≈ 1989

$$P(x,y) = -\sum\!\!\!\!\!\!\int_a \psi_a(x)\,\overline{\psi_a(y)}$$ kernel of the fermionic projector

observations

(1) contains all information on fermionic wave functions

$$P(x,y) = P^{sea}(x,y) - \sum_a \psi_a(x)\,\overline{\psi_a(y)}$$ particle states

$$+ \sum_b \phi_b(y)\,\overline{\phi_b(x)}$$ antiparticle states

(2) contains all information on bosonic fields A

$$P^{sea}(x,y) = \int \frac{d^4k}{(2\pi)^4}\,(\slashed{k}+m)\,\delta(k^2-m^2)\,\Theta(-k^0)\,e^{-ik(x-y)}$$

$$(i\slashed{\partial}-m)\,P^{sea}(x,y) = 0$$

$$(i\slashed{\partial}+\slashed{A}-m)\,\tilde{P}^{sea}(x,y) = 0$$

$P^{sea}(x,y)$ determines A

(3) contains the causal structure

$P(x,y)$ is singular if $(x-y)^2 = 0$

disregard: Dirac eqn
Maxwell & Einstein eqns
Minkowski space

instead formulate eqns directly with $P(x,y)$

Let M be a discrete set

$x \in M$: $(S_x, \langle .|.\rangle_x)$ spin space

$P(x,y) : S_y \to S_x$

$$\Vert$$

$$\sum_a |\psi_a(x)\rangle\langle\psi_a(y)|$$

$$\psi_a : M \to SM := \dot{\bigcup_{x\in M}} S_x$$

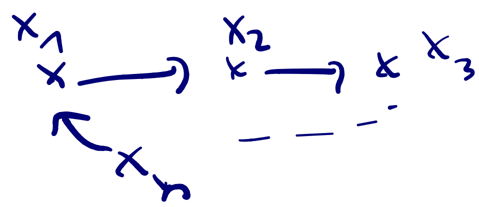
$$x \mapsto \psi_a(x) \in S_x$$

principle of the fermionic projector (≈ 1990)

guiding principles

- variational approach, action principle
- the Dirac sea vacuum should be a critical point
- Maxwell + Einstein eqns should come out in a specific limiting case
- Dirac eqn should hold in a specific limiting case

n-point action



$x_1, \dots, x_n \in M$

closed chain $\quad P(x_1, x_n)\, P(x_n, x_{n-1}) \cdots P(x_3, x_2)\, P(x_2, x_1) : S_{x_1} \circlearrowleft$

$A_{x_1,\dots,x_n} :=$

$\mathcal{L}(A_{x_1,\dots,x_n})$ Lagrangian

$$S = \sum_{x_1,\dots,x_n \in M} \mathcal{L}(A_{x_1,\dots,x_n})$$

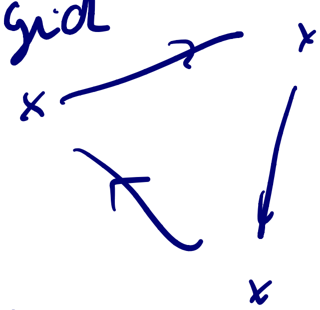
gauge phases $\quad P^{vac}(x,y)\, e^{-i\int_x^y A_j (y-x)^j} + \dots$

$$= P^{vac}(x,y)\, e^{i(\Lambda(y)-\Lambda(x))}$$

Lagrangian is gauge invariant.

$n=1$: $\quad \mathcal{L}(P(x,x))$ too simple

$n \geq 2$: $\quad$ too rigid



flux contribution

Remark 6.2.5 in PFP-book 2006

two-point actions

$$\mathcal{L}(A_{xy}), \qquad A_{xy} = P(x,y)\, P(y,x)$$

closed chain

$$S = \sum_{x,y \in M} \mathcal{L}(A_{xy})$$

first attempt: choose $\mathcal{L}$ as a

$\rightarrow$ polynomial in traces of powers of A_{xy}

e.g. $\quad \mathrm{Tr}(A_{xy}^2) - c\,(\mathrm{Tr}(A_{xy}))^2$

1996
1997 $\Big\}$ preprints in German

basic difficulty: eigenvalues of A_{xy}
involve phase factors

$\approx$ 1999: take absolute values of λ_i^{xy}

two advantages : – phases of λ_i^{xy} drop out

– $\mathcal{S} \geq 0$: minimization possible

$$\mathcal{L}(x,y) = \sum_{i,j} \left(|\lambda_i^{xy}|^p - |\lambda_j^{xy}|^p \right)^2, \quad p \geq 1$$

2002-03 : state stability gives $p = 1$

details in book of 2006.

since then : constraints

generalize to $\mathcal{S}$ measure on local correlation operator