Constructions of conformal QFTs from Borchers triples: non-local examples

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joint work with Charley Scotford [arXiv:2111.03172]

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  Single (von Neumann) algebra → infinite collection of observable algebras

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  Key tool: Modular theory.
- The map Borchers triples → QFTs might result in pathological/ non-local QFTs (that we want to avoid).
- Will show explicit examples that are "very non-local" (not known before); these are constructed with the help of a deformation procedure inspired by quantization.

Principles of QFT in minimal setting ("spacetime = $\mathbb{R}$")

1. Locality
2. Covariance
3. Vacuum

Borchers triples

Definition

A (one-dimensional) **Borchers triple** $(\mathcal{M}, T, \Omega)$ consists of a von Neumann algebra $\mathcal{M} \subset \mathcal{B}(\mathcal{H})$ and a unitary rep. T of $\mathbb{R}$ on $\mathcal{H}$ s.t.

- T has positive generator. The T-invariant vectors are $\mathbb{C}\Omega$.
- $T(x)\mathcal{M}T(-x) \subset \mathcal{M}$ for $x \geq 0$.
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- $T(x)\mathcal{M}T(-x) \subset \mathcal{M}$ for $x \geq 0$.
- **Ω is cyclic for $\mathcal{M}$ and for $\mathcal{M}'$. —> modular theory!**

This implies that Ω *separates* $\mathcal{M}$: Let $A \in \mathcal{M}, B' \in \mathcal{M}'$.

$$A\Omega = 0 \quad \Rightarrow \quad 0 = B'A\Omega = AB'\Omega \quad \Rightarrow \quad A = 0.$$

A cyclic+separating vector is called a "standard vector".

Modular theory in 5 minutes

$(\mathcal{M}, \Omega)$: von Neumann algebra with standard vector.

- The map

$$S : \mathcal{M}\Omega \ni A\Omega \longmapsto A^*\Omega \in \mathcal{M}\Omega$$

is a well-defined, densely defined, closable antilinear operator.

- Has polar decomposition

$$S = J\Delta^{1/2}.$$

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Tomita's Theorem:

$$J\mathcal{M}J = \mathcal{M}', \qquad \Delta^{it}\mathcal{M}\Delta^{-it} = \mathcal{M}, \quad t \in \mathbb{R}.$$

Modular theory provides us with a natural one-parameter group of automorphisms of $\mathcal{M}$, and a conjugation exchanging $\mathcal{M}$ and $\mathcal{M}'$.

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  Modular theory provides us with a natural one-parameter group of automorphisms of $\mathcal{M}$, and a conjugation exchanging $\mathcal{M}$ and $\mathcal{M}'$.

- In context of Borchers triple: **Borchers' Theorem:**

$$JT(x)J = T(-x), \qquad \Delta^{it}T(x)\Delta^{-it}$$

  The "modular data" J, Δ extend the representation T from $\mathbb{R}$ to the affine group ("$ax + b$ group"). J =TCP, Δ^{it} = dilations

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- May compactify $\mathbb{R}$ to S^1, get all structures of a CFT on S^1.

Theorem ([Longo,Guido,Wiesbrock 98])

- *In standard situation, this construction yields a conformal net on S^1.*
- *There exists a bijection between (strongly additive) conformal nets and standard Borchers triples.*

Degrees of non-locality: the local subspace $\mathcal{H}_{\mathrm{loc}} \subset \mathcal{H}$

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Theorem ([Bostelmann,GL,Morsella 11])

This space is independent of I and invariant under the net $\mathcal{A}$.

Three cases:

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   - Here the construction works as in (1) after restriction to $\mathcal{H}_{\text{loc}}$.
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Does case (3) occur? Answer from 2019: Yes. **[Longo, Tanimoto, Ueda 19]** have free probability construction to get an example of (3).

The algebra at infinity

Let $(\mathcal{M}, T, \Omega)$ be a BT, write $\alpha_x = \mathrm{Ad}T(x)$, $\sigma_t = \mathrm{Ad}\Delta^{it}$, $\mathcal{N} = \alpha_1(\mathcal{M})$.

The algebra at infinity:

$$\mathscr{X} := \bigcap_{t \in \mathbb{R}} \sigma_t(\mathcal{N} \vee J\mathcal{N}J) = \bigcap_{I \in \mathcal{I}} \mathcal{A}(I)'.$$

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Remarks/Lemmas:

- The larger $\mathscr{X}$, the smaller $\mathcal{H}_{\mathsf{loc}}$.
- $\mathscr{X} = \mathbb{C}1 \iff \mathcal{H}_{\mathsf{loc}} = \mathcal{H}$ (standard case).
- $\mathscr{X} = \mathcal{B}(\mathcal{H}) \iff \mathcal{H}_{\mathsf{loc}} = \mathbb{C}\Omega$ (singular case)

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$$\begin{aligned} A \in \mathcal{A}(I) \quad &\Rightarrow A\Omega = AP_\Omega\Omega = P_\Omega A\Omega = \langle\Omega, A\Omega\rangle \cdot \Omega \\ &\Rightarrow \quad A = \langle\Omega, A\Omega\rangle \cdot 1 \end{aligned}$$

