$$
\begin{aligned}
\chi_L\, P(x,y) = &\frac{i}{2}\,\chi_L\, e^{-i\Lambda_L^{xy}}\, \not{\xi}\, T^{(-1)} && \text{(B.2.2)}\\
&-\frac{1}{2}\,\chi_L\, \not{\xi}\, \xi_i \int_x^y [0,0\,|\,1]\, j_L^i\, T^{(0)} && \text{(B.2.3)}\\
&+\frac{1}{4}\,\chi_L\, \not{\xi} \int_x^y F_L^{ij}\, \gamma_i\gamma_j\, T^{(0)} && \text{(B.2.4)}\\
&-\chi_L\, \xi_i \int_x^y [0,1\,|\,0]\, F_L^{ij}\, \gamma_j\, T^{(0)} && \text{(B.2.5)}\\
&-\chi_L\, \xi_i \int_x^y [0,1\,|\,1]\, \not{\partial} j_L^i\, T^{(1)} && \text{(B.2.6)}\\
&-\chi_L \int_x^y [0,2\,|\,0]\, j_L^i\, \gamma_i\, T^{(1)} && \text{(B.2.7)}\\
&-im\,\chi_L\, \xi_i \int_x^y Y A_R^i\, T^{(0)} && \text{(B.2.8)}\\
&+\frac{im}{2}\,\chi_L\, \not{\xi} \int_x^y (Y \not{A}_R - \not{A}_L Y)\, T^{(0)} && \text{(B.2.9)}\\
&+im\,\chi_L\, \xi_i \int_x^y [0,0\,|\,1]\, Y\, j_R^i\, T^{(1)} && \text{(B.2.10)}\\
&-\frac{im}{2}\,\chi_L \int_x^y [1,0\,|\,0]\, Y F_R^{ij} \gamma_i\gamma_j\, T^{(1)} && \text{(B.2.11)}\\
&-\frac{im}{2}\,\chi_L \int_x^y [0,1\,|\,0]\, F_L^{ij} \gamma_i\gamma_j\, Y\, T^{(1)} && \text{(B.2.12)}\\
&+im\,\chi_L \int_x^y [0,1\,|\,0]\, \Big(Y (\partial_j A_R^j) - (\partial_j A_L^j)\, Y \Big)\, T^{(1)} && \text{(B.2.13)}\\
&+\frac{m^2}{2}\,\chi_L\, \not{\xi}\, \xi_i \int_x^y [1,0\,|\,0]\, YY A_L^i\, T^{(0)} && \text{(B.2.14)}\\
&+\frac{m^2}{2}\,\chi_L\, \not{\xi}\, \xi_i \int_x^y [0,1\,|\,0]\, A_L^i YY\, T^{(0)} && \text{(B.2.15)}\\
&-m^2\,\chi_L\, \xi_i \int_x^y [0,0\,|\,1]\, YY F_L^{ij} \gamma_j\, T^{(1)} && \text{(B.2.16)}\\
&-m^2\,\chi_L\, \xi_i \int_x^y [0,2\,|\,0]\, F_L^{ij} \gamma_j\, YY\, T^{(1)} && \text{(B.2.17)}\\
&+m^2\,\chi_L \int_x^y [1,0\,|\,0]\, YY \not{A}_L\, T^{(1)} && \text{(B.2.18)}\\
&-m^2\,\chi_L \int_x^y [0,0\,|\,0]\, Y \not{A}_R\, Y\, T^{(1)} && \text{(B.2.19)}\\
&+m^2\,\chi_L \int_x^y [0,1\,|\,0]\, \not{A}_L\, YY\, T^{(1)} && \text{(B.2.20)}\\
&+\not{\xi}\,(\deg < 1) + (\deg < 0) + \mathcal{O}(A_{L/R}^2)\,,
\end{aligned}
$$

$$
\begin{aligned}
P(x,y) \asymp \frac{i}{48}\, R_{jk}\, \xi^j \xi^k\, \not{\xi}\, T^{(-1)} && (4.5.3) \\
+ \frac{i}{24} R_{jk}\, \xi^j \gamma^k\, T^{(0)} + \not{\xi}\,(\deg \leq 1) + (\deg < 1)\,, && (4.5.4)
\end{aligned}
$$

