Def: A <u>topological manifold</u> of dimension n is a Hausdorff topological space $\mathcal{M}$ with

- σ-compact (i.e. an at most countable union of compact subsets)

$\forall x \in \mathcal{M} \ \exists\, U \in \mathcal{O}$ with $x \in U$ and

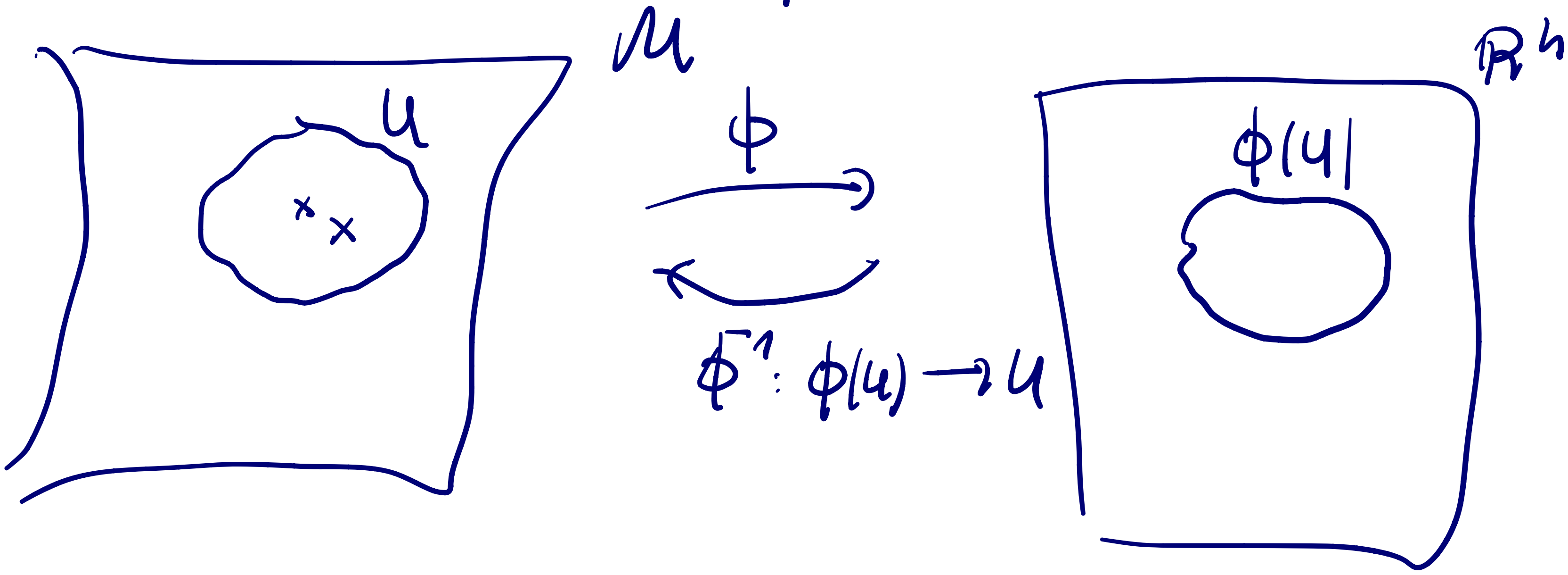
$\phi : U \to \mathbb{R}^n$ s.t.

