TRACE ANOMALY FOR CHIRAL FERMIONS

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OUTLINE

INTRODUCTION
OVERVIEW

- Anomalies are violations of symmetries in quantum field theory which hold in classical theories.
- By the mid 1960's, it was observed that the dominant decay mode of the neutral pion $\pi^0 \to 2\gamma$, was different than the expected (theoretical) one.
- In 1969, Jackiw and Bell found out that the source of this disagreement was the violation of the chiral symmetry.

INTRODUCTION

TYPES OF ANOMALIES

- The **chiral anomaly** is the quantum mechanical violation of the classically conserved chiral current j_μ, i.e. $\partial^\mu j_\mu \neq 0$.

- **Conformal (or trace) anomalies** occur when the classical conformal invariance of a certain theory is broken by quantum effects.

INTRODUCTION

METHODS TO CALCULATE ANOMALIES

- **Fujikawa Method:** it recognizes the anomaly as arising from the non-invariance of the path integral measure.
- **Heat Kernel Expansion:** the anomaly is written in terms of the the HaMiDeW coefficients of the trace of the heat kernel.
- **Hadamard Subtraction:** the anomaly is calculated by using point splitting and then subtracting the Hadamard parametrix.
- **Feynman Diagrams Calculation:** direct calculation using expectation values.

TRACE ANOMALY

OVERVIEW

- Conformal or trace anomalies are manifested by the trace of the stress-energy tensor.
- In four dimensions, the conformal anomaly takes the form [1]

$$\boldsymbol{\mathcal{A}} = a\mathcal{E}_4 + bR + cC^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + dR^*R + e\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}R^{\alpha\beta}{}_{\rho\sigma}$$

Where $\mathcal{E}_4$ is the Euler invariant and $C^{\mu\nu\rho\sigma}$ is the Weyl tensor.

INTRODUCTION

HISTORY OF TRACE ANOMALY

- Trace anomaly was discovered in 1973 by British physicists Michael Duff and Derek Capper.
- They announced their discovery at The First Oxford Quantum Gravity Conference held in Chilton, UK in 1974.
- The physics community rejected these findings by large. “*Something is wrong*”, said Christensen while Adler, Liberman and Ng asserted: “*We find no evidence of conformal trace anomalies*”.

INTRODUCTION

EXAMPLE: TRACE ANOMALY DRIVEN INFLATION

- As proposed by Alen Guth in 1981 [2], inflation seems to be the most convincing (if not the only) explanation of some observed features of our universe.
- In1984, Starobinsky suggested that inflation is driven by the trace anomaly of a large number of matter fields [3].
- We take the semi-classical Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$$

INTRODUCTION

EXAMPLE: TRACE ANOMALY DRIVEN INFLATION

We work in de Sitter space where $R_{\mu\nu\rho\sigma} = H^2(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})$, and take $\langle T_{\mu\nu}\rangle = \frac{1}{4}g_{\mu\nu}g^{\rho\sigma}\langle T_{\rho\sigma}\rangle = \frac{1}{4}g_{\mu\nu}\mathcal{A}$.

The Einstein equations now read:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 2\pi G\, g_{\mu\nu}g^{\rho\sigma}\langle T_{\rho\sigma}\rangle.$$

- Using the value of $\mathcal{A}$ we computed: Inflation exists.
- Trace-anomaly driven inflation has been supported by recent cosmological data [4][5][6][7]. You can read more about it in [8].

TRACE ANOMALY FOR CHIRAL FERMIONS

MOTIVATION

- Bonora et al. (2014) claim that an **imaginary** term appears in the trace of the renormalized stress tensor [9]. $H = \int T^{00}(x) d^4x$
- Bastianelli and Martelli (2016) recovered the standard results using Pauli-Villars regularization and Fujikawa's method [10].
- Bonora et. al. (2017, 2018) hit back, pointing out some possible inconsistencies in Bastianelli and Martelli's work, and re-derive the same result they originally had, using dimensional regularization [11][12].
- M. Fröb and J. Zahn (2019) do the same calculation using Hadamard subtraction, and show that the imaginary term vanishes [13].
- Bonora et. al. (2019) comment on that [14].

TRACE ANOMALY FOR CHIRAL FERMIONS
DISCUSSION I

$$\mathcal{A} = a\mathcal{E}_4 + bR + cC^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + dR^2 + e\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}R^{\alpha\beta}{}_{\rho\sigma}$$

Claim: The Pontryagin density should vanish.

Applying a CPT transformation to the trace should leave it invariant:

$$(CPT)\mathcal{A}(CPT)^{-1} =$$

$$a^*\mathcal{E}_4 + b^*R + c^*C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + d^*R^2 - e^*\epsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}R^{\alpha\beta}{}_{\rho\sigma} = \mathcal{A}.$$

This gives

$$a^* = a, \qquad b^* = b, \qquad c^* = c, \qquad d^* = d, \qquad e^* = -e.$$

$\Rightarrow e$ should vanish.

TRACE ANOMALY FOR CHIRAL FERMIONS

DISCUSSION II

- **Problem:** dimensional regularization and chiral theories:
  $\{\gamma^\mu, \gamma_*\} = 0$ only in $n = 4$ dimensions.
- **Solution 1:** Thompson and Yu's proposal [15]:
  Non-vanishing expression for $\{\gamma^\mu, \gamma_*\}$.
- **Solution 2:** Breitenlohner-Maison scheme [16]:
  - Split the n-dimensional Minkowski space into a product of a four- and an $(n-4)$-dimensional one.
  - Denote four-dimensional quantities by a bar, and $(n-4)$-dimensional ones by a hat.
  - $\{\gamma^\mu, \gamma_*\} = \{\hat{\gamma}^\mu, \gamma_*\} = 2\hat{\eta}_{\mu\nu}$

TRACE ANOMALY FOR CHIRAL FERMIONS
CALCULATION

- **Aim:** compute the trace anomaly for chiral fermions: $\mathcal{A} = g^{\mu\nu}\langle T_{\mu\nu}\rangle$

- **Method:**
  - We work in *n* dimensions and use dimensional regularization.
  - Start from the curved space action of Weyl fermions.
  - Calculate $T^{\mu\nu}$ by evaluating the metric variation of the action.
  - Expand $T^{\mu\nu}$ and S to second order around flat spacetime.
  - Calculate the interacting expectation value $\langle T_{\mu\nu}\rangle$.
  - Compute $g^{\mu\nu}\langle T_{\mu\nu}\rangle$.

