \otimes \mathbb{O}$ $\quad 1,\ i,\ e_1,\ e_2,\ e_3,\ e_4,\ e_5,\ e_6,\ e_7$ $\quad SL(2, \mathbb{O})$



■ A non-associative algebra

$\mathbb{C} \otimes \mathbb{O} \not\longrightarrow$ Clifford Algebra

$$p, m, n, f \in A$$

$$mf = f' \qquad m : f \mapsto f'$$

$$n(mf) = f'' \qquad \overleftarrow{nm} : f \mapsto f''$$

$$p(n(mf)) = f''' \qquad \overleftarrow{pnm} : f \mapsto f'''$$

■ Maps are associative

$$F \circ (G \circ H) = (F \circ G) \circ H$$

- Most general left action map of C x O on itself :

$$Mf = C_{abc}\overleftarrow{e_a e_b e_c} f + C_{ij}\overleftarrow{e_i e_j} f + C_k \overleftarrow{e_k} f + C_0 f$$

$$\overleftarrow{e_i e_j} f = -\overleftarrow{e_j e_i} f \qquad i \neq j \qquad\qquad \overleftarrow{e_i e_j} f = -f \qquad i = j$$

- Hence octonionic chains exhibit Clifford algebra structure.

- The chains have a total of sixty-four complex degrees of freedom.

- Hence they can be represented by 8x8 C matrices.
- Endomorphism on C X O
- Faithful representation of complex Clifford algebra Cl(6)

$$\alpha_1 = \frac{1}{2}[-e_5 + ie_4]; \qquad \alpha_2 = \frac{1}{2}[-e_3 + ie_1]; \qquad \alpha_3 = \frac{1}{2}[-e_6 + ie_2];$$

$$\alpha_1^\dagger = \frac{1}{2}[e_5 + ie_4]; \qquad \alpha_2^\dagger = \frac{1}{2}[e_3 + ie_1]; \qquad \alpha_3^\dagger = \frac{1}{2}[e_6 + ie_2]$$

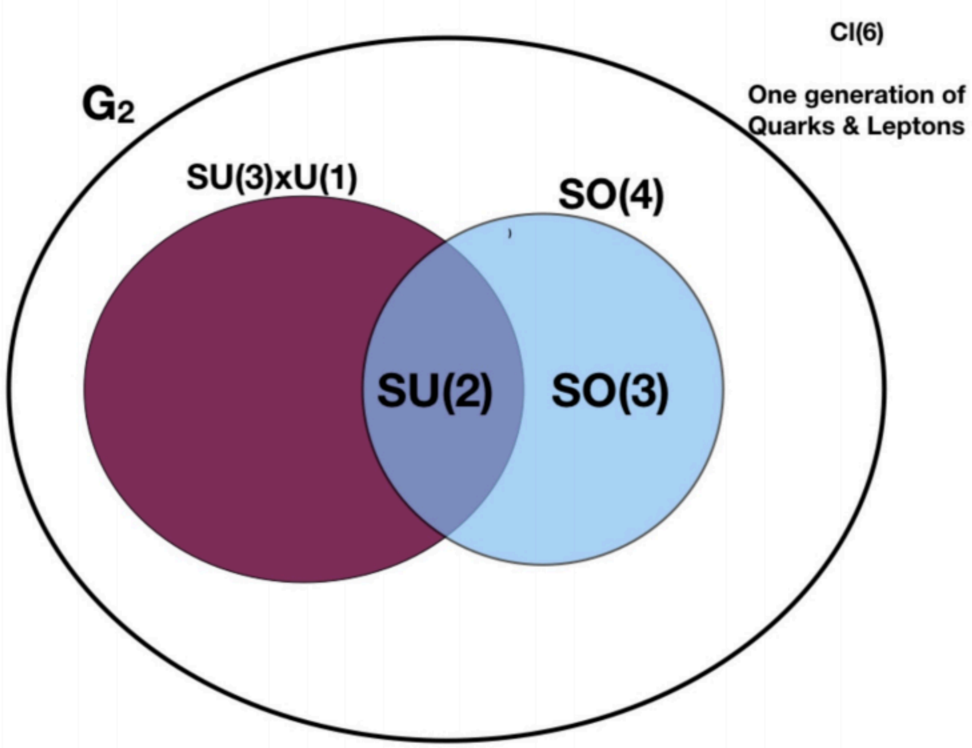
$$\{\alpha_i, \alpha_j\} = 0; \qquad \{\alpha_i^\dagger, \alpha_j^\dagger\} = 0; \qquad \{\alpha_i, \alpha_j^\dagger\} = \delta_{ij}$$

Number operator: $N = \sum_1^3 \alpha_i^\dagger \, \alpha_i$

Continued…

- The ladder operators have a unitary symmetry which acts on them: U(3)
- Rotates lowering (raising) operators into lowering (raising) operators.

$$U(3) = SU(3) \times U(1)/\mathbb{Z}_3$$



- G_2 is the automorphism group of the octonions.
- SU(3) is the element preserver sub-group of G_2
- U(1) is generated by a number operator.

