Let $(\mathcal{H}, \mathcal{F}, \rho)$ be a causal fermion system and ρ a minimizer of the causal action under the constraints

$M := \operatorname{supp} \rho$

<u>Thm</u> $\operatorname{tr}(x) = c$ on M.

treat the boundedness constraint with a Lagrange multiplier κ,

$$\mathcal{L}_\kappa(x,y) := \mathcal{L}(x,y) + \kappa \, |xy|^2 .$$

assume that ρ is <u>regular</u> in the sense that x regular $\forall x \in M$

$\hookrightarrow$ (i.e. $\dim x(\mathcal{H}) = 2n$)

<u>Thm</u>: $\mathcal{F}_c^{reg} := \{ x \in \mathcal{F} \text{ regular and } \operatorname{tr}(x) = c \} \subset L(\mathcal{H})$

is a manifold

(more precisely, if $\dim \mathcal{H} < \infty$, it is a noncompact manifold

if $\dim \mathcal{H} = \infty$, it is an infinite-dimensional Banach manifold).

Then ρ is a critical point of the causal variational principle

$$\mathcal{S} = \int_{\mathcal{F}_c^{reg}} d\rho(x) \int_{\mathcal{F}_c^{reg}} d\rho(y) \; \mathcal{L}_\kappa(x,y)$$

Minimize over all regular Borel measures on $\mathcal{F}_c^{reg}$

under the volume constraint $\rho(F_c^{reg}) = \text{const}$.