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: STRESS TENSOR

- We start from the action of Weyl fermions in curved spacetime

$$S = -\int \bar{\psi} P_- \gamma^\mu \nabla_\mu P_+ \psi \sqrt{-g}\, d^4x$$

where $\nabla_\mu \equiv \partial_\mu + \frac{1}{4}\omega_{\mu\rho\sigma}\gamma^{\rho\sigma}$ is the spinor covariant derivative and $P_\mp$ are the chiral projectors which satisfy $\psi = P_+\psi$ and $\bar{\psi} = \bar{\psi} P_-$.

- We compute the stress-energy tensor

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}},$$

and get

$$T^{\mu\nu} = \frac{1}{2} \bar{\psi} \gamma^{(\mu} \overleftrightarrow{\nabla}^{\nu)} P_+ \psi + \frac{1}{2} g^{\mu\nu} \left[\nabla_\mu \bar{\psi} \gamma^\mu P_+ \psi - \bar{\psi} P_- \gamma^\mu \nabla_\mu \psi \right].$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: EXPANSION

- Expand $T^{\mu\nu}$ and S to second order around flat spacetime, using:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}{}_{\alpha} h^{\alpha\nu} + O(\kappa^3)$$

$$e_{\mu}{}^{a} = e_{\rho}{}^{a}(\eta^{\rho\mu} - \frac{1}{2}\kappa h^{\rho\mu} + \frac{3}{8}\kappa^2\, h^{\mu}{}_{\alpha} h^{\rho\alpha}) + O(\kappa^3)$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: EXPANSION

- The following was obtained, with $\boldsymbol{\Psi}^{\boldsymbol{\mu\nu}} = \bar{\psi}\gamma^{\mu}\partial^{\nu}\psi - \partial^{\nu}\bar{\psi}\gamma^{\mu}\psi$ and $\boldsymbol{j}^{\boldsymbol{\mu}} = \bar{\psi}\gamma^{\mu}\psi$.

$$\begin{aligned} \boldsymbol{T}^{\boldsymbol{\mu\nu}} = \; & \frac{1}{2}\Psi^{(\mu\nu)} - \frac{1}{2}\eta^{\mu\nu}\Psi^{\alpha}{}_{\alpha} + \boldsymbol{\kappa}\left(\frac{1}{2}h^{\mu\nu}\Psi^{\alpha}{}_{\alpha} - \frac{1}{4}h^{\alpha(\nu}\Psi_{\alpha}{}^{\mu)} - \frac{1}{2}h^{(\nu}{}_{\alpha}\Psi^{\mu)\alpha} + \frac{1}{4}\eta^{\mu\nu}h_{\alpha\beta}\Psi^{\alpha\beta} + \frac{1}{4}j^{\alpha\beta(\mu}\partial_{\beta}h^{\nu)}{}_{\alpha}\right) \\ & + \boldsymbol{\kappa}^{2}\left(-\frac{1}{2}h^{\mu\beta}h^{\nu}{}_{\beta}\Psi^{\alpha}{}_{\alpha} + \frac{3}{16}h^{\alpha}{}_{\beta}h^{\beta(\mu}\Psi_{\alpha}^{\nu)} + \frac{1}{2}h_{\alpha\beta}h^{\beta(\mu}\Psi^{\nu)\alpha} - \frac{1}{8}j_{\alpha\beta}{}^{\delta}h^{\alpha(\mu}\partial_{\delta}h^{\nu)\beta} + \frac{1}{16}\eta^{\mu\nu}h^{\alpha\beta}j_{\beta\delta\lambda}\partial^{\lambda}h_{\alpha}{}^{\delta} \right. \\ & + \frac{1}{32}j^{(\nu}{}_{\beta\delta}\left(-4h^{\mu)\alpha}\partial^{\delta}h_{\alpha}{}^{\beta} + 2\partial_{\alpha}h^{\mu)\delta}h^{\alpha\beta} - 2\partial^{\delta}h^{\mu)}{}_{\alpha}h^{\alpha\beta} - \partial^{\mu)}h_{\alpha}{}^{\delta}h^{\alpha\beta}\right) \\ & \left. + \frac{1}{4}\left(-h_{\alpha\beta}h^{\mu\nu} + h^{(\mu}{}_{\beta}h^{\nu)}{}_{\alpha} - \frac{3}{4}\eta^{\mu\nu}h_{\alpha}{}^{\delta}h_{\beta\delta}\right)\Psi^{\alpha\beta}\right) \end{aligned}$$

$$\begin{aligned} \boldsymbol{S} = \; & \int\left[-\frac{1}{2}\Psi^{\alpha}{}_{\alpha} + \boldsymbol{\kappa}\left(-\frac{1}{4}h^{\beta}{}_{\beta}\Psi^{\alpha}{}_{\alpha} + \frac{1}{4}h_{\alpha\beta}\Psi^{\alpha\beta}\right)\right. \\ & \left. + \boldsymbol{\kappa}^{2}\left(\frac{1}{8}h_{\beta\delta}h^{\beta\delta}\Psi^{\alpha}{}_{\alpha} - \frac{1}{16}h^{\beta}{}_{\beta}h^{\delta}{}_{\delta}\Psi^{\alpha}{}_{\alpha} - \frac{3}{16}h_{\alpha}{}^{\delta}h_{\beta\delta}\Psi^{\alpha\beta} + \frac{1}{8}h_{\alpha\beta}h^{\delta}{}_{\delta}\Psi^{\alpha\beta} + \frac{1}{16}h^{\alpha\beta}j_{\beta\delta\lambda}\partial^{\lambda}h_{\alpha}{}^{\delta}\right)\right]\mathrm{d}^{n}y \end{aligned}$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: EXPANSION