Minimal Left Ideals of Cl(6)

$V = \alpha_1\alpha_2\alpha_3\alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger$ Projector / Idempotent / “Vacuum”

$S^u = (\mathbb{C}\otimes\mathbb{O})V = Cl_6 V$	$S^d = (\mathbb{C}\otimes\mathbb{O})V^* = Cl_6 V^*$
V	V^*
$\alpha_1^\dagger V \quad \alpha_2^\dagger V \quad \alpha_3^\dagger V$	$\alpha_1 V^* \quad \alpha_2 V^* \quad \alpha_3 V^*$
$\alpha_3^\dagger\alpha_2^\dagger V \quad \alpha_1^\dagger\alpha_3^\dagger V \quad \alpha_2^\dagger\alpha_1^\dagger V$	$\alpha_2\alpha_3 V^* \quad \alpha_3\alpha_1 V^* \quad \alpha_1\alpha_2 V^*$
$\alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger V$	$\alpha_1\alpha_2\alpha_3 V^*$

Transformations on ladder operators $\Longrightarrow$ Transformations on basis vectors

Under SU(3) and Under U(1)

Transformations under SU(3)

$V = \alpha_1\alpha_2\alpha_3\alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger$ Projector / Idempotent / “Vacuum”

$S^u = (\mathbb{C}\otimes\mathbb{O})V = Cl_6 V$				$S^d = (\mathbb{C}\otimes\mathbb{O})V^* = Cl_6 V^*$			
	V		$\underline{1}$		V^*		$\underline{1}$
$\alpha_1^\dagger V$	$\alpha_2^\dagger V$	$\alpha_3^\dagger V$	$\underline{3}^*$	$\alpha_1 V^*$	$\alpha_2 V^*$	$\alpha_3 V^*$	$\underline{3}$
$\alpha_3^\dagger\alpha_2^\dagger V$	$\alpha_1^\dagger\alpha_3^\dagger V$	$\alpha_2^\dagger\alpha_1^\dagger V$	$\underline{3}$	$\alpha_2\alpha_3 V^*$	$\alpha_3\alpha_1 V^*$	$\alpha_1\alpha_2 V^*$	$\underline{3}^*$
	$\alpha_3^\dagger\alpha_2^\dagger\alpha_1^\dagger V$		$\underline{1}$		$\alpha_1\alpha_2\alpha_3 V^*$		$\underline{1}$

Transformations on ladder operators $\Longrightarrow$ Transformations on basis vectors

Under SU(3) and Under U(1)

Transformations under U(1)

$$NS^u = nS^u \qquad n = \{0, 1, 1, 1, 2, 2, 2, 3\} \qquad Q \equiv N/3$$

Q	$S^u = (\mathbb{C} \otimes \mathbb{O})V = Cl_6 V$			Rep.	Q	$S^d = (\mathbb{C} \otimes \mathbb{O})V^* = Cl_6 V^*$			Rep.
0		V		$\underline{1}$	0		V^*		$\underline{1}$
$1/3$	$\alpha_1^\dagger V$	$\alpha_2^\dagger V$	$\alpha_3^\dagger V$	$\underline{3}^*$	$-1/3$	$\alpha_1 V^*$	$\alpha_2 V^*$	$\alpha_3 V^*$	$\underline{3}$
$2/3$	$\alpha_3^\dagger \alpha_2^\dagger V$	$\alpha_1^\dagger \alpha_3^\dagger V$	$\alpha_2^\dagger \alpha_1^\dagger V$	$\underline{3}$	$-2/3$	$\alpha_2 \alpha_3 V^*$	$\alpha_3 \alpha_1 V^*$	$\alpha_1 \alpha_2 V^*$	$\underline{3}^*$
1		$\alpha_3^\dagger \alpha_2^\dagger \alpha_1^\dagger V$		$\underline{1}$	-1		$\alpha_1 \alpha_2 \alpha_3 V^*$		$\underline{1}$

Quarks and Leptons [Furey, 2016]

$$Q \equiv N/3 \; = 0, 1/3, 2/3, 1$$

$$S^u = (\mathbb{C} \otimes \mathbb{O})V = Cl_6 V$$

Q				Rep.
0		V		$\underline{1}$
$1/3$	$\alpha_1^\dagger V$	$\alpha_2^\dagger V$	$\alpha_3^\dagger V$	$\underline{3}^*$
$2/3$	$\alpha_3^\dagger \alpha_2^\dagger V$	$\alpha_1^\dagger \alpha_3^\dagger V$	$\alpha_2^\dagger \alpha_1^\dagger V$	$\underline{3}$
1		$\alpha_3^\dagger \alpha_2^\dagger \alpha_1^\dagger V$		$\underline{1}$

ν

$\overline{d}$

u

e^+

$$S^d = (\mathbb{C} \otimes \mathbb{O})V^* = Cl_6 V^*$$

Q				Rep.
0		V^*		$\underline{1}$
$-1/3$	$\alpha_1 V^*$	$\alpha_2 V^*$	$\alpha_3 V^*$	$\underline{3}$
$-2/3$	$\alpha_2 \alpha_3 V^*$	$\alpha_3 \alpha_1 V^*$	$\alpha_1 \alpha_2 V^*$	$\underline{3}^*$
-1		$\alpha_1 \alpha_2 \alpha_3 V^*$		$\underline{1}$

$\overline{\nu}$

d

$\overline{u}$

e^-

<u>Thus far</u> :

- Start with complex octonions.
- Have them act on themselves.
- That generates the Clifford algebra Cl(6)
- Spinors constructed to obtain states,
- which transform as one generation of standard model particles under the unbroken $SU(3)_c \times U(1)_{em}$

Ratios of electric charge are a consequence of symmetries of the octonion algebra.