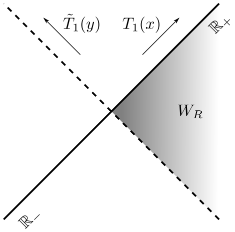
How to construct elements in $\mathscr{X}$

Let $A \in \mathcal{M}$, $B' \in \mathcal{M}'$, and let L be a weak limit point of $\sigma_t(\alpha_1(A)\alpha_{-1}(B'))$ as $t \to -\infty$. Then $L \in \mathscr{X}$. ✎

1d vs 2d theories, holography

Plan: Find a BT such that $\sigma_t(\alpha_1(A)\alpha_{-1}(B')) \to P_\Omega$ weakly as $t \to -\infty$.

- This will rely on a representation $T(x,y)$ of **two-dimensional** translation symmetry



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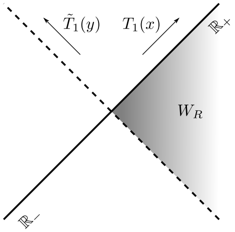

  - Need to extend QFT $(I \mapsto \mathcal{A}(I), U, \Omega)$ to 2d theory
  - Not known in general, but always possible in free field theory (free $U(1)$-current)

$$T_1(x) = e^{ixP}, \quad \tilde{T}_1(y) = e^{iyP'}. \qquad P' = \text{sec.quant.}(P^{-1})$$

Warped convolution in 5 minutes [Buchholz,GL,Summer 2011]

Setup: Hilbert space $\mathcal{H}$ with unitary rep. T of $\mathbb{R}^d$, $d \geq 2$. Write $\alpha_x = \mathrm{Ad}T(x)$ for action on $\mathcal{B}(\mathcal{H})$.

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- Fix a deformation parameter Q, an antisymmetric $(d \times d)$-matrix.
- Define space of smooth vectors $\mathcal{H}^\infty$ and space of smooth operators $\mathcal{C}^\infty \subset \mathcal{B}(\mathcal{H})$ as usual.
- For $A \in \mathcal{C}^\infty$, $\Psi \in \mathcal{H}^\infty$, define deformed ("warped") operator

$$A_Q \Psi := (2\pi)^{-d} \iint e^{-i(p,x)} \alpha_{Qp}(A) T(x) \Psi \, dp \, dx$$

  "deformation quantization for operators"

- **Facts:** A_Q extends to a bounded operator. $A \mapsto A_Q$ is a faithful representation of the Rieffel-deformed C^*-algebra $(\mathcal{C}_Q, \times_Q, \| \cdot \|_Q)$.

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- If $(\mathcal{M}, T, \Omega)$ is a 2d Borchers triple, set

$$\mathcal{M}_Q := \{A_Q : A \in \mathcal{M}^\infty\}''.$$

Then also $(\mathcal{M}_Q, T, \Omega)$ is a Borchers triple if Q is "positive".

In 2d setting, "Q positive" means

$$Q_\kappa = \begin{pmatrix} 0 & \kappa \\ \kappa & 0 \end{pmatrix}, \qquad \kappa \geq 0$$

- $\kappa = 0$ is the undeformed situation = free field theory, standard situation, $\mathcal{H}_{\text{loc}} = \mathcal{H}$, many local fields/observables.

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Theorem ([GL/Scotford 2021])

Consider the free field triple $(\mathcal{M}, T, \Omega)$, and let $\kappa > 0$. Then, for any $A \in \mathcal{M}_{Q_\kappa}$ and any $B' \in \mathcal{M}_{Q_\kappa}{}'$,

$$\underset{t\to-\infty}{\text{w-lim}}\, \sigma_t(\alpha_1(A)\alpha_{-1}(B')) = \omega(AB)P_\Omega + \omega(A)\omega(B)P_\Omega^\perp$$

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proof partially relies on a Riemann-Lebesgue type argument

$$\int dp_1 \cdots dp_n\, dq'\, \overline{\Phi_n(p)}\Psi_n(p) f^+(q')\overline{g^+(q')} \prod_{l=1}^{n} e^{i(p_l, Q_\kappa \Lambda_t q')} \longrightarrow 0 \quad \text{as } t \to -\infty$$

Outlook, open questions, conjectures

Our examples show:

- Singular case (3) exists and can be realized by deformation.
- The local subspace $\mathcal{H}_{\text{loc}}$ varies discontinuously with κ ($\mathcal{H}_{\text{loc}} = \mathcal{H}$ for $\kappa = 0$, but $\mathcal{H}_{\text{loc}} = \mathbb{C}\Omega$ for $\kappa > 0$)

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- These non-local examples complement **local** constructive approaches (inverse scattering programme for 2d integrable models)
- should inform novel **local** constructions.
- currently under investigation in Erlangen: Does this non-local behaviour show up in, for example, entropic or thermal properties? Aim at criteria avoiding the non-local case.