$\vec{x} \in \mathbb{R}^3$ space

$t \in \mathbb{R}$ time

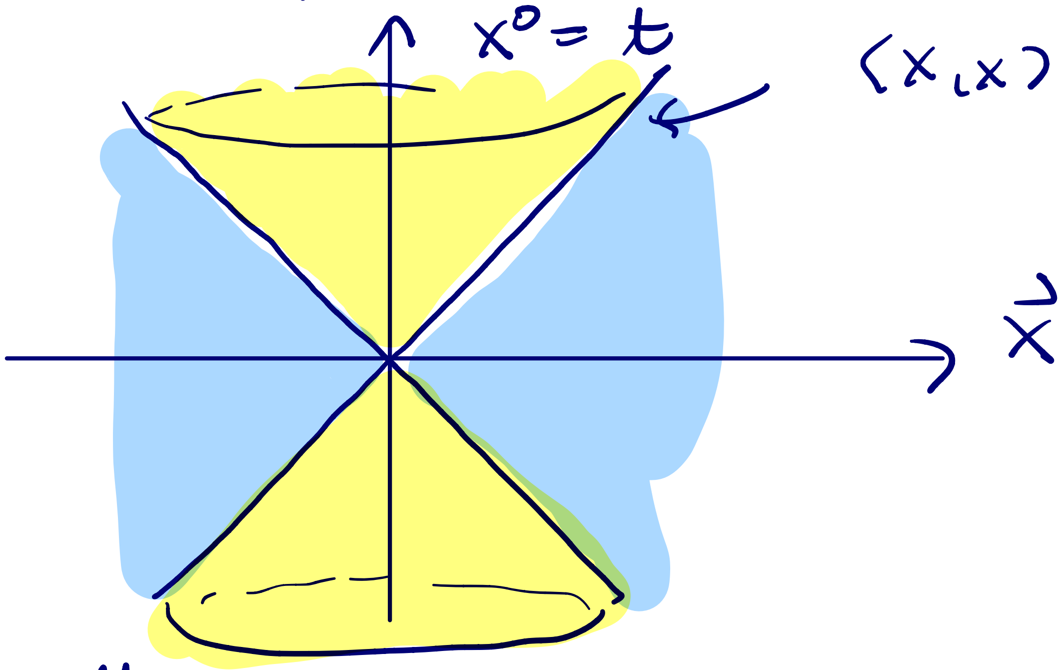
spacetime $x = (t, \vec{x}) \in \mathbb{R}^4$, $x^0 = t$

$c = 1$ speed of light

$$\langle x, y \rangle = x^0 y^0 - \langle \vec{x}, \vec{y} \rangle_{\mathbb{R}^3}$$
$$= x^0 y^0 - \sum_{\alpha=1}^{3} x^\alpha y^\alpha$$

<u>Def</u>: $x \in \mathbb{R}^4$ is $\qquad \langle x, x \rangle = (x^0)^2 - |\vec{x}|^2$

- timelike if $\langle x, x \rangle > 0$
- spacelike if $\langle x, x \rangle < 0$
- lightlike if $\langle x, x \rangle = 0$



<u>Def</u>: Let M be a 4-dim real vector space endowed with an indefinite inner product $\langle \cdot, \cdot \rangle$ of signature $(1,3)$.
$(M, \langle \cdot, \cdot \rangle)$ is <u>Minkowski space</u>.

an indefinite inner product is a non-degenerate, bilinear form

$\hookrightarrow \langle u, v \rangle = 0 \ \forall u \in M \Rightarrow v = 0$

The positive (negative) signature of an inner product is the maximal dimension of a positive (negative) definite subspace.

Let $(e_i)_{i=0,\dots,3}$ be a pseudo-orthonormal basis, i.e.

$$\langle e_i, e_j \rangle = \delta_{ij}\, s_i \quad \text{where}$$

$$s_0 = 1, \qquad s_1 = s_2 = s_3 = -1.$$

$$x = \sum_{i=0}^{3} x^i e_i \quad \text{basis representation}$$

$x^0 = t$

$x^{1,2,3}$ spatial coordinates

} of observer corresponding to (e_i)

The transformations from one pseudo-ONB to another are the Lorentz transformations.

$u \in M$ is $\left\{\begin{array}{l}\text{timelike}\\ \text{spacelike}\\ \text{lightlike}\end{array}\right\}$ just as above

$$J^\vee = \{ x \mid \langle x, x\rangle > 0 \text{ and } x^0 > 0 \}$$

$$\mathcal{J}^\vee = \{ x \mid \langle x, x\rangle \geq 0 \quad \text{—〃—} \quad x^0 \geq 0 \}$$

$$J^\wedge, \quad \mathcal{J}^\wedge$$

Let $\mu(\tau)$ be a curve in M ("world line")

$$\mu : J \longrightarrow M$$

J: generalized interval

$\mu'(\tau)$ four-velocity

<u>Causality</u>: no information can travel faster than with the speed of light

If $\gamma(\tau)$ is a regular curve ($\gamma'(\tau) \neq 0$), then

$\gamma'(\tau)$ is $\underbrace{\text{either timelike or lightlike}}_{\text{non-spacelike}}$ $\forall \tau \in J$

Such curves are called <u>causal curves</u>.