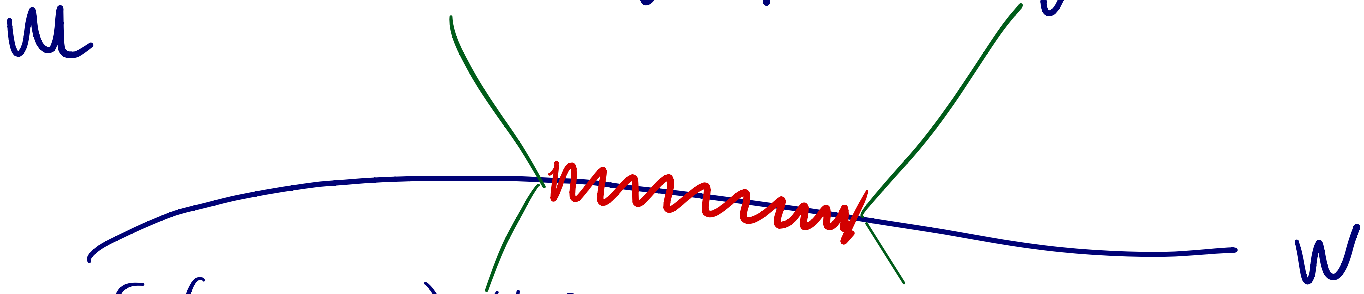
$(\mathcal{M}, g)$

$(\mathcal{D} - m)\,\psi = 0$ with $m \in \mathcal{I} = (m_a, m_b)$ and $m_a, m_b > 0$.

Let $\mathcal{N}$ be a Cauchy surface, e.g. $\mathcal{N} = \{t = \text{const}\}$



$$\begin{cases} (\mathcal{D} - m)\,\psi(x, m) = 0 \\ \psi|_{\mathcal{N}}(x, m) = \psi_0(x, m) \in C^\infty_0(\mathcal{N} \times \mathcal{I}, S\mathcal{M}) \end{cases}$$ Cauchy problem

$$\psi_0(\,\cdot\,, m) \in C^\infty_0(\mathcal{N}, S\mathcal{M})$$

$$\psi_0 \in C^\infty_0(\mathcal{N} \times \mathcal{I}, S\mathcal{M})$$

$$\psi(\,\cdot\,, m) \in C^\infty_{sc}(\mathcal{M}, S\mathcal{M})$$

and smooth and compactly supported in m

$$\psi \in C^\infty_{sc,0}(\mathcal{M} \times \mathcal{I}, S\mathcal{M}).$$

$$\psi_m(\,\cdot\,) = \psi(\,\cdot\,, m)$$

$\psi = (\psi_m)_{m \in \mathcal{I}}$ is a family of solutions in $C^\infty_{sc,0}$

$$(\psi | \phi) := \int_{\mathcal{I}} (\psi_m | \phi_m)_m \, dm$$

defines a scalar product on such families of solutions

Taking the completion gives the Hilbert space

$$(\mathcal{H}, \|\cdot\|).$$

Def let $\mathcal{H}^\infty \subset C^\infty_{sc,0}(\mathcal{M}\times J, S\mathcal{M}) \cap \mathcal{H}$ be a subspace with the following properties:

(i) $\mathcal{H}^\infty$ is invariant under multiplication by smooth functions in the mass parameter, i.e.

$$(\eta(m)\,\psi_m)_{m\in J} \in \mathcal{H}^\infty \qquad \forall\, \psi \in \mathcal{H}^\infty \text{ and } \eta \in C^\infty(J,\mathbb{R})$$

(ii) $\forall\, m \in J$, the set $\mathcal{H}^\infty_m := \{\psi_m \mid \psi \in \mathcal{H}^\infty\}$ is dense in $\mathcal{H}_m$.

This subspace is referred to as the <u>domain</u> for the mass oscillation properties.

Introduce

$$T : \mathcal{H} \to \mathcal{H}, \qquad (T\psi)_m = m\,\psi_m$$

is bounded and symmetric $\quad (\|T\| \le m_b)$

Moreover, from (i)

$$T : \mathcal{H}^\infty \to \mathcal{H}^\infty$$

$$\mathfrak{p} : \mathcal{H}^\infty \longrightarrow C^\infty_{sc}(\mathcal{M}, S\mathcal{M})$$

no Dirac solutions

$$(\mathfrak{p}\psi)(x) := \int_J \psi_m(x)\, dm$$

Def: The Dirac operator $\mathcal{D}$ has the <u>weak mass oscillation property</u> if

(i) $\forall\, \psi, \phi \in \mathcal{H}^\infty, \qquad \langle \mathfrak{p}\psi | \mathfrak{p}\phi \rangle_x \in L^1(\mathcal{M}, \overset{d^4x}{\overset{\parallel}{d\mu_{\mathcal{M}}}})$

Moreover, $\forall\, \psi \in \mathcal{H}^\infty \;\exists\; c = c(\psi)$ s.t.

$$|\langle \mathfrak{p}\psi | \mathfrak{p}\phi \rangle| \le c\, \|\phi\| \qquad \forall\, \phi \in \mathcal{H}^\infty$$

(ii) $\forall\, \psi, \phi \in \mathcal{H}^\infty$

$$\langle \rho T\psi \mid \rho\phi\rangle = \langle \rho\psi \mid \rho T\phi\rangle .$$

Remark: $\mathcal{D}\psi := (\mathcal{D}\psi_m)_{m\in J} = (m\psi_m)_{m\in J} = T\psi$

Def The Dirac operator $\mathcal{D}$ has the strong mean oscillation property if $\exists\, c>0$ s.t.

$$\left|\langle \rho\psi \mid \rho\phi\rangle\right| \leq c \int_J \|\psi_m\|_m \|\phi_m\|_m \, dm$$

$$\forall \psi, \phi \in \mathcal{H}$$