Do mass ratios also result from some symmetry of the octonion algebra?

Division Algebras and the Weak Symmetry

- Cl(4) and right minimum ideals used to demonstrate correct action of $SU(2)_L$ on one generation of leptons.

  - Furey (2018)

- Two copies of Cl(6): one for electro-color and one for the Weak-Lorentz symmetry: one generation of quarks and leptons.

  - Stoica (2018)

- However, the Cl(4) of weak symmetry can be made from the Cl(6) of the electro-colour symmetry !

- How to describe three fermion generations using division algebras? Need to consider the next two exceptional Lie groups F_4 and E_6.

Gravitation: Left-Right symmetric fermions and sterile neutrinos from complex split biquaternions and bioctonions

$$Cl(0,3) \qquad \{1, e_1, e_2, e_3, e_1e_2, e_2e_3, e_3e_1, e_1e_2e_3\}$$

$$(1, e_1, e_2, e_1e_2), \quad (e_1e_2e_3, e_2e_3, e_3e_1, e_3)$$

$\omega \equiv e_1e_2e_3$ Split complex number $\omega^2 = 1 \;\; \tilde{\omega} = -\omega$

$$(1, e_1, e_2, e_1e_2), \quad \omega(1, -e_1, -e_2, -e_1e_2)$$

Complex Split Biquaternions $Cl(3)$

$$\mathbb{C} \otimes (\mathbb{H} \oplus \mathbb{H}) \cong \mathbb{C} \otimes \mathbf{D} \otimes \mathbf{H} \qquad \mathbf{D} \equiv (1, \omega)$$

RH Leptons

$$\alpha = (e_1 + ie_2)/2$$

$$\overline{\mathcal{V}}_R = \frac{1 + ie_1e_2}{2} \qquad \mathcal{V}_L = \frac{1 - ie_1e_2}{2}$$

$$e_R^+ = \frac{-e_1 + ie_2}{2} \qquad e_L^- = \frac{-e_1 - ie_2}{2}$$

LH Leptons

$$\alpha = \omega(-e_1 - ie_2)/2$$

$$\overline{\mathcal{V}}_L = \frac{1 + ie_1e_2}{2} \qquad \mathcal{V}_R = \frac{1 - ie_1e_2}{2} \qquad e_L^+ = \omega\frac{(e_1 - ie_2)}{2} \qquad e_R^- = \omega\frac{(e_1 + ie_2)}{2}$$

$$Q = \alpha^\dagger \alpha$$

Complex split bioctonions and Cl(7) & L-R symmetric model

Writing Cl(7) as a sum of two copies of C X O with opposite parity.

$$Cl(0,7) \text{ from } \quad (1, e_1, e_2, e_3, e_4, e_5, e_6, e_8) \oplus \omega(1, -e_1, -e_2, -e_3, -e_4, -e_5, -e_6, -e_8)$$

$$e_8 = \overleftarrow{e_1 e_2 e_3 e_4 e_5 e_6} \qquad \omega = \overleftarrow{e_1 e_2 e_3 e_4 e_5 e_6 e_7}$$

$$\mathbb{C} \otimes (\mathbb{O} \oplus \mathbb{O}) \cong \ \mathbb{C} \otimes \mathbf{D} \otimes \mathbf{O} \qquad \mathbf{D} \equiv (1, \omega) \qquad Cl(7)$$

One generation of L-R symmetric fermions

$$\alpha_1 = \frac{-e_5 + ie_4}{2}, \quad \alpha_2 = \frac{-e_3 + ie_1}{2}, \quad \alpha_3 = \frac{-e_6 + ie_2}{2}$$

$$\Omega = \alpha_1 \alpha_2 \alpha_3 \qquad \Omega\Omega^\dagger = \alpha_1 \alpha_2 \alpha_3 \alpha_3^\dagger \alpha_2^\dagger \alpha_1^\dagger$$

$$\alpha_1 = -\omega \frac{-e_5 + ie_4}{2}, \alpha_2 = -\omega \frac{-e_3 + ie_1}{2}, \alpha_3 = -\omega \frac{-e_6 + ie_2}{2}$$

Constructing the L-R basis states in Cl(7) :

U(1)em

Left-handed Particles

$$\overline{\mathcal{V}} = \frac{ie_8+1}{2} \qquad \mathcal{V} = \frac{-ie_8+1}{2}$$

$$V_{ad1} = \frac{(e_5+ie_4)}{2} \qquad V_{d1} = \frac{(e_5-ie_4)}{2}$$

$$V_{ad2} = \frac{(e_3+ie_1)}{2} \qquad V_{d2} = \frac{(e_3-ie_1)}{2}$$

$$V_{ad3} = \frac{(e_6+ie_2)}{2} \qquad V_{d3} = \frac{(e_6-ie_2)}{2}$$

$$V_{u1} = \frac{(e_4+ie_5)}{2} \qquad V_{au1} = \frac{(e_4-ie_5)}{2}$$

$$V_{u2} = \frac{(e_1+ie_3)}{2} \qquad V_{au2} = \frac{(e_1-ie_3)}{2}$$

$$V_{u3} = \frac{(e_2+ie_6)}{2} \qquad V_{au3} = \frac{(e_2-ie_6)}{2}$$

$$V_{e+} = -\frac{(i+e_8)}{2} \qquad V_{e-} = -\frac{(-i+e_8)}{2}$$

U(1)grav

Right-handed Particles

$$\overline{\mathcal{V}} = \frac{ie_8+1}{2} \qquad \mathcal{V} = \frac{-ie_8+1}{2}$$

