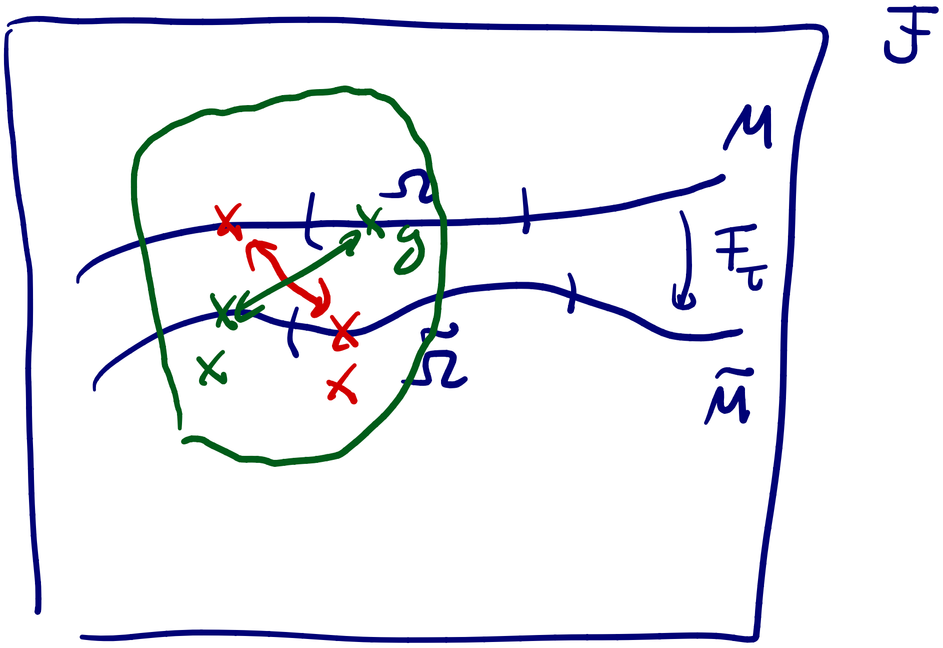
let $\left\{ \begin{array}{ll} \rho & \text{(vacuum)} \\ \tilde{\rho} & \text{(interacting system)} \end{array} \right\}$ be measures on $\mathcal{F}$

let $\Omega \subset M := \operatorname{supp} \rho$ be compact
$\tilde{\Omega} \subset \tilde{M} := \operatorname{supp} \tilde{\rho}$

nonlinear surface layer integral

$$\gamma^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \rho) := \Big(\int_{\tilde{\Omega}} d\tilde{\rho}(x) \int_{M \setminus \Omega} d\rho(y) - \int_{\tilde{M} \setminus \tilde{\Omega}} d\tilde{\rho}(x) \int_{\Omega} d\rho(y) \Big)\, \mathcal{L}(\underset{\in \tilde{M}}{x}, \underset{\in M}{y})$$



assume that

$\tilde{\rho} = (F_\tau)_* (f_\tau \, \rho)$, $\quad F_\tau : M \to \tilde{M}$

$\tilde{\Omega} = F_\tau(\Omega)$ $\qquad \int_{\tilde{\Omega}} d\tilde{\rho}(x)\, g(x) = \int_\Omega d\rho(x)\, f_\tau(x)\, g(F_\tau(x))$

$$\Rightarrow \gamma^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \rho) = \Big(\int_\Omega d\rho(x) \int_{M \setminus \Omega} d\rho(y) - \int_{M \setminus \Omega} d\rho(x) \int_\Omega d\rho(y) \Big)$$
$$f_\tau(x)\, \mathcal{L}(F_\tau(x), y)$$

linear in τ : $\quad$ conserved one-form $\quad \gamma^\Omega_\rho(\underline{v})$

quadratic in τ : $\quad$ surface layer inner product $\quad (\underline{v}, \underline{v})^\Omega_\rho + \ldots$

$\underline{v} = \frac{d}{d\tau} (f_\tau, F_\tau) \big|_{\tau=0}$

conservation law

identify M and $\tilde{M}$

$$F : M \to \tilde{M}$$

$$\Phi := F^{-1} : \tilde{M} \to M$$

choose $\tilde{\Omega} = F(\Omega)$

goal: choose F such that $\mathcal{J}^{\tilde{\Omega},\Omega}(\tilde{\rho},\rho) = 0 \quad \forall\, \Omega \subset M$ compact

$\Phi_* \tilde{\rho}$ measure on M

$$\Rightarrow \mathcal{J}^{\tilde{\Omega},\Omega}(\tilde{\rho},\rho) = \int_\Omega d(\Phi_* \tilde{\rho})(x) \int_{M\setminus\Omega} d\rho(y)\; \mathcal{L}(F(x),y)$$

$$- \int_\Omega d\rho(x) \int_{M\setminus\Omega} d(\Phi_* \tilde{\rho})\; \mathcal{L}(x, F(y))$$

Using a symmetry argument,

$$= \int_\Omega d(\Phi_* \tilde{\rho})(x) \int_M d\rho(y)\; \mathcal{L}(F(x),y)$$

$$- \int_\Omega d\rho(x) \int_M d(\Phi_* \tilde{\rho})\; \mathcal{L}(x, F(y))$$

$$= \int_\Omega d(\Phi_* \tilde{\rho}) \int_M d\rho(y)\; \mathcal{L}(F(x),y) \;=\; (\Phi_* \tilde{\nu})(\Omega)$$

$$- \int_\Omega d\rho(x) \int_{\tilde{M}} d\tilde{\rho}(y)\; \mathcal{L}(x,y) \;=\; \nu(\Omega)$$

Introduce

$$d\nu(x) = \left(\int_{\tilde{M}} \mathcal{L}(x,y)\, d\tilde{\rho}(y) \right) d\rho(x) \qquad \text{measure on } M$$

$$d\tilde{\nu}(x) = \left(\int_M \mathcal{L}(x,y)\, d\rho(y) \right) d\tilde{\rho}(x) \qquad \text{measure on } \tilde{M}$$

correlation measures

$$= \left(\left(\Phi_* \tilde{\nu} \right) - \nu \right)(\Omega) \overset{!}{=} 0 \qquad \forall\, \Omega \subset M \text{ compact}$$

Thus we need to arrange that

$$\Phi_* \tilde{\nu} = \nu$$

General problem:

given ν on M

$\tilde{\nu}$ on $\tilde{M}$

is there $\Phi : \tilde{M} \to M$ s.t. $\Phi_* \tilde{\nu} = \nu$?

Answer: Yes, under general assumptions

Greene and Shiohama — rather abstract

Moser's theorem — infinitesimally, then integrate

Causal fermion systems

$(\mathcal{H}, \mathcal{F}, \rho)$, $(\tilde{\mathcal{H}}, \tilde{\mathcal{F}}, \tilde{\rho})$

identify $\mathcal{H}$ with $\tilde{\mathcal{H}}$

Choose $V : \mathcal{H} \to \tilde{\mathcal{H}}$ unitary

not canonical

freedom: $V \to V\mathcal{U}$ with $\mathcal{U} \in \mathrm{U}(\mathcal{H})$

integrate over $\mathcal{U}$

partition function

$$Z^{\Omega}(\beta, \tilde{\rho}) = \int_{\mathcal{G} \subset \mathrm{U}(\mathcal{H})} e^{\beta\, \mathcal{L}^{\tilde{\Omega}, \Omega}(\tilde{\rho}, \mathcal{U}\rho)}\, d\mathcal{U}$$

gives rise to quantum state

compact Lie group

Haar measure