external potential in Minkowski space

$(i\partial\!\!\!/ + \mathcal{B} - m)\,\psi = 0$

- $\mathcal{B}$ smooth and decays suitably for large times

  $\Rightarrow$ strong mass oscillation property holds on $\mathcal{H}^\infty = C^\infty_{sc,0}(M, SM) \cap \mathcal{H}$

  $\langle p\psi | p\phi \rangle = \int_y (\psi_m | \underline{S_m} \phi_m)_m \, dm$

- if $\mathcal{B}$ is not too large, $P(x,y)$ is well-defined and is of Hadamard form

methods of scattering theory

Lippmann-Schwinger eqn

integrate by parts in m

$$\int_y \hat{\psi}_m \, e^{-i\omega t} \, dm \qquad ; \; \omega(m) = \sqrt{k^2+m^2}$$

$$A = \int_y \hat{\psi}_m \left(\frac{1}{-it} \frac{\partial}{\partial \omega} e^{-i\omega t} \right) dm$$

$$\omega \, d\omega = m \, dm$$

$$\Rightarrow \frac{\partial}{\partial \omega} = \frac{\omega(m)}{m} \frac{\partial}{\partial m}$$

$$\Rightarrow A = \frac{1}{-it} \int_y \hat{\psi}_m \frac{\omega(m)}{m} \frac{\partial}{\partial m} e^{-i\omega(m) t} \, dm$$

integrate by parts

$$= \frac{1}{it} \underbrace{\int_y \frac{\partial}{\partial m}\left(\hat{\psi}_m \frac{\omega(m)}{m} \right)}_{|\cdots| \,\leq\, |y| \sup \left| \frac{\partial}{\partial m}(\quad) \right|} e^{-i\omega(m) t} \, dm$$

Rindler spacetime
de Sitter
Schwarzschild
plane wave in Minkowski space

methods: case by case, use symmetries

general result:

symmetries of spacetime ← Lie group $\mathcal{G}$ which acts locally as an isomorphism of SM

$\Downarrow$

symmetries of S_m ← corresponding U of $\mathcal{H}_m$

$$US_mU^{-1} = S_m$$

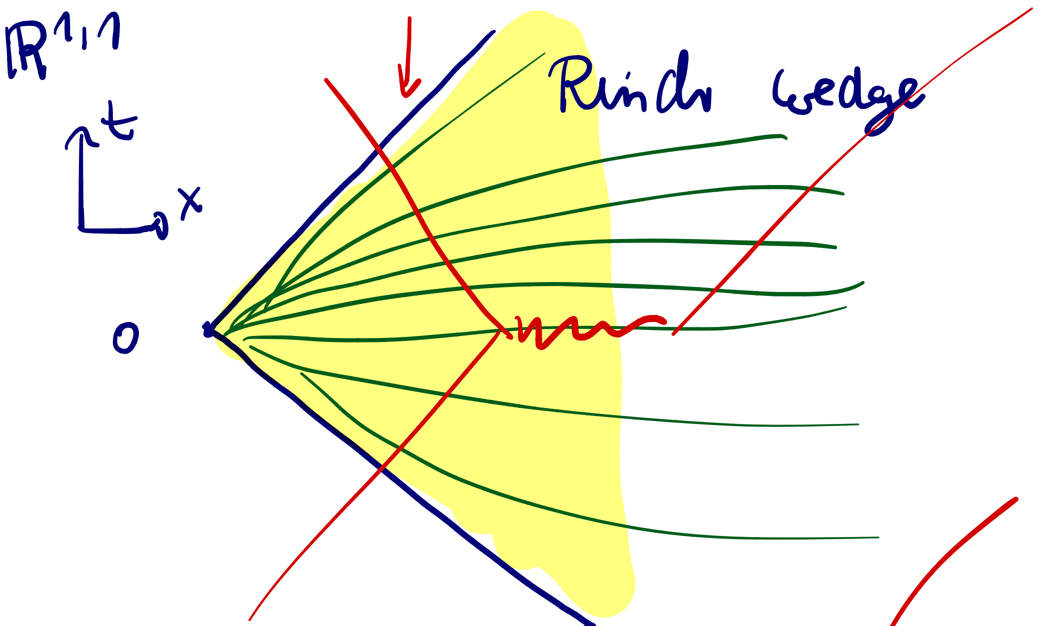
& A s.a. on $\mathcal{H}_m$

$$[A, S_m] = 0$$

<u>Note</u>: If time direction is reversed, then $S_m \longrightarrow -S_m$

<u>event horizon</u>:

2-dim Rindler spacetime



densely defined unbounded operator

$$\langle \psi_m | \phi_m \rangle = (\psi_m | S_m \phi_m)_m \quad , \forall \psi_m, \phi_m \in C^\infty_{sc}(M, SM) \cap \mathcal{H}_m$$

Schwarzschild: exterior region $\mathcal{M}$

mass oscillation properties do not hold.

instead: $\Psi, \Phi \in C^\infty_{sc,0}(\mathcal{M}\times\mathcal{I}, S\mathcal{M}) \cap \mathcal{H}$

$$
\langle \mathfrak{p}\Psi | \mathfrak{p}\Phi\rangle = \int_{\mathcal{I}} (\Psi_m | S_m \Phi_m)\, dm
$$

$$
+ \frac{i}{\pi} \int_{\mathcal{I}} dm \int_{\mathcal{I}} dm' \; \frac{\mathrm{PP}}{m-m'} \; \mathcal{D}(\Psi_m, \Phi_{m'})
$$

related to the flux
of the Dirac waves through event horizon

S_m symmetric and bounded.