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Title: Ramified local isometric embeddings of singular Riemannian metrics

Summary: We shall discuss the existence of local isometric embeddings into Euclidean space for analytic Riemannian metrics g defined on a domain~$U\subset \R^n$, which are singular in the sense that the determinant of the metric tensor is allowed to vanish in a rather precise sense at an isolated point. The metrics being thus considered are degenerate, but in a fairly mild sense and in exactly one direction. Specifically, we show that under suitable technical assumptions, there exists a local analytic isometric embedding $\bu$ from (U',Π^*g) into Euclidean space $\mathbb{E}^{(n^2+3n-4)/2}$, where $\Pi:U' \to U\backslash\{0\}$ is a finite Riemannian branched cover of a deleted neighborhood of the origin. Our result can thus be thought of as a generalization of the classical Cartan-Janet Theorem to the singular setting in which the metric tensor is degenerate at an isolated point. Our proof uses Leray's ramified Cauchy-Kovalevskaya Theorem for analytic differential systems, in the form established by Choquet-Bruhat for non-linear systems. This is joint work with Alberto Enciso (ICMAT, Madrid).