- The expectation value of the stress-energy tensor was evaluated using the Gell-Mann and Low theorem:

$$\langle T^{\mu\nu}(x)\rangle_{\text{int}} = \frac{\int T^{\mu\nu} e^{\mathrm{i}S_{\text{int}}} D\psi D\bar{\psi}}{\int e^{\mathrm{i}S_{\text{int}}} D\psi D\bar{\psi}} = \frac{\left\langle T_{\mu\nu} e^{\mathrm{i}S_{\text{int}}}\right\rangle_0}{\left\langle e^{\mathrm{i}S_{\text{int}}}\right\rangle_0}$$

and the following was obtained:

$$\langle T^{\mu\nu}(x)\rangle = \left\langle T^{\mu\nu(0)}\right\rangle +$$

$$\kappa\left(\left\langle T^{\mu\nu(1)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(0)}S^{(1)}\right\rangle - \mathrm{i}\left\langle T^{\mu\nu(0)}\right\rangle\left\langle S^{(1)}\right\rangle\right) +$$

$$\kappa^2\left(\begin{array}{l}\frac{1}{2}\left\langle T^{\mu\nu(1)}\right\rangle\left\langle {S^{(1)}}^2\right\rangle - \left\langle T^{\mu\nu(0)}\right\rangle\left\langle S^{(1)}\right\rangle^2 - \frac{1}{2}\left\langle T^{\mu\nu(0)}{S^{(1)}}^2\right\rangle + \left\langle T^{\mu\nu(0)}S^{(1)}\right\rangle\left\langle S^{(1)}\right\rangle - \\ \mathrm{i}\left\langle T^{\mu\nu(0)}\right\rangle\left\langle S^{(2)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(0)}S^{(2)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(1)}S^{(1)}\right\rangle - \mathrm{i}\left\langle T^{\mu\nu(1)}\right\rangle\left\langle S^{(1)}\right\rangle + \left\langle T^{\mu\nu(2)}\right\rangle\end{array}\right)$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT FIRST ORDER

The trace anomaly at first order reads

$$\mathcal{A}^{(1)} = g_{\mu\nu}\langle T^{\mu\nu}(x)\rangle^{(1)}$$

where

$$\langle T^{\mu\nu}(x)\rangle^{(1)} = \left\langle T^{\mu\nu\,(1)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu\,(0)} S^{(1)}\right\rangle - \mathrm{i}\left\langle T^{\mu\nu\,(0)}\right\rangle \left\langle S^{(1)}\right\rangle$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ONE-POINT FUNCTIONS

- <u>One-Point Function:</u>

$$\langle \Psi^{\mu\nu} \rangle = \left\langle \bar{\psi}(x)\gamma^{\mu}\partial_x^{\nu}\psi(x) - \partial_x^{\nu}\bar{\psi}(x)\gamma^{\mu}\psi(\mathrm{x}) \right\rangle$$

$$= \left\langle \bar{\psi}_a(x)P_{-ab}\gamma^{\mu}_{bc}\partial_x^{\nu}P_{+cd}\psi_d(x)\right\rangle - \left\langle \partial_x^{\nu}\bar{\psi}_a(x)P_{-ab}\gamma^{\mu}_{bc}P_{+cd}\psi_d(x)\right\rangle$$

$$= \lim_{x'\to x}\partial_{x'}^{\nu}P_{-ab}\gamma^{\mu}_{bc}P_{+cd}\left\langle \bar{\psi}_a(x)\psi_d(x')\right\rangle - \lim_{x\to x'}\partial_x^{\nu}P_{-ab}\gamma^{\mu}_{bc}P_{+cd}\left\langle \bar{\psi}_a(x)\psi_d(x')\right\rangle$$

$$= -i\lim_{x'\to x}\partial_{x'}^{\nu}P_{-ab}\gamma^{\mu}_{bc}P_{+cd}G_{da}(x',x) + i\lim_{x\to x'}\partial_x^{\nu}P_{-ab}\gamma^{\mu}_{bc}P_{+cd}G_{da}(x',x)$$

$$= -i\lim_{x'\to x}\partial_{x'}^{\nu}tr[P_{-}\gamma^{\mu}P_{+}G(x',x)] + i\lim_{x\to x'}\partial_x^{\nu}tr[P_{-}\gamma^{\mu}P_{+}G(x',x)]$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ONE-POINT FUNCTIONS

In Fourier space

$$G(x', x) = \int \tilde{G}(p) e^{ip(x'-x)} \frac{\mathrm{d}^n p}{(2\pi)^n} \qquad \text{where} \qquad \tilde{G}(p) = \mathrm{i} \frac{\gamma^\nu p_\nu}{p^2}.$$

This gives

$$\langle \Psi^{\mu\nu} \rangle = \mathrm{i}\, \mathrm{tr}(P_- \gamma^\mu P_+ \gamma^\rho) \int \frac{p_\rho p^\nu}{p^2} \frac{d^n p}{(2\pi)^n} + (\text{second term}).$$

This integral vanishes in dimensional regularization, so we are left with:

$$\langle \Psi^{\mu\nu} \rangle = 0$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ONE-POINT FUNCTIONS

The expectation value of the stress-energy tensor can be now written as

$$\langle T^{\mu\nu}(x)\rangle = \kappa\, \mathrm{i} \left\langle T^{\mu\nu\,(0)} S^{(1)} \right\rangle + \kappa^2 \left(-\frac{1}{2} \left\langle T^{\mu\nu\,(0)} S^{(1)} S^{(1)} \right\rangle + \mathrm{i} \left\langle T^{\mu\nu\,(0)} S^{(2)} \right\rangle + \mathrm{i} \left\langle T^{\mu\nu\,(1)} S^{(1)} \right\rangle \right)$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: TWO-POINT FUNCTIONS

