input: – specify the vacuum
– resulting causal fermion system should
satisfy the EL eqns in the continuum limit
gives a few conditions on the regularizations
we do not put in the form of the interaction
output: – structure of interactions (classical bosonic fields)
– chiral gauge fields
– gauge groups
– coupling ; field eqns
– linearized gravity

1st step (Chapter 3)

only charged leptons (e, μ, τ)
in a Dirac sea configuration called a sector

result: – axial gauge field
– is massive
– no gravity
– no Higgs field

2nd step (Chapter 4)

charged leptons and neutrinos in two sectors
neutrinos break chiral symmetry
results: – $SU(2)_L$ gauge field
– is massive
– linearized gravity

3rd step (Chapter 5)

charged leptons and neutrinos : 2 sectors
+ quark sector 3 × 2 = 6 sectors

8 sectors

results : – effective gauge group

$SU(2)_L \times U(1) \times SU(3)$

massive

– Lagrangian of standard model after spontaneous symmetry breaking
– correct relative charges and couplings
– linearized gravity
– scalar fields corresponding to Higgs, but no field eqns yet.

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one sector

m_e, m_μ, m_τ free parameters

$$P_m(x,y) = \int \frac{d^4k}{(2\pi)^4} (\not k + m)\, \delta(k^2 - m^2)\, \Theta(-k^0)\, e^{-ik(x-y)}$$

$$P^\varepsilon(x,y) = \sum_{\alpha=1}^{3} P^\varepsilon_{m_\alpha}(x,y)$$

interactions : $$P^{aux}(x,y) := \bigoplus_{\alpha=1}^{3} P_{m_\alpha}(x,y)$$

$$\Bigg(i\not\partial - \underbrace{\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}}_{mY}\Bigg) P^{aux}(x,y) = 0$$

sectorial projection $$P(x,y) := \sum_{\alpha,\beta=1}^{3} P^\alpha_\beta(x,y)$$

$$(i\partial\!\!\!/ + \mathscr{B} - m Y_i)\, \tilde{P}^{\text{aux}}(x,y) = 0$$

$\tilde{P}(x,y)$ is again obtained by taking the sectorial projection

most general: $\mathscr{B}$ nonlocal operator

"conclusion": $\mathscr{B}$ must be local

$$\mathscr{B}(x) = \chi_L A\!\!\!/_R + \chi_R A\!\!\!/_L + \Lambda_{ij}\, \sigma^{ij} + \Phi + i\gamma^5\, \Xi$$

<u>gauge invariance</u>, massive gauge fields

local gauge transformations of causal fermion system

$$\psi(x) \longrightarrow U(x)\, \psi(x), \qquad U(x) = U(S_x) \simeq U(2,2)$$

$$U(1) \subset U(2,2) \quad ; \quad \psi(x) \to e^{-i\Lambda(x)}\, \psi(x)$$

$$A \longrightarrow A + i\partial\Lambda$$

local gauge freedom of electrodynamics

$$U(1)_L \times U_R(1) \quad ; \quad \psi(x) \longrightarrow \underbrace{\left(\chi_L\, e^{-i\Lambda_L(x)} + \chi_R\, e^{-i\Lambda_R(x)}\right)}_{U(x)} \psi(x)$$

$$\chi_L = \frac{1}{2}(\mathbb{1} \mp \gamma^5)$$

$$U(x) \not\in U(S_x) \simeq U(2,2)$$

$$U(x)^* = \chi_R\, e^{i\Lambda_L(x)} + \chi_L\, e^{i\Lambda_R(x)} \neq U(x)^{-1}$$

$$\chi_L^* = \chi_R$$

$P(x,y)$ $P(y,x)$ and λ_i^{xy} depend on relative phases $\Lambda_L - \Lambda_R$

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corrections to field eqns on Compton scale

$P(x,y) \sim \quad j \cdot T^{(1)}$

$\quad\quad\quad\quad\quad\quad$ logarithmic pole on light cone

logarithm must be compensated by

microlocal chiral transformation

works only for 3 generations

Compute the coupling constant in examples of simple regularizations.

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two sectors:

nontrivial regularization effects for the right-handed component in the neutrino sector

$\begin{cases} \text{general vacuum states} \\ \text{class of vacuum states} \end{cases}$

$\Rightarrow$ $SU(2)_L$

linearized Einstein eqns $\quad\quad \kappa \sim \varepsilon^2$

usually $\varepsilon\, |\vec{x}-\vec{y}|$

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8 sectors

spontaneous block formation: sectors form pairs

$$\lambda^{xy}_a = e^{-i(\Lambda_L - \Lambda_R)} \lambda^{xy}_{vac,a}$$

drops out of $\mathcal{L}(x,y)$

$+$ (other phase factors) (lower order on the lightcone)

$\Rightarrow$ $|\lambda^{xy}_a|$ depends on relative phases