$$V_{ad1} = \omega\frac{(-e_5-ie_4)}{2} \qquad V_{d1} = \omega\frac{(-e_5+ie_4)}{2}$$

$$V_{ad2} = \omega\frac{(-e_3-ie_1)}{2} \qquad V_{d2} = \omega\frac{(-e_3+ie_1)}{2}$$

$$V_{ad3} = \omega\frac{(-e_6-ie_2)}{2} \qquad V_{d3} = \omega\frac{(-e_6+ie_2)}{2}$$

$$V_{u1} = \frac{(e_4+ie_5)}{2} \qquad V_{au1} = \frac{(e_4-ie_5)}{2}$$

$$V_{u2} = \frac{(e_1+ie_3)}{2} \qquad V_{au2} = \frac{(e_1-ie_3)}{2}$$

$$V_{u3} = \frac{(e_2+ie_6)}{2} \qquad V_{au3} = \frac{(e_2-ie_6)}{2}$$

$$V_{e+} = \omega\frac{(i+e_8)}{2} \qquad V_{e-} = \omega\frac{(-i+e_8)}{2}$$

- Cl(7) gives one generation of L-R symmetric fermions
- Each set has SU(3) x U(1) symmetry.
- $SU(2)_L$ and $SU(2)_R$ act only on left (right) handed particles.
- This is the L-R symmetric extension of SM: $SU(3)c \times SU(2)_L \times SU(2)_R \times U(1)$

Why is the square-root-mass ratio for down / up / electron

(1 : 2/3 : 1/3)

in reverse order of their electric charge ratio

(1/3 : 2/3 : 1)

?

E_6 as a symmetry group for unification of standard model with `would-be-gravity'   [Complexified exceptional Jordan Algebra]

$\tilde{H}_1 = [SU(3) \times SU(3) \times SU(3)]/\mathbf{Z_3}, \tilde{H}_2 = Spin(10)$ $\text{Intersection}: SU(3) \times SU(2)_R \times SU(2)_L \times U(1)$

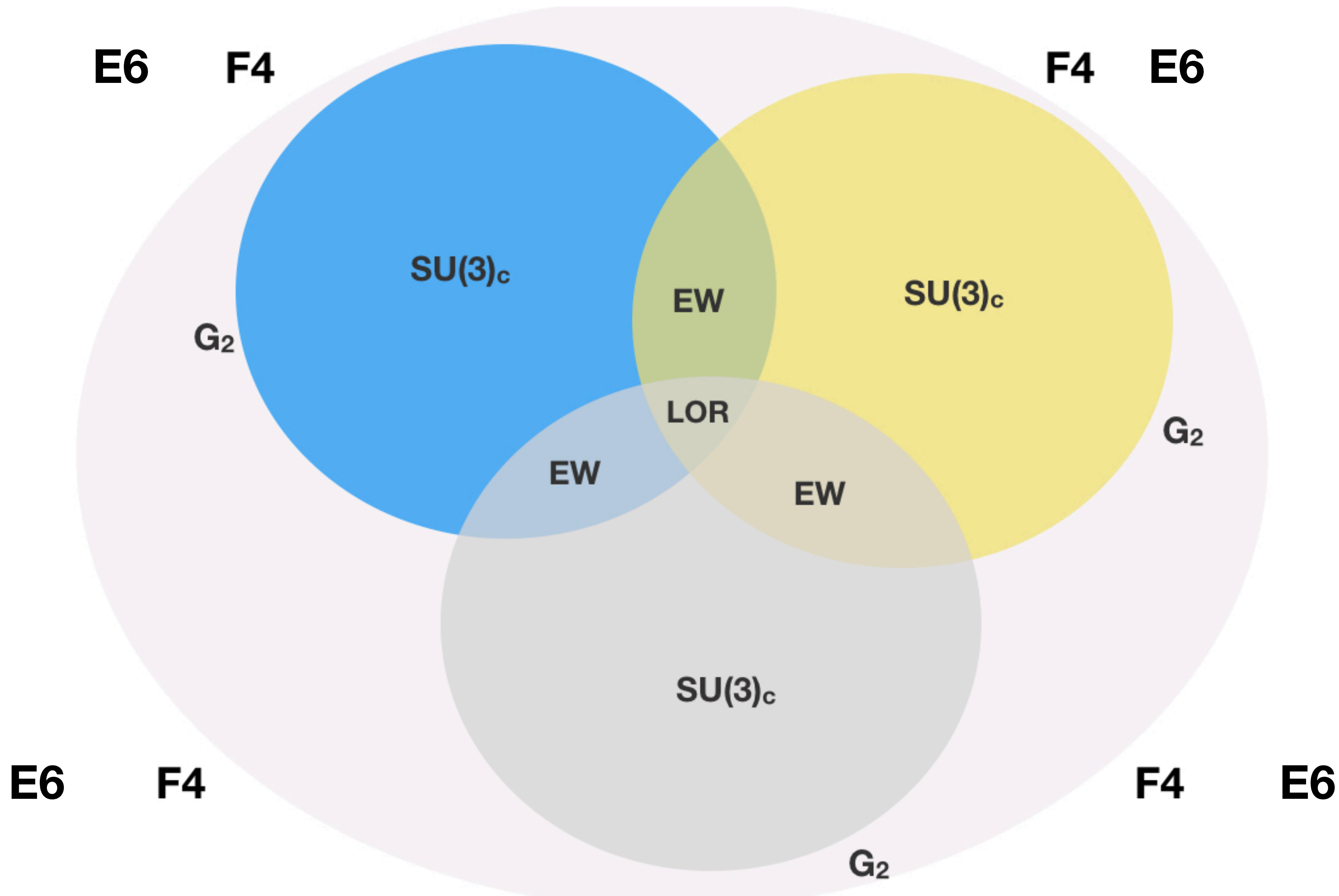
Outside the intersection: SU(3) X SU(3) and Spin(6) ~ SO(1,5) ~ SL(2,H)