$\phi(U)$ open in $\mathbb{R}^n$

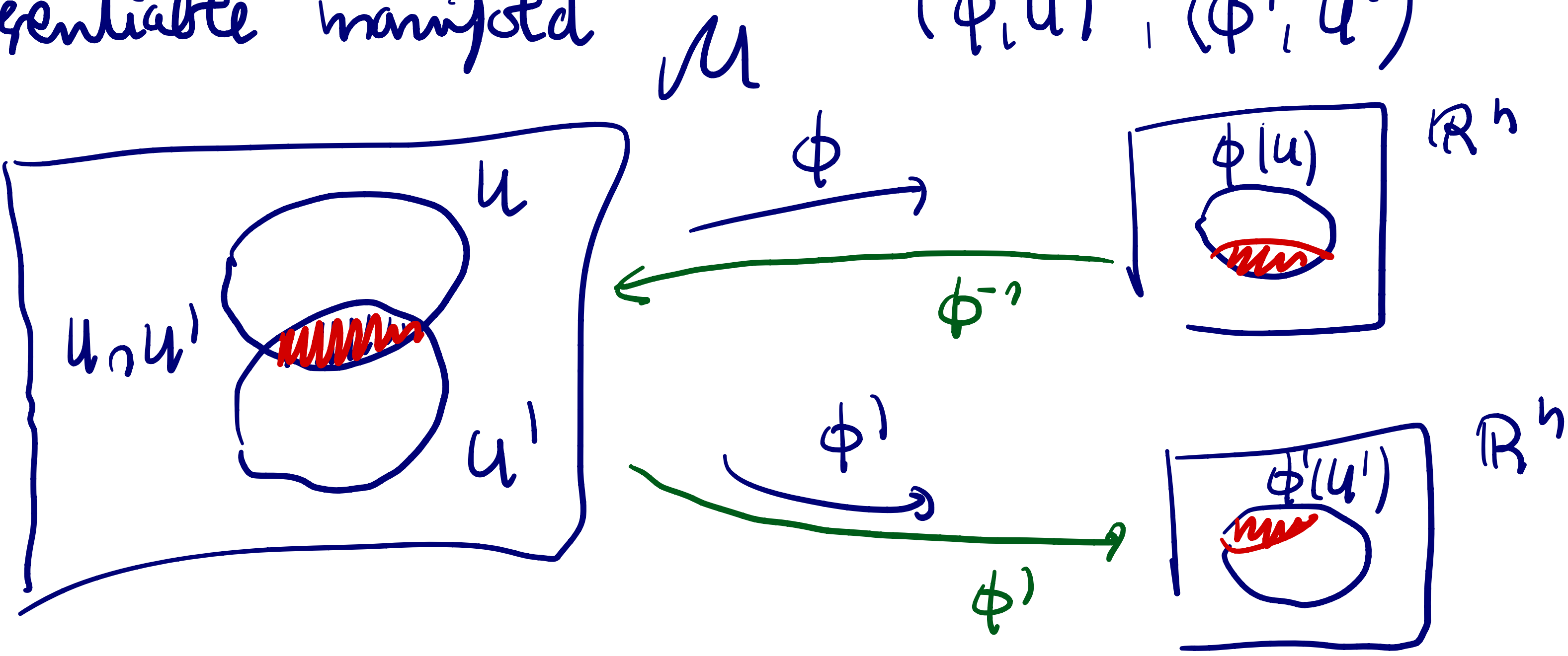
$\phi : U \to \phi(U)$ is a homeomorphism (i.e. bijective and ϕ, ϕ^{-1} are continuous)

(ϕ, U) are called <u>charts</u>

The collection of charts defines an <u>atlas</u> $\mathcal{A}$.



differentiable manifold $\qquad (\phi, U), \ (\phi', U')$



$$\phi|_{U\cap U'} : \quad U\cap U' \xrightarrow{\text{homeomorphism}} \phi(U\cap U') \overset{\text{open}}{\subset} \mathbb{R}^n$$

$$\phi'|_{U\cap U'} : \quad U\cap U' \xrightarrow{\text{homeo}} \phi'(U\cap U') \overset{\text{open}}{\subset} \mathbb{R}^n$$

<u>transition map</u>

$$\phi'|_{U\cap U'} \circ \phi|_{U\cap U'}^{-1} : \quad \begin{array}{ccc} \phi(U\cap U') & \longrightarrow & \phi'(U\cap U') \\ \cap & & \cap \\ \mathbb{R}^n & & \mathbb{R}^n \end{array}$$

is homeomorphism

<u>Def</u>: $(M, \mathcal{A})$ is a differentiable manifold if all transition maps are diffeomorphisms.

$(M, \mathcal{A})$ is a smooth manifold if all transition maps (and their inverses) are smooth.

<u>Def</u>: (topological vector bundle)

Let $\mathcal{B}$ and M be topological spaces and

$\pi : \mathcal{B} \longrightarrow M$ continuous and surjective

Let Y be a (real or complex) vector space and

$G \subset GL(Y)$ a group acting on Y.

assume: $\forall x \in M$ $\exists$ open neighborhood U and a homeomorphism $\phi_U : \pi^{-1}(U) \longrightarrow U \times Y$ (local trivialization) such that the following diagram commutes

$$\begin{tikzcd} \pi^{-1}(U) \arrow[r, "\phi_U"] \arrow[dr, "\pi"'] \arrow[d, phantom, "\cap\ \text{open}"] & U \times Y \arrow[d, "(x,y) \mapsto x"] \\ \mathcal{B} & U \end{tikzcd}$$

Let ϕ_u, ϕ_v be local trivializations and $x \in U \cap V$

$$\phi_u \circ \phi_v^{-1}\Big|_{\{x\} \times Y} = g_{uv}(x) : \quad \{x\} \times Y \circlearrowleft$$

Assume that $g_{uv}(x) \in G$ and depends continuously on x, i.e.

$$g_{uv} : U \cap V \longrightarrow G \quad \text{continuous.}$$

$(\mathcal{B}, \mathcal{M}, (\phi_u))$ is referred to as a <u>topological vector bundle</u> with <u>fibre</u> Y and <u>structure group</u> G.

<u>example:</u> $\mathcal{B} = \mathcal{M} \times Y$.