- <u>Two-Point Functions:</u>

$$\left\langle \Psi^{\mu\nu}(x)\Psi^{\alpha\beta}(y)\right\rangle = \left\langle (\bar{\psi}(x)\gamma^{\mu}\partial_x^{\nu}\psi(x) - \partial_x^{\nu}\bar{\psi}(x)\gamma^{\mu}\psi(\mathrm{x}))(\bar{\psi}(y)\gamma^{\alpha}\partial_y^{\beta}\psi(y) - \partial_y^{\beta}\bar{\psi}(y)\gamma^{\alpha}\psi(\mathrm{y}))\right\rangle$$

$$\left\langle \Psi^{\mu\nu}(x)\mathrm{j}^{\alpha\beta\lambda}(y)\right\rangle = \left\langle (\bar{\psi}(x)\gamma^{\mu}\partial_x^{\nu}\psi(x) - \partial_x^{\nu}\bar{\psi}(x)\gamma^{\mu}\psi(\mathrm{x}))(\bar{\psi}(y)\gamma^{\alpha\beta\lambda}\psi(x)\right\rangle$$

Following the same steps as before, we get

$$\left\langle \Psi^{\mu\nu}(x)\Psi^{\alpha\beta}(y)\right\rangle = 2\mathrm{tr}(P_+\gamma^{\rho}P_-\gamma^{\mu}P_+\gamma^{\sigma}P_-\gamma^{\alpha}) \int I(q)\, e^{\mathrm{i}q(x-y)} A^{\nu\beta}{}_{\rho\sigma}(q) \frac{\mathrm{d}^n q}{(2\pi)^n}$$

$$\left\langle \Psi^{\mu\nu}(x)\mathrm{j}^{\alpha\beta\lambda}(y)\right\rangle = 2\mathrm{i}\,\mathrm{tr}\big(P_-\gamma^{\mu}P_+\gamma^{\rho}P_-\gamma^{\alpha\beta\lambda}P_+\gamma^{\sigma}\big) \int I(q)\, e^{\mathrm{i}q(x-y)} B^{\nu}{}_{\rho\sigma}(q) \frac{\mathrm{d}^n q}{(2\pi)^n}$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: TWO-POINT FUNCTIONS

where

$$I(q) = \int \frac{1}{p^2(q+p)^2} \frac{\mathrm{d}^n p}{(2\pi)^n},$$

and

$$\mathrm{tr}(P_+\gamma^\rho P_-\gamma^\mu P_+\gamma^\sigma P_-\gamma^\alpha)^{\mathrm{naive}} = \frac{1}{2}\mathrm{tr}(\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\alpha) + \frac{1}{2}\mathrm{tr}(\gamma_*\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\alpha),$$

$$\mathrm{tr}(P_+\gamma^\rho P_-\gamma^\mu P_+\gamma^\sigma P_-\gamma^\alpha)^{\mathrm{BM}} = \mathrm{tr}(P_+\gamma^\rho P_-\gamma^\mu P_+\gamma^\sigma P_-\gamma^\alpha)^{\mathrm{naive}} + 2(\bar{\eta}^{\alpha\rho}\bar{\eta}^{\mu\sigma} - \eta^{\alpha\rho}\eta^{\mu\sigma} + \bar{\eta}^{\alpha\sigma}\bar{\eta}^{\mu\rho} - \eta^{\alpha\sigma}\eta^{\mu\rho} + \bar{\eta}^{\alpha\mu}\bar{\eta}^{\rho\sigma} - \eta^{\alpha\mu}\eta^{\rho\sigma})$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: TWO-POINT FUNCTIONS

$$I(q) = \int \frac{1}{p^2(q+p)^2} \frac{\mathrm{d}^n p}{(2\pi)^n}$$

Solving $I(q)$ gives:

$$I(q) = \frac{\mathrm{i}}{(4\pi)^{\frac{n}{2}}} \frac{\Gamma\left(\frac{4-n}{2}\right)\Gamma\left(\frac{n-2}{2}\right)^2}{\Gamma(n-2)} (q^2 - \mathrm{i}\varepsilon)^{\frac{n-4}{2}}$$

Expanding around $n = 4$ to first order, we get:

$$I(q) = \frac{\mathrm{i}}{(4\pi)^2}\left[-\frac{2}{(n-4)} + 2 - \gamma + \ln(4\pi) - \ln(\mu^2) - \ln\left(\frac{q^2 - \mathrm{i}\varepsilon}{\mu^2}\right)\right] + O(n-4)$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT FIRST ORDER

Reminder: $\langle T^{\mu\nu}(x)\rangle = \kappa\,\mathrm{i}\left\langle T^{\mu\nu\,(0)} S^{(1)}\right\rangle + \kappa^2\left(-\frac{1}{2}\left\langle T^{\mu\nu\,(0)} S^{(1)} S^{(1)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu\,(0)} S^{(2)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu\,(1)} S^{(1)}\right\rangle\right)$

$$
\begin{aligned}
\langle T^{\mu\nu}(x)\rangle^{(1)} &= \mathrm{i}\left\langle T^{\mu\nu\,(0)} S^{(1)}\right\rangle \\
&= \mathrm{i}\left\langle\left(\frac{1}{2}\Psi^{(\mu\nu)} - \frac{1}{2}\eta^{\mu\nu}\Psi^{\alpha}{}_{\alpha}\right)\int\left(-\frac{1}{4}h^{\beta}{}_{\beta}\Psi^{\alpha}{}_{\alpha} + \frac{1}{4}h_{\alpha\beta}\Psi^{\alpha\beta}\right)\mathrm{d}^n y\right\rangle \\
&= \frac{\mathrm{i}}{8}\int h_{\alpha\beta}(y)\left\langle\left(\Psi^{(\mu\nu)} - \eta^{\mu\nu}\Psi^{\alpha}{}_{\alpha}\right)\left(-\eta^{\alpha\beta}\Psi^{\delta}{}_{\delta} + \Psi^{\alpha\beta}\right)\right\rangle \mathrm{d}^n y \\
&= \frac{\mathrm{i}}{16}\int h_{\alpha\beta}(y)\left\langle\Psi^{(\mu\nu)}(x)\Psi^{\alpha\beta}(y)\right\rangle \mathrm{d}^n y
\end{aligned}
$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT FIRST ORDER

$$I(q) = \frac{\mathrm{i}}{(4\pi)^2}\left[-\frac{2}{(n-4)} + 2 - \gamma + \ln(4\pi) - \ln(\mu^2) - \ln\left(\frac{q^2 - \mathrm{i}\varepsilon}{\mu^2}\right)\right]$$