Why three fermion generations?

- Automorphism group of sedenions:

$$\mathrm{Aut}(\mathbb{S}) = \mathrm{Aut}(\mathbb{O}) \times S_3$$

S_3 is isomorphic to SO(8): Triality



L-R Symmetry and its Breaking

Square-root mass

- Before: U(1) charge is : Qgem = $(0, 1/3, 2/3, 1)$
- After: Two DIFFERENT U(1) charges: Q and Q_{grav}

$$\ln \alpha_{unif} \propto 2 \ln Q_{gem} \equiv \ln(\alpha\beta) = \ln \alpha + \ln \beta \propto Q + Q_{grav}$$

- Before: Unification of standard model with gravity

Idempotent is Dirac neutrino $(\nu_L + \nu_R)/2$

Theory is L-R Symmetric !!

8D Octonionic Space-time

Dirac Neutrino

LH(anti-down-quark)-RH(positron) $Q_{gem} = 1/3$

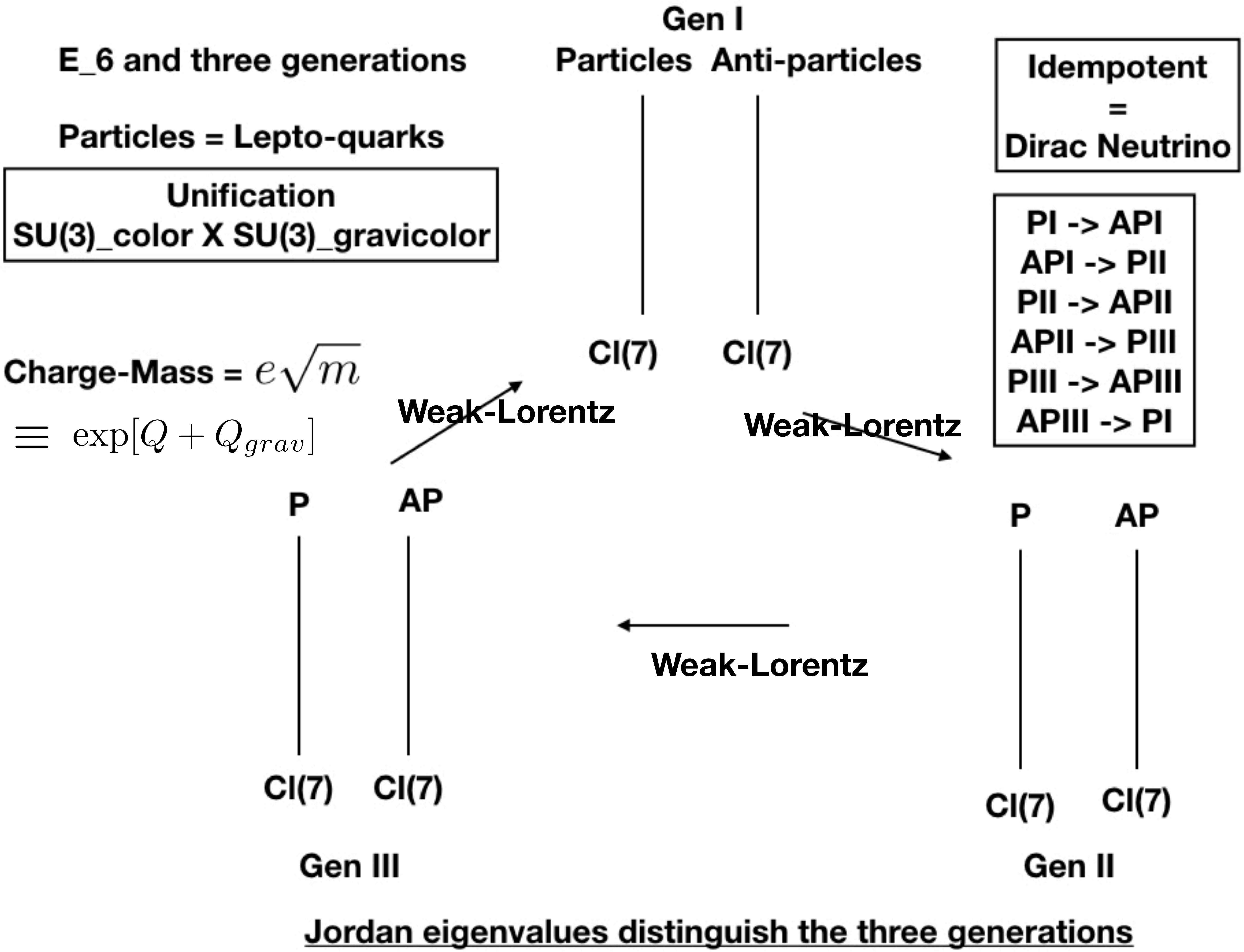
LH(up-quark)-RH(up-quark) $Q_{gem} = 2/3$

