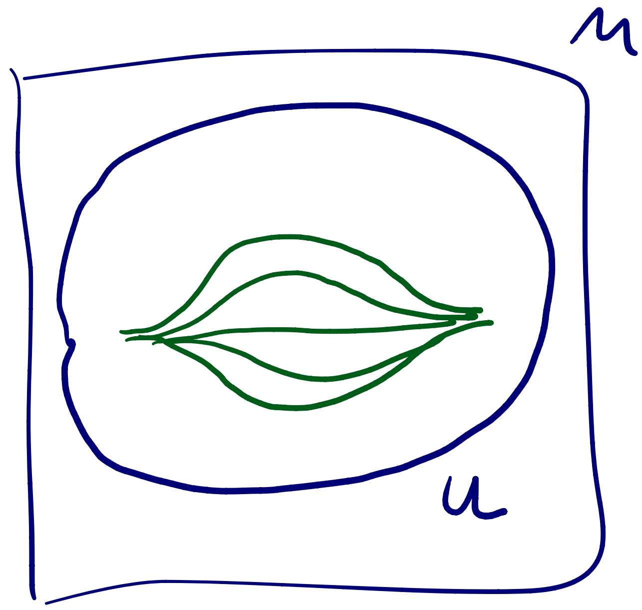
$\mathcal{F}$ smooth manifold

$\mathcal{L} \in C^\infty(\mathcal{F} \times \mathcal{F}, \mathbb{R}^+_0)$

let ρ be a minimizing measure

$M := \operatorname{supp} \rho$ spacetime



jet $\underline{u} = (a, u) \in \mathfrak{J}$

$a \in C^\infty(M, \mathbb{R})$

$u \in \Gamma(M, T\mathcal{F})$

$$\langle \underline{u}, \Delta \underline{v} \rangle(x) = \nabla_{\underline{u}} \Big(\int_M \big(\nabla_{1,\underline{v}} + \nabla_{2,\underline{v}} \big) \mathcal{L}(x,y)\, d\rho(y) - \nabla_{\underline{v}}\, s \Big)$$

linearized field equations

$$\langle \underline{u}, \Delta \underline{v} \rangle(x) = \langle \underline{u}, \underline{w} \rangle(x) \qquad \forall\, \underline{u} \in \mathfrak{J}^{\text{test}}$$
$$\underline{w} \in \mathfrak{J}^*$$

Def: Let $U \subset M$ be open and $I \subset \mathbb{R}$ a compact interval. Let $\eta \in C^\infty(I \times U, \mathbb{R})$ with $0 \leq \eta \leq 1$ which for every $t \in I$ has the following properties:

(i) $\theta(t, .) := \partial_t \eta(t, .)$ is ≥ 0 and compactly supported in U

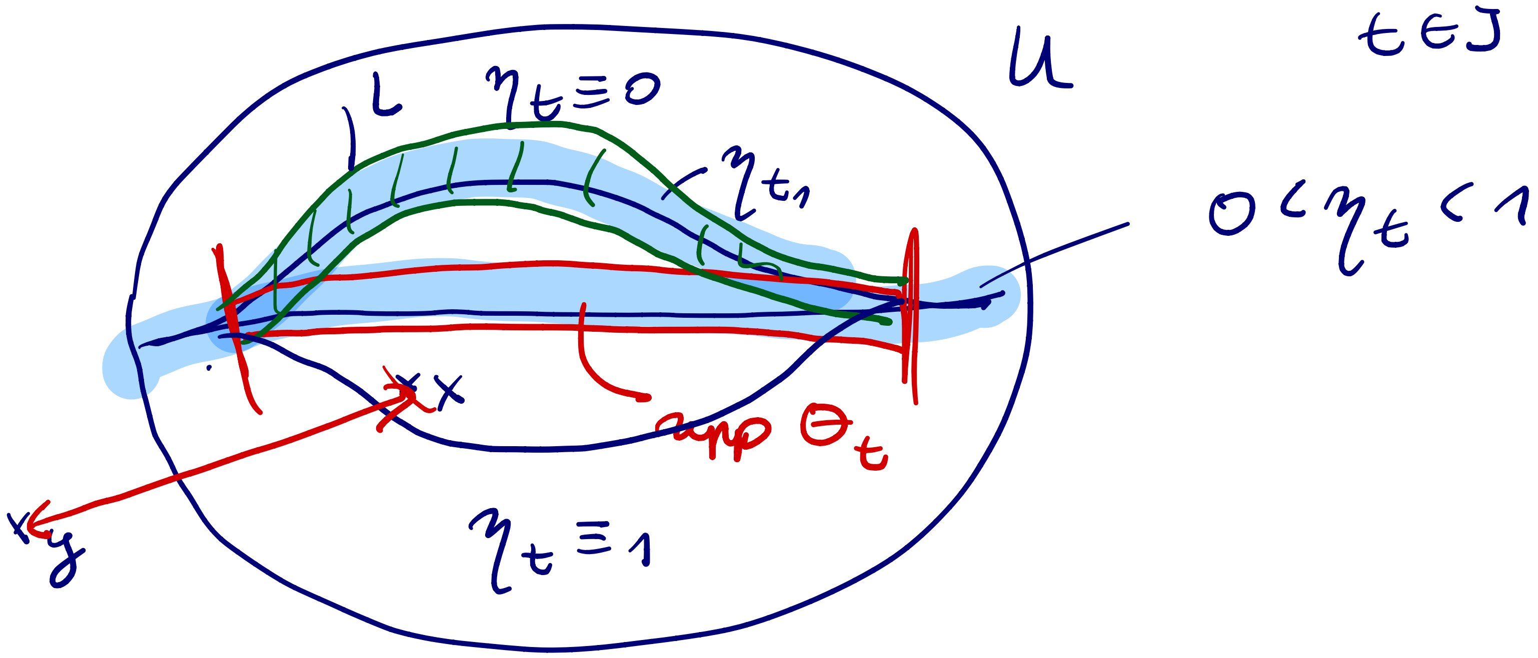
(ii) $\forall\, x \in \operatorname{supp} \theta(t, .)$ $\quad \forall\, y \in M \setminus U$,

$$\mathcal{L}(x,y) = 0 \quad \text{and}$$

$$\nabla \mathcal{L}(x,y) = \nabla^2 \mathcal{L}(x,y) = 0.$$

We also write $\eta_t(x) = \eta(t,x)$ and $\theta_t(x) = \theta(t,x)$

The family $(\eta_t)_{t\in J}$ is referred to as a <u>local foliation</u>



$L := \bigcup_{t\in J} \operatorname{supp} \Theta_t$ compact $\subset U$

$d\rho_t := \Theta_t(x)\, d\rho$