$$
\begin{aligned}
T_{m^2}(x,y) = -\frac{1}{8\pi^3} &\left(\frac{\mathrm{PP}}{(y-x)^2} + i\pi\delta\big((y-x)^2\big)\, \epsilon\big((y-x)^0\big) \right) \\
&+ \frac{m^2}{32\pi^3} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\,(j+1)!}\, \frac{\big(m^2(y-x)^2\big)^j}{4^j} \\
&\qquad \times \Big(\log\big|m^2(y-x)^2\big| + c_j + i\pi\, \Theta\big((y-x)^2\big)\, \epsilon\big((y-x)^0\big) \Big)
\end{aligned} \tag{2.2.3}
$$

$$
T_a^{\mathrm{reg}}(x,y) = T_a(x,y) - \frac{a}{32\pi^3}\, \log|a| \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\,(j+1)!}\, \frac{(a\xi^2)^j}{4^j} \tag{2.2.117}
$$

$$
T^{(l)} = \left(\frac{d}{da}\right)^l T^{\mathrm{reg}}_{a\,|\,a=0} \tag{2.2.118}
$$

$$
P(x,y) = \lim_{\varepsilon \searrow 0}\, i\partial\!\!\!/_x \left(\frac{U(x,y)}{\sigma_\varepsilon(x,y)} + V(x,y)\, \log\sigma_\varepsilon(x,y) + W(x,y) \right),
$$

Hadamard expansion / light-cone expansion

$$P(x,y) = \lim_{\varepsilon \searrow 0} i\partial\!\!\!/_x \left(\frac{U(x,y)}{\sigma_\varepsilon(x,y)} + V(x,y)\, \log \sigma_\varepsilon(x,y) + W(x,y) \right),$$

$$T_a(x,y) = \int \frac{d^4k}{(2\pi)^4} \delta(k^2 - a)\; \Theta(-k^0)\; e^{-ik(x-y)}$$

---

goal: analyse the EL-eqns of the causal action

$\nabla \mathcal{L}(x,y)$

$$A_{xy} = P(x,y)\, P(y,x)$$

compute its eigenvalues

In order to make sense of this pointwise product, we need to introduce an ultraviolet regularization

$$\tilde{A}^\varepsilon_{xy} = \tilde{P}^\varepsilon(x,y)\; \tilde{P}^\varepsilon(y,x)$$

1) regularized light-cone expansion

general result:

- smooth functions remain unchanged
- $T^{(n)} \xrightarrow{\text{replace}} T^{(n)}_{[p]},\ T^{(n)}_{\{p\}}$

depends on the pair of m

$T^{(n)}_\bullet$ are smooth, but singular as $\varepsilon \to 0$

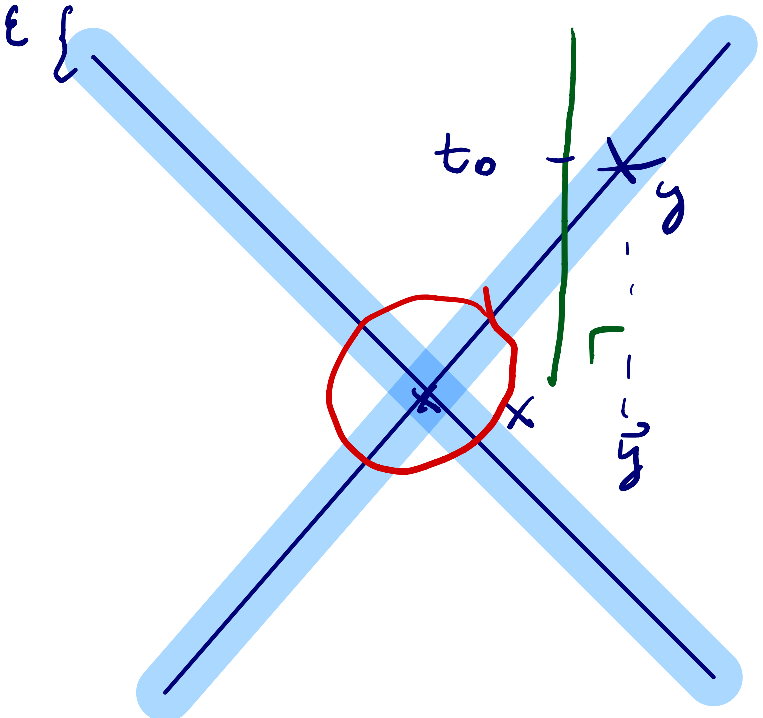
depend on the unknown microscopic structure of spacetime