LH(positron)-RH(anti-down-quark) $Q_{gem} = 1$

SU(3)_c X SU(3)_g Cl(7)

After L-R symmetry breaking (same as EW symmetry breaking)

LH Fermions	**RH Fermions**
U(1) is electric charge	U(1) is square-root mass
LH Majorana Neutrino	RH Majorana Neutrino
LH anti-down quark 1/3	RH positron 1/3
LH up quark 2/3	RH up quark 2/3
LH positron 1	RH anti-down quark 1
$SU(3)_c \times SU(2)_W \times U(1)_{em}$	$SU(3)_g \times SU(2)_R \times U(1)_g$



E_6 and three generations
Particles = Lepto-quarks
Unification
SU(3)_color X SU(3)_gravicolor
Charge-Mass = $e\sqrt{m}$
$\equiv \exp[Q + Q_{grav}]$
Gen I
Particles
Anti-particles
Cl(7)
Cl(7)
Idempotent
=
Dirac Neutrino
PI -> API
API -> PII
PII -> APII
APII -> PIII
PIII -> APIII
APIII -> PI
Weak-Lorentz
Weak-Lorentz
P
AP
P
AP
Weak-Lorentz
Cl(7)
Cl(7)
Cl(7)
Cl(7)
Gen III
Gen II
Jordan eigenvalues distinguish the three generations

The free particle Lagrangian

Define: $\dot{\tilde{Q}}_B = \frac{1}{L}(i\alpha q_B + L\dot{q}_B);$ $\dot{\tilde{Q}}_F = \frac{1}{L}(i\alpha q_F + L\dot{q}_F);$

$$\mathcal{L} = Tr\left[\frac{L_p^2}{L^2}\left(\dot{\tilde{Q}}_B^\dagger + \frac{L_p^2}{L^2}\beta_1\dot{\tilde{Q}}_F^\dagger\right)\left(\dot{\tilde{Q}}_B + \frac{L_p^2}{L^2}\beta_2\dot{\tilde{Q}}_F\right)\right] =$$

$$\mathcal{L} = Tr\left[\frac{L_p^2}{L^2}\left\{\dot{\tilde{Q}}_B^\dagger\dot{\tilde{Q}}_B + \frac{L_p^2}{L^2}\left(\beta_1\dot{\tilde{Q}}_F^\dagger\dot{\tilde{Q}}_B + \dot{\tilde{Q}}_B^\dagger\beta_2\dot{\tilde{Q}}_F\right) + \frac{L_p^4}{L^4}\beta_1\dot{\tilde{Q}}_F^\dagger\beta_2\dot{\tilde{Q}}_F\right\}\right]$$

Define: $\dot{\tilde{Q}}_{1sed}^\dagger = \dot{\tilde{Q}}_B^\dagger + \frac{L_p^2}{L^2}\beta_1\dot{\tilde{Q}}_F^\dagger;$ $\dot{\tilde{Q}}_{2sed} = \dot{\tilde{Q}}_B + \frac{L_p^2}{L^2}\beta_2\dot{\tilde{Q}}_F$

$$\mathcal{L} = Tr\left[\frac{L_p^2}{L^2}\dot{\tilde{Q}}_{1sed}^\dagger\;\dot{\tilde{Q}}_{2sed}\right]$$

$$\frac{S}{\hbar} = \frac{1}{2}\int\frac{d\tau}{\tau_{Pl}}\;Tr\left[\frac{L_p^2}{L^2}\dot{\tilde{Q}}_{1sed}^\dagger\;\dot{\tilde{Q}}_{2sed}\right]$$

Atoms of Space-time-matter

Mass Ratios & FSC from the Exceptional Jordan Algebra $J_3(\mathbb{O})$

- This is the algebra of 3 x 3 Hermitian matrices with octonionic entries

$$X(\xi, x) = \begin{bmatrix} \xi_1 & x_3 & \tilde{x}_2 \\ \tilde{x}_3 & \xi_2 & x_1 \\ x_2 & \tilde{x}_1 & \xi_3 \end{bmatrix}$$

- Automorphism group is F_4
- For complexified EJA: E_6

- Characteristic equation: $\lambda^3 - Tr(X)\lambda^2 + S(X)\lambda - Det(X) = 0$

$$Tr(X) = \xi_1 + \xi_2 + \xi_3 \ , \qquad Det(X) = \xi_1\xi_2\xi_3 + 2\ Re(x_1x_2x_3) - \sum_{i=1}^{3} \xi_i x_i \tilde{x}_i$$

$$S(x) = \xi_1\xi_2 + \xi_2\xi_3 + \xi_3\xi_1 - x_1\tilde{x}_1 - \tilde{x}_2 x_2 - x_3\tilde{x}_3$$

- We will determine mass ratios from roots of this cubic equation.