Plugging everything in we obtain

$$\langle T^{\mu\nu}(x)\rangle^{(1)}_{\mathrm{reg}} = \frac{\mathrm{i}}{8}\mathrm{tr}(P_+\gamma^\rho P_-\gamma^\mu P_+\gamma^\sigma P_-\gamma^\alpha)\int h_{\alpha\beta}(y)\int I(q)\, e^{\mathrm{i}q(x-y)} A^{\nu\beta}{}_{\rho\sigma}(q)\,\frac{\mathrm{d}^n q}{(2\pi)^n}\,\mathrm{d}^n y$$

We renormalize using the $\overline{\mathrm{MS}}$ scheme by subtracting the divergent part then replacing n by 4 in the expression

$$\langle T^{\mu\nu}(x)\rangle^{(1)}_{\mathrm{ren}} = \langle T^{\mu\nu}(x)\rangle^{(1)}_{\mathrm{reg}} - \langle T^{\mu\nu}(x)\rangle^{(1)}_{\mathrm{div}}$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT FIRST ORDER

$$\langle T^{\mu\nu}(x)\rangle^{(1)}_{\text{div}} = -\frac{1}{960\pi^2(n-4)}\iint (3q^4h^{\mu\nu} - 6q^2q^\alpha q^{(\mu}\, h^{\nu)}_{\ \ \alpha} + q^2q^\mu q^\nu h$$

$$+ 2q^\alpha q^\beta q^\mu q^\nu h_{\alpha\beta} - \eta^{\mu\nu}q^4h^\alpha_{\ \alpha} + \eta^{\mu\nu}q^2q^\alpha q^\beta h_{\alpha\beta})e^{\mathrm{i}q(x-y)}\frac{\mathrm{d}^nq}{(2\pi)^n}\mathrm{d}^ny.$$

Solving, then contracting with $g_{\mu\nu}$, we get

$$\mathcal{A}^{(1)}_{\text{div}} = g_{\mu\nu}\langle T^{\mu\nu}(x)\rangle^{(1)}_{\text{div}} = -\frac{1}{960\pi^2}\nabla^2\mathrm{R}$$

With which we find the renormalized trace anomaly at first order to be the same up to sign:

$$\mathcal{A}^{(1)}_{\text{ren}} = \frac{1}{960\pi^2}\nabla^2\mathrm{R}$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT SECOND ORDER

At second order, the trace anomaly reads:

$$\mathcal{A}^{(2)} = \eta_{\mu\nu}\langle T^{\mu\nu}\rangle^{(2)} + h_{\mu\nu}\langle T^{\mu\nu}\rangle^{(1)}$$

$$= \eta_{\mu\nu}\left(-\frac{1}{2}\left\langle T^{\mu\nu(0)} {S^{(1)}}^2\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(0)} S^{(2)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(1)} S^{(1)}\right\rangle\right) + h_{\mu\nu}\mathrm{i}\left\langle T^{\mu\nu(0)} S^{(1)}\right\rangle.$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT SECOND ORDER

$$\langle T^{\mu\nu}_{(0)} S^{2}_{(1)} \rangle = \frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\mu\nu\alpha\beta\delta\xi}\, h(y)_{\alpha\beta}\, h(z)_{\delta\xi} + \frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\nu\mu\alpha\beta\delta\xi}\, h(y)_{\alpha\beta}\, h(z)_{\delta\xi} - \frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\mu\nu\alpha}{}_{\alpha}{}^{\delta\xi}\, h(y)^{\beta}{}_{\beta}\, h(z)_{\delta\xi} -$$

$$\frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\nu\mu\alpha}{}_{\alpha}{}^{\delta\xi}\, h(y)^{\beta}{}_{\beta}\, h(z)_{\delta\xi} - \frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\mu\nu\alpha\beta\delta}{}_{\delta}\, h(y)_{\alpha\beta}\, h(z)^{\xi}{}_{\xi} - \frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\nu\mu\alpha\beta\delta}{}_{\delta}\, h(y)_{\alpha\beta}\, h(z)^{\xi}{}_{\xi} +$$

$$\frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\mu\nu\alpha}{}_{\alpha}{}^{\delta}{}_{\delta}\, h(y)^{\beta}{}_{\beta}\, h(z)^{\xi}{}_{\xi} + \frac{1}{64} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\nu\mu\alpha}{}_{\alpha}{}^{\delta}{}_{\delta}\, h(y)^{\beta}{}_{\beta}\, h(z)^{\xi}{}_{\xi} - \frac{1}{32} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\alpha}{}_{\alpha}{}^{\beta\delta\xi\theta}\, h(y)_{\beta\delta}\, h(z)_{\xi\theta}\, \eta^{\mu\nu} +$$

$$\frac{1}{32} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\alpha}{}_{\alpha}{}^{\beta}{}_{\beta}{}^{\xi\theta}\, h(y)^{\delta}{}_{\delta}\, h(z)_{\xi\theta}\, \eta^{\mu\nu} + \frac{1}{32} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\alpha}{}_{\alpha}{}^{\beta\delta\xi}{}_{\xi}\, h(y)_{\beta\delta}\, h(z)^{\theta}{}_{\theta}\, \eta^{\mu\nu} - \frac{1}{32} \langle \Phi(x)\, \Phi(y)\, \Phi(z) \rangle^{\alpha}{}_{\alpha}{}^{\beta}{}_{\beta}{}^{\xi}{}_{\xi}\, h(y)^{\delta}{}_{\delta}\, h(z)^{\theta}{}_{\theta}\, \eta^{\mu\nu};$$

