$(\mathcal{M}, \langle .,. \rangle)$ Minkowski space

$(i\partial\!\!\!/ - m)\psi = 0$, $\quad (\mathcal{H}_m , (.|.)_m)$

$\mathcal{H} \subset \mathcal{H}_m$ closed subspace, $\quad \langle .|. \rangle_{\mathcal{H}} := (.|.)_m|_{\mathcal{H}\times\mathcal{H}}$

$\mathfrak{R}^\varepsilon : \mathcal{H} \to C^0(\mathcal{M}, S\mathcal{M})$

local correlation operator

$$\text{defined by} \quad \langle u | F^\varepsilon(x)\, v \rangle_{\mathcal{H}} = - \overline{(\mathfrak{R}^\varepsilon u)(x)}\, (\mathfrak{R}^\varepsilon v)(x) \qquad (*)$$

can be constructed as follows: $\quad$ Let $v \in \mathcal{H}$

$$\phi : \mathcal{H} \longrightarrow \mathbb{C}$$
$$u \longmapsto - \overline{(\mathfrak{R}^\varepsilon u)(x)}\, (\mathfrak{R}^\varepsilon v)(x)$$

is anti-linear and bounded

(because $u \mapsto (\mathfrak{R}^\varepsilon u)(x)$ has finite rank)

The Fréchet-Riesz theorem gives a unique vector $w \in \mathcal{H}$ with

$$\phi(u) = \langle u | w \rangle_{\mathcal{H}} \qquad \forall\, u \in \mathcal{H}$$

The mapping

$$F^\varepsilon(x) : \begin{array}{ccc} v & \longmapsto & w \\ \mathcal{H} & \longrightarrow & \mathcal{H} \end{array}$$

is linear and satisfies our defining equation $(*)$.

push-forward measure $\quad \rho := (F^{\varepsilon})_* \mu$

in more detail

On $\mathcal{M}$: measure $d\mu = d^4x$ Lebesgue measure

$$\mathfrak{M}_\mu \underset{\neq}{\supset} \mathcal{B}_{\mathcal{M}}$$

$\mathfrak{M}_\mu$: Lebesgue-measurable sets; $\quad \mathcal{B}_{\mathcal{M}}$: Borel algebra

let $F : \mathcal{M} \longrightarrow \mathcal{F}$

$$\mathfrak{M}_\rho := \{ A \subset \mathcal{F} \mid F^{-1}(A) \in \mathfrak{M}_\mu \}$$

is a σ-algebra $\longleftarrow$ no proof

On $\mathfrak{M}_\rho$ we define ρ by

$$\rho(\Omega) := \mu(F^{-1}(\Omega))$$

is a measure $\longleftarrow$ no proof

Assume that F is continuous.

Then for any $\Omega \subset \mathcal{F}$ open, $\quad F^{-1}(\Omega)$ is open in $\mathcal{M}$

$$\Rightarrow F^{-1}(\Omega) \in \mathfrak{M}_\mu \Longrightarrow \Omega \in \mathfrak{M}_\rho$$

$$\Longrightarrow \mathcal{B}_{\mathcal{F}} \subset \mathfrak{M}_\rho$$

Thus $\rho|_{\mathcal{B}_{\mathcal{F}}}$ is a Borel measure on $\mathcal{F}$

$F: \mathcal{M} \to \mathcal{F}, \qquad \rho := F_* \mu \qquad (F \text{ continuous})$

$M := \operatorname{supp} \rho$

<u>Lemma:</u> $M = \overline{F(\mathcal{M})}$

<u>Proof:</u> Let $A = F(x)$

Let $\mathcal{U} \subset \mathcal{F}$ be an open neighborhood of A

$\Rightarrow F^{-1}(\mathcal{U}) \subset \mathcal{M}$ is open and $\neq \emptyset$

$\Rightarrow \mu(F^{-1}(\mathcal{U})) \neq 0 \Rightarrow \rho(\mathcal{U}) \neq 0$