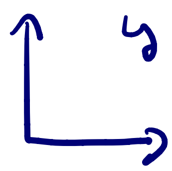
- analyse composite expressions symbolically, using certain computation rules
- get a finite (small) number of free parameters

- EL eqns $\Longrightarrow$ classical field eqns

$P^\varepsilon(x,y)$ is smooth;
thus pointwise computations can be performed

weak evaluation on the light cone:





$$R^\varepsilon(x,y) \; \big(= \nabla \mathcal{L}(x,y), \ldots\big)$$

$$R^\varepsilon(x,.) \xrightarrow[\text{loc}]{\varepsilon \to 0} R(x,.) \text{ smooth}$$

$$\Gamma = (t, \vec{y}) \qquad \vec{y} \text{ fixed} \qquad \vec{y} \neq \vec{x}$$

regularization parameter

$$\int_\Gamma \eta(y)\, R^\varepsilon(x,y)\, dt = \eta(t,\vec{y}) \,\frac{c_{reg}}{(i\,|\vec{y}-\vec{x}|)^L}\; \frac{\log^r(\varepsilon|\vec{y}-\vec{x}|)}{\varepsilon^{L-1}}$$

$$+ \text{ (error terms)}$$

$\xi = (y-x)$;
$$P^\varepsilon(x,y) \asymp i \,\xi\!\!\!/ \; T_0^{(1)} + \ldots$$

$$A^\varepsilon(x,y) \asymp \underbrace{\xi_i\, \xi^i}_{= \xi^2 \text{ vanishes on the light cone!}} \ldots + \ldots$$

taking this into account leads to the so-called contraction rules:

outer factors: are contracted with other smooth tensors (not other ξ's);
no regularization necessary

inner factors are contracted with other factors $\mathcal{G}$

$$\mathcal{G}\, T^{(n)} \xrightarrow{\text{regularized}} \mathcal{G}^{(n)}_{\bullet} \qquad T^{(n)}_{[p]} \text{ or } T^{(n)}_{\{p\}}$$

$$\langle \mathcal{G}^{(n)}_{[p]} , \mathcal{G}^{(m)}_{[q]} \rangle = \frac{1}{2} \left(z^{(n)}_{[p]} + z^{(m)}_{[q]} \right)$$

$\Downarrow$ $\mathcal{G}^2$ with regularization

$$z^{(n)}_{[p]} \cdot T^{(n)}_{[p]} = -4 \left(T^{(n+1)}_{[p]} + T^{(n+2)}_{\{p\}} \right)$$

$\overline{\mathcal{G}^{[n]}_{[p]}}$ complex conjugation has to be taken into account

$$\langle \mathcal{G}^{(n)}_{[p]} , \overline{\mathcal{G}^{(m)}_{[q]}} \rangle = \frac{1}{2} \left(z^{(n)}_{[p]} + \overline{z^{(m)}_{[q]}} \right)$$

$$R(x,y) = \sum (\text{smooth}) \underbrace{\frac{T^{(b_1)} \cdots T^{(a_p)}\, \overline{T^{(b_1)} \cdots T^{(a_q)}}}{T \cdots T \; \overline{T \cdots T}}}_{\text{simple fraction}}$$

---

integration-by-parts rules
give equations between regularization parameters
proved that these are the only equations between c_{ij}'s.
but inequality constraints remain

$$|T^{(-1)}_{[0]}|^2 \geq 0$$

In order to make sure that all inequality constraints are respected, one should realize the desired regularization parameters in explicit examples.

---

error of formalism:

consider only leading contribution in $\dfrac{\varepsilon}{|\vec{y}-\vec{x}|}$

gravitational field

linearized gravity: $K \sim S^2 \approx \varepsilon^2$

ε^2-term is taken into in the so-called

L-formalism.