$X(\xi, x)$

- Octonionic entries: States representing fermions.
- Diagonal entries: Electric charge.

Octonionic representation for three generations

■ Choose f=1, choose one color, and map states to real octonions:

Gen I (assuming Majorana neutrino)

$$V_\nu = \frac{i}{2}e_7 \longrightarrow \frac{1}{2}e_6$$

$$V_{ad} = \frac{1}{4}e_5 + \frac{i}{4}e_4 \longrightarrow \frac{1}{4}e_5 + \frac{1}{4}e_3$$

$$V_u = \frac{1}{4}e_4 + \frac{i}{4}e_5 \longrightarrow \frac{1}{4}e_4 + \frac{1}{4}e_2$$

$$V_{e^+} = -\frac{i}{4} - \frac{1}{4}e_7 \longrightarrow -\frac{1}{4}e_1 - \frac{1}{4}e_7$$

• Obtain second and third generation states by rotation in the respective fermionic plane, respectively by angles $2\pi/3 \text{ and } 4\pi/3$

Gen II

$$V_{\nu\mu}^M = -\frac{e_6 + \sqrt{3}}{4} \qquad V_{as} = \frac{-e_5 - e_3 - \sqrt{3} - \sqrt{3}e_2}{8}$$

$$V_c = \frac{-e_4 - e_2 - \sqrt{3} - \sqrt{3}e_1}{8} \qquad V_{a\mu} = \frac{e_1 + e_7 + \sqrt{3} - \sqrt{3}e_3}{8}$$

Gen III

$$V_{\nu\tau}^M = -\frac{e_6 - \sqrt{3}}{4} \qquad V_{ab} = \frac{-e_5 - e_3 + \sqrt{3} + \sqrt{3}e_2}{8}$$

$$V_t = \frac{-e_4 - e_2 + \sqrt{3} + \sqrt{3}e_1}{8} \qquad V_{a\tau} = \frac{e_1 + e_7 - \sqrt{3} + \sqrt{3}e_3}{8}$$

Jordan matrices and Jordan eigenvalues

\

$$X_\nu = \begin{bmatrix} 0 & V_\tau & \tilde{V_\mu} \\ \tilde{V_\tau} & 0 & V_\nu \\ V_\mu & \tilde{V_\nu} & 0 \end{bmatrix} \quad X_{e^+} = \begin{bmatrix} 1 & V_{a\tau} & \tilde{V_{a\mu}} \\ \tilde{V_{a\tau}} & 1 & V_{e^+} \\ V_{a\mu} & \tilde{V_{e^+}} & 1 \end{bmatrix} \quad X_u = \begin{bmatrix} \frac{2}{3} & V_t & \tilde{V_c} \\ \tilde{V_t} & \frac{2}{3} & V_u \\ V_c & \tilde{V_u} & \frac{2}{3} \end{bmatrix} \quad X_{ad} = \begin{bmatrix} \frac{1}{3} & V_{ab} & \tilde{V_{as}} \\ \tilde{V_{ab}} & \frac{1}{3} & V_{ad} \\ V_{as} & \tilde{V_{ad}} & \frac{1}{3} \end{bmatrix}$$

- Anti-neutrino has same eigenvalues as the neutrino!

The Jordan Eigenvalues

Neutrinos: Magntitude 3/4	$-\frac{\sqrt{3}}{2}$	**0**	$\frac{\sqrt{3}}{2}$
1/3 Quarks: Mag. 3/8	$\frac{1}{3} - \sqrt{\frac{3}{8}}$	$\frac{1}{3}$	$\frac{1}{3} + \sqrt{\frac{3}{8}}$
2/3 Quarks Mag. 3/8	$\frac{2}{3} - \sqrt{\frac{3}{8}}$	$\frac{2}{3}$	$\frac{2}{3} + \sqrt{\frac{3}{8}}$
Charged Leptons Mag. 3/8	$1 - \sqrt{\frac{3}{8}}$	**1**	$1 + \sqrt{\frac{3}{8}}$

These are all numbers in Base four !!

Charge eigenstates are NOT mass eigenstates

Mass ratios for charged fermions [Casimir of SO(1,5), along with spin]

Mass ratios: Square root of the mass of a charged fermion with respect to the down quark

Down quark	Strange quark	Bottom quark
1	$\dfrac{1+\sqrt{3/8}}{1-\sqrt{3/8}}$	$\dfrac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \dfrac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \dfrac{1+\sqrt{3/8}}{1}$
Up quark	**Charm quark**	**Top quark**
$2/3$	$\dfrac{2}{3} \times \dfrac{\frac{2}{3}+\sqrt{3/8}}{\frac{2}{3}-\sqrt{3/8}}$	$\dfrac{2}{3} \times \dfrac{\frac{2}{3}+\sqrt{3/8}}{\frac{2}{3}-\sqrt{3/8}} \times \dfrac{\frac{2}{3}}{\frac{2}{3}-\sqrt{3/8}}$
Positron	**Muon**	**Tau lepton**
$1/3$	$\dfrac{1}{3} \times \dfrac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \dfrac{1/3+\sqrt{3/8}}{\lvert 1/3-\sqrt{3/8}\rvert}$	$\dfrac{1}{3} \times \dfrac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \dfrac{1+\sqrt{3/8}}{1-\sqrt{3/8}} \times \dfrac{1/3+\sqrt{3/8}}{\lvert 1/3-\sqrt{3/8}\rvert}$