$$\langle T^{\mu\nu}_{(0)} S_{(2)} \rangle = -\frac{3}{32} \langle \Phi(x)\, \Phi(y) \rangle^{\mu\nu\alpha\beta}\, h(y)_{\alpha}{}^{\delta}\, h(y)_{\beta\delta} - \frac{3}{32} \langle \Phi(x)\, \Phi(y) \rangle^{\nu\mu\alpha\beta}\, h(y)_{\alpha}{}^{\delta}\, h(y)_{\beta\delta} + \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\mu\nu\alpha\beta}\, h(y)_{\alpha\beta}\, h(y)^{\delta}{}_{\delta} + \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\nu\mu\alpha\beta}\, h(y)_{\alpha\beta}\, h(y)^{\delta}{}_{\delta} -$$

$$\frac{1}{8} \langle \Phi(x)\, \Phi(y) \rangle^{\alpha}{}_{\alpha}{}^{\beta}{}_{\beta}\, h(y)_{\delta\xi}\, h(y)^{\delta\xi}\, \eta^{\mu\nu} + \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\alpha}{}_{\alpha}{}^{\beta}{}_{\beta}\, h(y)^{\delta}{}_{\delta}\, h(y)^{\xi}{}_{\xi}\, \eta^{\mu\nu} + \frac{1}{32} \langle \Phi(x)\, j^{3}(y) \rangle^{\mu\nu}{}_{\beta\delta\xi}\, h(y)^{\alpha\beta}\, \partial^{\xi} h(y)_{\alpha}{}^{\delta} + \frac{1}{32} \langle \Phi(x)\, j^{3}(y) \rangle^{\nu\mu}{}_{\beta\delta\xi}\, h(y)^{\alpha\beta}\, \partial^{\xi} h(y)_{\alpha}{}^{\delta} -$$

$$\frac{1}{16} \langle \Phi(x)\, j^{3}(y) \rangle^{\alpha}{}_{\alpha\delta\xi\theta}\, h(y)^{\beta\delta}\, \eta^{\mu\nu}\, \partial^{\theta} h(y)_{\beta}{}^{\xi};$$

$$\langle T^{\mu\nu}_{(1)} S_{(1)} \rangle = -\frac{1}{32} \langle \Phi(x)\, \Phi(y) \rangle_{\alpha}{}^{\nu\beta\delta}\, h(x)^{\alpha\mu}\, h(y)_{\beta\delta} - \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\nu}{}_{\alpha}{}^{\beta\delta}\, h(x)^{\alpha\mu}\, h(y)_{\beta\delta} - \frac{1}{32} \langle \Phi(x)\, \Phi(y) \rangle_{\alpha}{}^{\mu\beta\delta}\, h(x)^{\alpha\nu}\, h(y)_{\beta\delta} - \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\mu}{}_{\alpha}{}^{\beta\delta}\, h(x)^{\alpha\nu}\, h(y)_{\beta\delta} +$$

$$\frac{1}{32} \langle \Phi(x)\, \Phi(y) \rangle_{\alpha}{}^{\nu\beta}{}_{\beta}\, h(x)^{\alpha\mu}\, h(y)^{\delta}{}_{\delta} + \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\nu}{}_{\alpha}{}^{\beta}{}_{\beta}\, h(x)^{\alpha\mu}\, h(y)^{\delta}{}_{\delta} + \frac{1}{32} \langle \Phi(x)\, \Phi(y) \rangle_{\alpha}{}^{\mu\beta}{}_{\beta}\, h(x)^{\alpha\nu}\, h(y)^{\delta}{}_{\delta} + \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\mu}{}_{\alpha}{}^{\beta}{}_{\beta}\, h(x)^{\alpha\nu}\, h(y)^{\delta}{}_{\delta} -$$

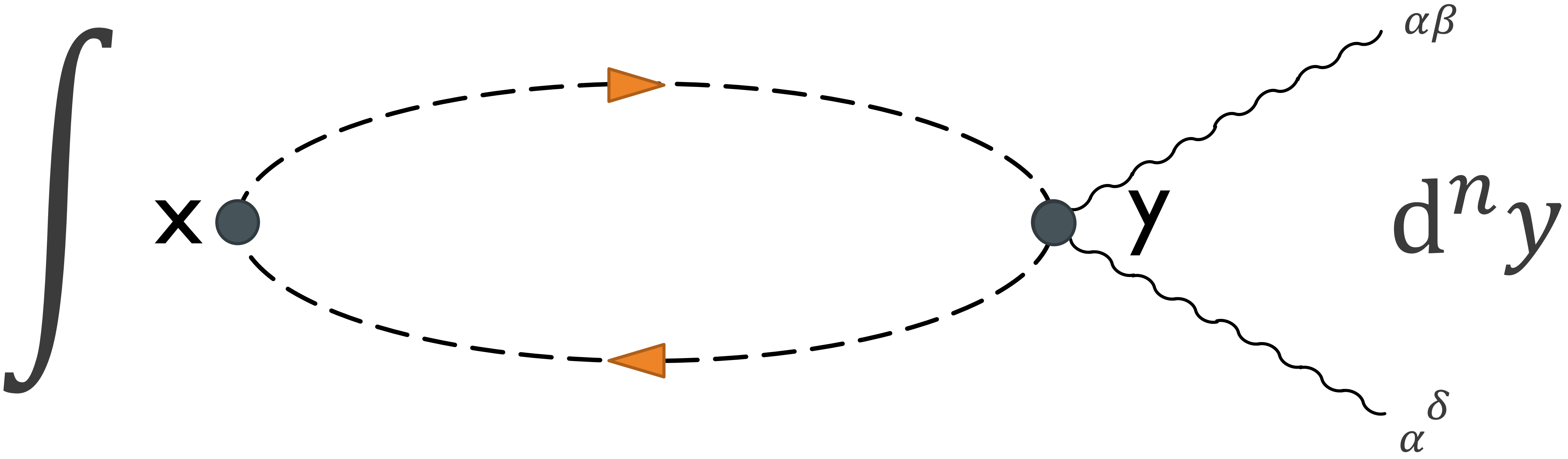
$$\frac{1}{8} \langle \Phi(x)\, \Phi(y) \rangle^{\alpha}{}_{\alpha}{}^{\beta}{}_{\beta}\, h(x)^{\mu\nu}\, h(y)^{\delta}{}_{\delta} + \frac{1}{16} \langle \Phi(x)\, \Phi(y) \rangle^{\alpha\beta\delta\xi}\, h(x)_{\alpha\beta}\, h(y)_{\delta\xi}\, \eta^{\mu\nu} + \frac{1}{32} \langle \Phi(y)\, j^{3}(x) \rangle^{\delta\xi\alpha\beta\nu}\, h(y)_{\delta\xi}\, \partial_{\beta} h(x)^{\mu}{}_{\alpha} - \frac{1}{32} \langle \Phi(y)\, j^{3}(x) \rangle^{\delta}{}_{\delta}{}^{\alpha\beta\nu}\, h(y)^{\xi}{}_{\xi}\, \partial_{\beta} h(x)^{\mu}{}_{\alpha} +$$

