Abstract

We address the problem of finding a *product picture* for QED – i.e. a reformulation in which (formally) it has a total Hamiltonian, arising as a sum of a free electromagnetic Hamiltonian, a free Dirac Hamiltonian and an interaction term, acting on a Hilbert space which is a subspace (the *physical subspace*) of the full tensor product of an electron/positron Hilbert space and an electromagnetic-field Hilbert space. The traditional Coulomb gauge formulation of QED (as presented e.g. in Weinberg's textbook [2]) isn't a product picture in this sense because, in it, the longitudinal part of the electric field is a function of the Dirac ψ-field. We exhibit a product picture and prove (at a formal level) its equivalence with Coulomb gauge QED. We also point out a number of its features: (i) In all states in the physical subspace (including the vacuum) the ψ field is entangled with longitudinal photons; (ii) Gauss's law holds as an operator equation; (iii) The electric field operator (and therefore also the full Hamiltonian) fails to be self-adjoint on the full tensor-product Hilbert space. However it is self-adjoint when restricted to the physical subspace and so that's OK; (iv) The product picture provides a temporal gauge quantization of QED which appears to be free from the difficulties which plagued previous approaches to temporal-gauge quantization; (v) In its nonrelativistic limit, it provides a reformulation of nonrelativistic (Schrödinger) many body theory in which the usual Coulomb potential is absent and, instead, a term representing the kinetic energy of longitudinal photons is present so that, say, the binding energy of the Hydrogen atom arises as a byproduct of the entanglement of the proton and electron with longitudinal photons. (Reference for all the above: Sections 3 and 4 of [2].)

As far as time allows, we shall also discuss the matter-gravity entanglement hypothesis (see e.g. [3,4]) which claims to offer a resolution to the black hole information loss puzzle and the black hole thermal atmosphere puzzle etc. The notion of matter-gravity entanglement seems to require a product picture for quantum gravity and it is hoped that this work on QED will provide a useful analogy for that.

[1] Weinberg, S.: The Quantum Theory of Fields, Volume 1. Cambridge University Press, Cambridge (1995)

[2] Kay, B.S.: Quantum electrostatics, Gauss's law, and a product picture for quantum electrodynamics; or, the temporal gauge revised, arXiv:2003.07473

[3] Kay, B.S.: The matter-gravity entanglement hypothesis. Foundations of Physics **48**, 542-557 (2018) [arXiv:1802.03635]

[4] Kay, B.S.: Remarks on matter-gravity entanglement, entropy, information loss and events. In: Finster, F, Giulini, D., Kleiner, J., Tolksdorf, J. (eds) Progress and Visions in Quantum Theory in View of Gravity, p 233 Birkhäuser, Cham (2020) [arXiv:1909.04963]