Since $\mathcal{U}$ is arbitrary, it follows that $A \in M$

Hence $M \supset F(\mathcal{M})$

Let $A \in C\, \overline{F(\mathcal{M})}$

Then $\exists\, \mathcal{U} \subset \mathcal{F}$ open neighborhood of A

with $\mathcal{U} \cap F(\mathcal{M}) = \emptyset$

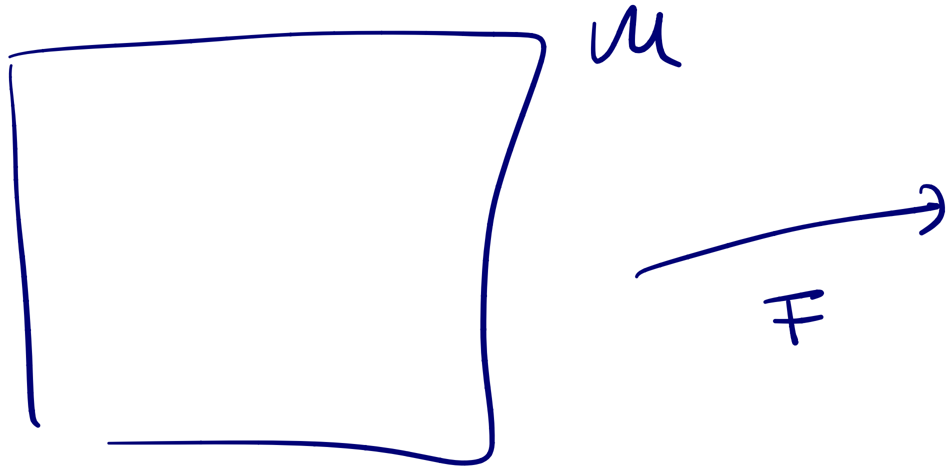
$\Rightarrow F^{-1}(\mathcal{U}) = \emptyset \Rightarrow \mu(F^{-1}(\mathcal{U})) = 0$

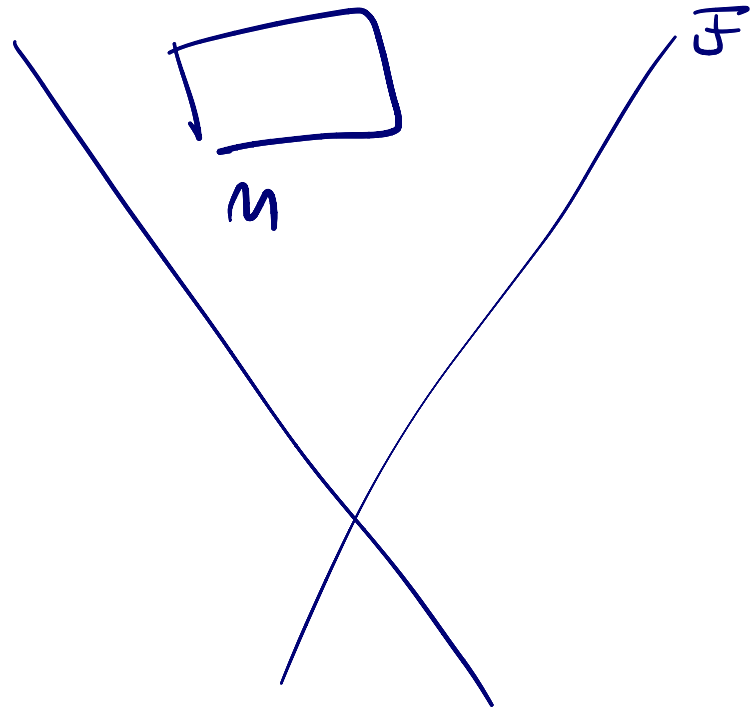
$\Rightarrow \rho(\mathcal{U}) = 0 \Rightarrow A \notin M.$

Hence $M \subset \overline{F(\mathcal{M})}$

By definition, the support of a measure is closed

$\Rightarrow M = \overline{M} = \overline{F(\mathcal{M})}.$ $\square$





$$F : \mathcal{M} \longrightarrow F(\mathcal{M}) \subset \mathcal{F}$$

$$M = \overline{F(\mathcal{M})}$$

We would like to identify x with $F(x)$

If – F is injective

– $F(\mathcal{M})$ is closed ?

then $F : \mathcal{M} \longrightarrow M$ is bijective

$$x \longmapsto F(x)$$

Let $A \in \overline{F(\mathcal{M})} \setminus F(\mathcal{M})$

Then $\exists\ A_n = F(x_n)$ with $A_n \longrightarrow A$