$$\frac{1}{32} \langle \Phi(y)\, j^{3}(x) \rangle^{\delta\xi\alpha\beta\mu}\, h(y)_{\delta\xi}\, \partial_{\beta} h(x)^{\nu}{}_{\alpha} - \frac{1}{32} \langle \Phi(y)\, j^{3}(x) \rangle^{\delta}{}_{\delta}{}^{\alpha\beta\mu}\, h(y)^{\xi}{}_{\xi}\, \partial_{\beta} h(x)^{\nu}{}_{\alpha};$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT SECOND ORDER

$$h_\alpha{}^\delta(y)h_{\beta\delta}(y)\left\langle\Psi^{\mu\nu}(x)\Psi^{\alpha\beta}(y)\right\rangle = \lim_{\substack{x'\to x\\ y'\to y}} \partial^\nu_{x'}\partial^\beta_{y'}\gamma^\mu\gamma^\alpha h_\alpha{}^\delta(y)h_{\beta\delta}(y)G(y',x)G(x',y)$$



TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: THREE-POINT FUNCTIONS

- Three-Point Functions:

$$\langle \Psi^{\mu\nu}(x)\Psi^{\alpha\beta}(y)\Psi^{\rho\sigma}(z)\rangle = \mathrm{i}\left(\mathcal{T}^{\mu\lambda\sigma\delta\alpha\tau} + \mathcal{T}^{\mu\tau\alpha\delta\sigma\lambda}\right)\iiint \frac{1}{q^2(q+p)^2(q-k)^2}$$

$$\times\, \mathrm{C}^{\beta\nu\rho}{}_{\delta\lambda\tau}(p,k)e^{\mathrm{i}p(x-y)}e^{\mathrm{i}k(x-z)}\frac{\mathrm{d}^n p}{(2\pi)^n}\frac{\mathrm{d}^n k}{(2\pi)^n}\frac{\mathrm{d}^n q}{(2\pi)^n}$$

where

$$\mathrm{C}^{\beta\nu\rho}{}_{\delta\lambda\tau}(p,k) = \left(p^\beta + 2q^\beta\right)q_\delta(p_\lambda + q_\lambda)(k^\nu - p^\nu - 2q^\nu)(k^\rho - 2q^\rho)(k_\tau - q_\tau)$$

and

$$\mathcal{T}^{\mu\lambda\sigma\delta\alpha\tau} = \mathrm{tr}\left(P_-\gamma^\mu P_+\gamma^\lambda P_-\gamma^\sigma P_+\gamma^\delta P_-\gamma^\alpha P_+\gamma^\tau\right) = \frac{1}{2}\mathrm{tr}\left(\bar{\gamma}^\mu\bar{\gamma}^\lambda\bar{\gamma}^\sigma\bar{\gamma}^\delta\bar{\gamma}^\alpha\bar{\gamma}^\tau\right) - \frac{1}{2}\mathrm{tr}\left(\bar{\gamma}_*\bar{\gamma}^\mu\bar{\gamma}^\lambda\bar{\gamma}^\sigma\bar{\gamma}^\delta\bar{\gamma}^\alpha\bar{\gamma}^\tau\right)$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: THREE-POINT LOOP INTEGRALS

Expanding the momenta in $C^{\beta\nu\rho}{}_{\delta\lambda\tau}(p,k)$ will give us integrals of the form

$$\int p^{\alpha_1} \dots p^{\alpha_r}\, e^{ip(x-y)} \frac{d^n p}{(2\pi)^n} \int k^{\beta_1} \dots k^{\beta_s}\, e^{ik(x-z)} \frac{d^n k}{(2\pi)^n} \int \frac{q^{\mu_1} \dots q^{\mu_t}}{q^2(q+p)^2(q-k)^2} \frac{d^n q}{(2\pi)^n}.$$

So we will need to evaluate three-point loop integrals of the form

$$I^{\mu_1 \dots \mu_t}(p,k) = \int \frac{q^{\mu_1} \dots q^{\mu_t}}{q^2(q+p)^2(q-k)^2} \frac{d^n q}{(2\pi)^n}$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: THREE-POINT LOOP INTEGRALS

- One way to evaluate such integrals is using a recursive method first introduced by Davidychev [17][18] and further developed by Godazgar and Nicolai [19].
- We developed a simpler method using Feynman parameters

$$\frac{1}{A_1 \dots A_k} = (k-1)! \int_0^0 \dots \int_0^1 \frac{\delta(x_1 + \cdots + x_{k-1})}{[x_1 A_1 + \cdots + x_k A_K]^k} \mathrm{d}x_1 \dots \mathrm{d}x_k$$

with which we express the integrals as

$$I^{\mu_1 \dots \mu_k}(p,k) = 2 \int_0^1 \int_0^{1-y} \int \frac{(q+yp+xk)^{\mu_1} \dots (q+yp+xk)^{\mu_k}}{[q^2 + y(1-y)p^2 + x(1-x)k^2 - 2xy(pk)]^3} \frac{\mathrm{d}^n q}{(2\pi)^n} \mathrm{d}x\mathrm{d}y$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: THREE-POINT LOOP INTEGRALS

