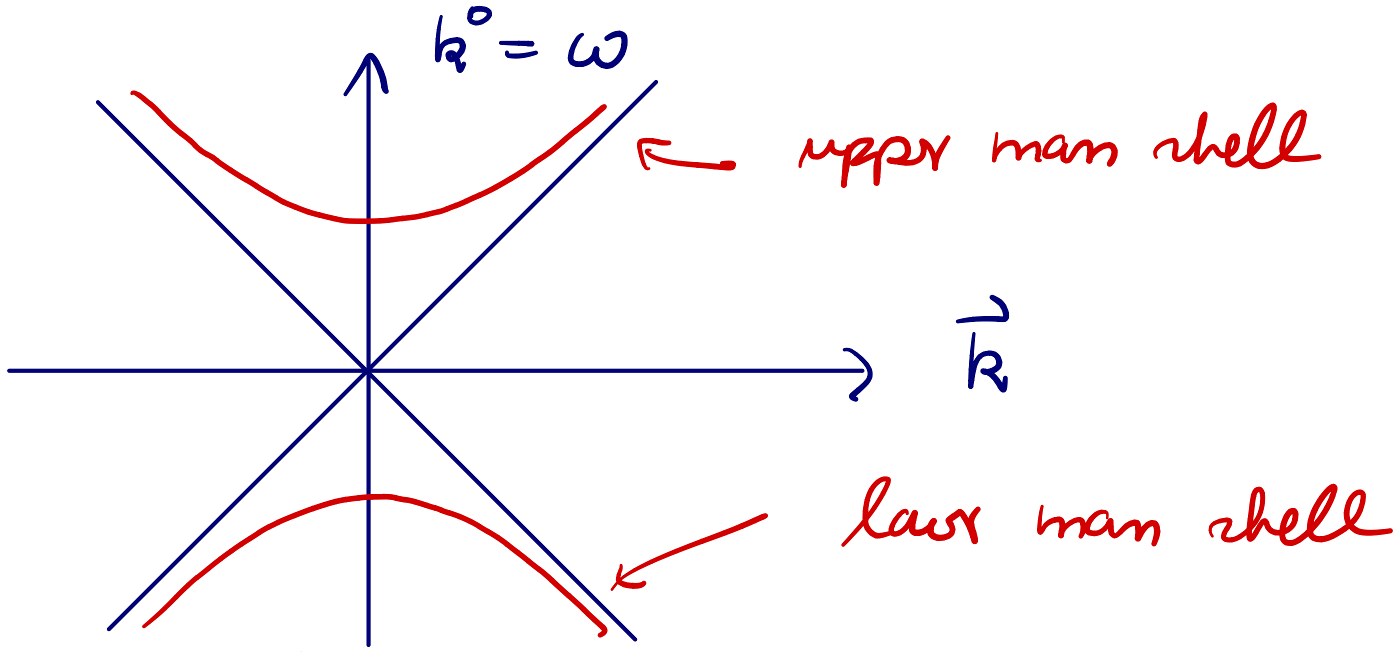
$$(i\not\partial - m)\,\psi = 0$$

$$\psi(x) = \int \frac{d^4k}{(2\pi)^4}\, \hat\psi(k)\, e^{-i\underbrace{kx}_{\langle k,x\rangle}}$$

$$\Leftrightarrow \left(\not k - m\right)\hat\psi(k) = 0 \qquad \Big|\times(\not k + m)$$

$$(\not k + m)(\not k - m) = (k^2 - m^2)\,\mathbb{1} \qquad \left(k^2 = \langle k, k\rangle\right)$$

$$\Rightarrow \quad (k^2 - m^2)\,\hat\psi(k) = 0 \qquad \omega^2 - |\vec k|^2 - m^2 = 0$$

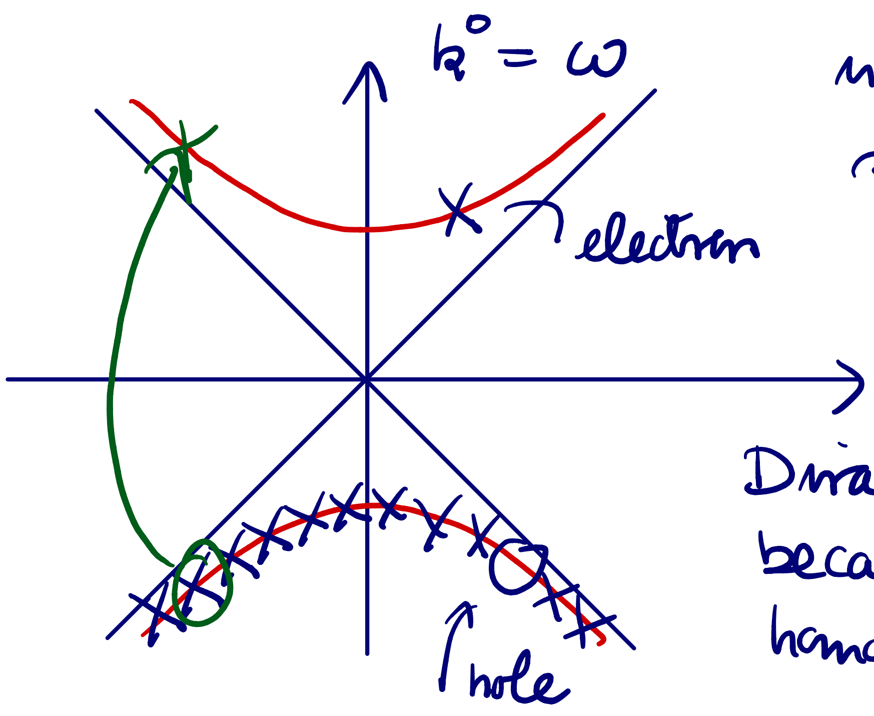


$$\begin{cases} \hat\psi \text{ is a tempered distribution} \\ \psi(x) \text{ will typically be smooth} \end{cases}$$

general solution $\qquad \hat\psi(k) = (\not k + m)\,\delta(k^2 - m^2)\,\hat\phi(k)$

$E = h\nu = \hbar\omega \qquad (\text{here } \hbar = 1)$

Dirac sea



in vacuum, all the states of negative energy are filled

Dirac sea, invisible because it is homogeneous and isotropic gives rise to anti-matter and pair creation

The hole appears as a particle of positive energy but positive charge

positron

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basic drawbacks:

- sea has an infinite charge density
- infinite negative energy density

in modern QFT the Dirac sea no longer appears because of a suitable "redefinition" of the vacuum.

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starting point of causal fermion systems:

Take the Dirac sea picture seriously, but formulate other equations which do not suffer from the above drawbacks.