These have the same fundamental status as electric charge ratios

Theory vs Experiment [assuming Majorana neutrino]

Square root mass ratios			
Particles	Theoretical mass ratio	Minimum experimental value	Maximum experimental value
muon/electron	14.10	14.37913078	14.37913090
taun/electron	58.64	58.9660	58.9700
charm/up	23.57	21.04	26.87
top/up	289.26	248.18	310.07
strange/down	4.16	4.21	4.86
bottom/down	28.44	28.25	30.97

Comparison of theoretically predicted square-root mass ratio with experimentally known range

Mass ratios: Majorana neutrino vs. Dirac neutrino

$$V_\nu^M = ie_7/2 \qquad\qquad V_\nu^D = (1+ie_7)/2$$

Square root mass ratios				
Particles	Theoretical mass ratio (D)	Theoretical mass ratio (M)	Minimum experimental value	Maximum experimental value
muon/electron	17.30	14.10	14.37913078	14.37913090
taun/electron	171.27	58.64	58.9660	58.9700
charm/up	3.39	23.57	21.04	26.87
top/up	4.05	289.26	248.18	310.07
strange/down	9.89	4.16	4.21	4.86
bottom/down	218.00	28.44	28.25	30.97

Fine structure constant

$$\frac{e^2}{\hbar c} \equiv \alpha_{fsc} = \frac{L_P^4}{L^4}\alpha^2 \; ; \quad \frac{L_P}{L} = \frac{1}{2}\sqrt{\frac{3}{8}} \; ; \quad \ln\alpha = \frac{1}{3} \times \left(\frac{1}{3} - \sqrt{\frac{3}{8}}\right)$$

$$\alpha_{fsc} = \frac{9}{1024}\exp\left[(1/3 - \sqrt{3/8}) \times 2/3\right] \sim 0.00729714 = \frac{1}{137.04006}$$

Experiment

$$0.0072973525693(11) = 1/137.035999084(21)$$

Spinor spacetime vs. Minkowski spacetime

The Karolyhazy correction

The Koide formula

- Koide observed for the charged leptons that:

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 0.666661(7) \approx \frac{2}{3}$$

- Using the mass-ratios we have found [assuming Majorana neutrino]

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = 0.669163 \approx \frac{2}{3}$$

- Remarkably, assuming Dirac neutrino, the Koide ratio we get is

$$\frac{(1 + \sqrt{3/2})^2 + (1)^2 + (1 - \sqrt{3/2})^2}{3^2} = \frac{2}{3}$$

- Strongly supports: Quantum systems are L-R symmetric and live in spinor space-time, prior to a quantum measurement.

The Cabibbo angle

- Koide proposed the following relation for the Cabibbo angle

$$\tan\theta_c = \sqrt{3}\frac{\sqrt{m_\mu}-\sqrt{m_e}}{2\sqrt{m_\tau}-\sqrt{m_\mu}-\sqrt{m_e}} = 0.225$$

- Our theoretical mass ratios (assuming Majorana neutrino) give 0.222

- Assuming Dirac neutrino, the answer is 0.09

- Can the other angles be predicted?

- What about the other dimensionless constants of the standard model?

Can we understand neutrino masses?

- Jordan eigenvalues assuming Majorana neutrino

$$-\sqrt{3}/2, 0, \sqrt{3}/2$$

Jordan eigenvalues assuming Dirac neutrino

$$-1/2-\sqrt{3}/2, \quad 1, \quad -1/2+\sqrt{3}/2$$

Sterile neutrinos

- Interact only via gravity.

- This cannot be classical gravity.

- When we introduce sterile neutrinos, we bring quantum gravity on par with the standard model interactions.

- Introducing sterile neutrinos requires unification of standard model interactions with gravitation.

  - Only after this is done, can one make reliable predictions for experimental signatures of sterile neutrinos.

What causes Left-Right symmetry breaking?

- Critical entanglement amongst fermions, leading to the quantum - classical transition, and emergence of 4D classical curved space-time. $SU(2)_L$ X $SU(2)_R$ breaks.

- Only classical systems live in 4D spacetime [compactification without compactification].

- Quantum systems obey L-R symmetry (group E_6).

- The mechanism in the early universe is the same as is in the laboratory today: quantum entanglement.

- Does this theory give the theoretical basis for MOND?

Summary

- There must exist a reformulation of quantum field theory without classical time.
- Such a formulation has been developed as a matrix-valued Lagrangian dynamics.
- Elementary particles live in the non-commutative octonionic space-time.
- This space-time determines the standard model, properties of elementary particles, and dimensionless constants of the standard model.
- The Left-Right symmetric extension of the standard model unifies it with gravitation.
- We predict that the neutrino is a Majorana particle.
- We predict three right-handed sterile neutrinos, a dark photon, and two spin-one Lorentz bosons.
- Prior to measurement, a quantum system obeys L-R symmetry.
- Only classical systems live in 4D classical spacetime. Quantum systems live in an octonionic space-time.
- Unification of standard model with gravitation is required at all energy scales, not just at the Planck energy scale.