$$
\begin{aligned}
I^{\mu\nu\rho\sigma\alpha\beta}(p,k) \;=\; & \frac{1}{192}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\eta^{\rho\sigma}\eta^{\alpha\beta)}\Big[3(k^2)^2-6k^2(pk)+4(pk)^2+5k^2p^2-6(pk)p^2+3(p^2)^2\Big]\Big[\mathcal{D}+\frac{3}{2}\Big] \\
& -\frac{1}{16}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\eta^{\rho\sigma}\Big[p^\alpha p^{\beta)}[3p^2+2k^2-3(pk)]+k^\alpha k^{\beta)}[2p^2+3k^2-3(pk)]+p^\alpha k^{\beta)}[3p^2+3k^2- \\
& 4(p\kappa)]\Big][\mathcal{D}+1] \\
& +\frac{1}{4}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}(p^\alpha p^\beta p^\rho p^{\sigma)}+p^\alpha p^\beta p^\rho k^{\sigma)}+p^\alpha p^\beta k^\rho k^{\sigma)}+p^\alpha k^\beta k^\rho k^{\sigma)}+k^\alpha k^\beta k^\rho k^{\sigma)})\mathcal{D} \\
& -\frac{15}{16}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\eta^{\rho\sigma}\eta^{\alpha\beta)}\Big[(p^2)^2\big(G_{04}(p,k)+G_{02}(p,k)-2G_{03}(p,k)\big)+(k^2)^2\big(G_{40}(p,k)+G_{20}(p, \\
& k)-2G_{30}(p,k)\big)+4p^2(pk)\big(G_{13}(p,k)-G_{12}(p,k)\big)+4k^2(pk)\big(G_{31}(p,k)-G_{21}(p,k)\big)+ \\
& 2p^2k^2\big(G_{11}(p,k)-G_{12}(p,k)+G_{22}(p,k)-G_{21}(p,k)\big)\Big] \\
& +\frac{45}{2}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\eta^{\rho\sigma}p^\alpha k^{\beta)}\Big[p^2(G_{12}(p,k)-G_{13}(p,k))+k^2(G_{21}(p,k)-G_{31}(p,k))-2(pk)\,G_{22}(p,k)\Big] \\
& +\frac{45}{4}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\eta^{\rho\sigma}k^\alpha k^{\beta)}\Big[p^2(G_{21}(p,k)-G_{22}(p,k))+k^2(G_{30}(p,k)-G_{40}(p,k))-2(pk)\,G_{31}(p,k)\Big] \\
& +\frac{45}{4}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\eta^{\rho\sigma}p^\alpha p^{\beta)}\Big[p^2(G_{03}(p,k)-G_{04}(p,k))+k^2(G_{12}(p,k)-G_{22}(p,k))-2(pk)G_{13}(p,k)\Big] \\
& -\frac{15}{2}\frac{i}{(4\pi)^2}\eta^{(\mu\nu}\Big(p^\rho p^\sigma p^\alpha p^{\beta)}G_{04}(p,k)+4p^\rho p^\sigma p^\alpha k^{\beta)}G_{13}(p,k)+6p^\rho p^\sigma k^\alpha k^{\beta)}G_{22}(p,k)+ \\
& 4p^\rho k^\sigma k^\alpha k^{\beta)}G_{31}(p,k)+k^\rho k^\sigma k^\alpha k^{\beta)}G_{40}(p,k)\Big) \\
& +\frac{i}{(4\pi)^2}\Big[p^\mu p^\nu p^\rho p^\sigma p^\alpha p^\beta F_{06}(p,k)+6p^{(\mu}p^\nu p^\rho p^\sigma p^\alpha k^{\beta)}F_{15}(p,k)+15p^{(\mu}p^\nu p^\rho p^\sigma k^\alpha k^{\beta)}F_{24}(p, \\
& k)+20p^{(\mu}p^\nu p^\rho k^\sigma k^\alpha k^{\beta)}F_{33}(p,k)+15p^{(\mu}p^\nu k^\rho k^\sigma k^\alpha k^{\beta)}F_{42}(p,k)+6p^{(\mu}k^\nu k^\rho k^\sigma k^\alpha k^{\beta)}F_{51}(p, \\
& k)+k^\mu k^\nu k^\rho k^\sigma k^\alpha k^\beta F_{60}(p,k)\Big]
\end{aligned}
\tag{6.24}
$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: THREE-POINT LOOP INTEGRALS

With

$$F_{ab}(p,k) = \int_0^1 \int_0^{y-1} \frac{x^a y^b}{y(1-y)p^2 + x(1-x)k^2 - 2xy(pk)} \mathrm{d}x\mathrm{d}y$$

$$G_{ab}(p,k) = \int_0^1 \int_0^{y-1} x^a y^b \ln\left[\frac{y(1-y)p^2 + x(1-x)k^2 - 2xy(pk)}{\mu^2}\right] \mathrm{d}x\mathrm{d}y$$

and

$$\mathcal{D} = -\frac{2}{(n-4)} + \ln(4\pi) - \gamma - \ln(\mu^2)$$

TRACE ANOMALY OF CHIRAL FERMIONS

CALCULATION: ANOMALY AT SECOND ORDER

Recall:

$$\mathcal{A}^{(2)} = \eta_{\mu\nu}\left(-\frac{1}{2}\left\langle T^{\mu\nu(0)} S^{(1)^2}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(0)} S^{(2)}\right\rangle + \mathrm{i}\left\langle T^{\mu\nu(1)} S^{(1)}\right\rangle\right) + h_{\mu\nu}\mathrm{i}\left\langle T^{\mu\nu(0)} S^{(1)}\right\rangle.$$

Plugging everything in, we find after a very long computation:

$$\mathcal{A}_{ren}^{(2)}(x) = \frac{1}{720(4\pi)^2}\left(-11\mathcal{E}_4 + 18C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + 12\nabla^2 R\right),$$

which is exactly half the trace anomaly for a Dirac spinor. 🙂

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